Wake decay constant for the infinite wind turbine array
Application of asymptotic speed deficit concept to existing engineering wake model

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WAKE DECAY CONSTANT FOR THE INFINITE WIND TURBINE ARRAY

Application of asymptotic speed deficit concept to existing engineering wake model.

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Summary

A wake decay constant is devised for wind farm models based on local momentum balance so that these models – deep inside the wind farm – will result in the same efficiency as for an infinite wind farm, when the effect of the turbines on the wind field is represented by an equivalent surface roughness. The equivalent roughness model reflects the interaction with the atmosphere and is considered more reliable for very large wind farms. The paper describes a work attempting to combine this model with an existing simple engineering wind farm wake model in a wind-resource software in order to get more realistic results for very large wind farms. This is accomplished by adjusting the expansion rule of the wake model whenever a wake overlaps with a turbine rotor, thereby approaching a speed deficit as that predicted for an infinite wind farm by the boundary layer method. Results from the adjusted wind farm wake model are compared to data from existing off-shore wind farms.

1 Background

For very large wind farms standard wake models seems to underpredict wake effects. Recent investigations by Sten Frandsen [1, 2] indicated strongly that the reason is the lack of accounting for the effect a large wind farm may have on the atmospheric boundary layer, e.g. by modifying the vertical wind profile.

In some way the effect of an extended “infinite” wind farm resembles that of a change in surface roughness, i.e. a change to an increased, equivalent roughness length which represents the additional shear stress in the wind flow caused by the wind farm turbines.

The idea of the work described in this paper is the following: While more detailed and more physics-based models are underway [3], it would be valuable to modify the existing engineering “Park” wind farm wake model, implemented in the WAsP wind resource software [4] to take this boundary-layer effect into account in an approximate way.

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2 Asymptotic speed deficit from boundary layer considerations

For an infinite wind farm the reduced wind speed at hub height can be determined by the modified boundary layer [1,2] as illustrated in figure 1:
A jump in friction velocity takes place at hub-height due to rotor thrust $t$: $\rho (u^*_{\text{eff}})^2 = \rho (u^*)^2 + t$. The approximation is made that the thrust is homogeneously distributed, i.e. described by a distributed thrust coefficient $c_t$ defined as:

$$c_t = \frac{\pi}{8} \frac{C_t}{\left( s_f / s_s \right)}$$

Here $s_f$ and $s_s$ are dimensionless turbine-turbine-distances (along- and across-wind), made dimensionless by $D_{\text{rotor}}$.

The wind profile is then assumed as follows:

- For $Z<h$: according to ground surface friction velocity $u^*_{\text{f}}$ / roughness $z_0$.
- For $Z>h$: according to increased friction velocity $u^*_{\text{eff}}(=u^*_{0})$ / roughness $z^*_{0\text{eff}}(=z^*_{0\text{f}})$.

This can be condensed into an equivalent, effective surface roughness:

$$z^*_{0\text{eff}} = h^* \cdot \exp \left(-\kappa / \sqrt{c_t + \left(\frac{\kappa}{\ln(h^* / z^*_{0\text{f}})}\right)^2}\right)$$

An approximate geostrophic drag-law is used: $G \approx \frac{u^*}{\kappa} \left( \ln \left( \frac{G}{f z^*_{0\text{f}}} \right) - A_{\text{f}} \right)$

Here $f$ is the latitude-dependent Coriolis parameter and $A_{\text{f}}$ is a modified geostrophic drag-law constant with a value of about 4.

Then the wind at hub-height can be expressed in the general form:

$$U(h^*) = \frac{G}{1 + \left( \ln \frac{G}{h^* \cdot f - A_{\text{f}}} \right) i}$$

where $i$ can be viewed as a kind of turbulent intensity taking the following values:
Free flow: \( i_0 = \frac{1}{\ln \frac{h}{z_0}} \); Over wind farm:

\[
i_{\text{Tot}} = \sqrt{i_0^2 + i_{\text{add}}^2}, \quad i_{\text{add}} = \frac{\sqrt{c_i}}{\kappa}
\]

This results in a relative speed deficit \( \varepsilon \):

\[
1 - \varepsilon = \frac{1 + \ln \left( \frac{G}{h \cdot f^t} \right) i_0}{1 + \ln \left( \frac{G}{h \cdot f^t} \right) i_{\text{Tot}}}
\]

2.1 Comparison with the Horns Rev off-shore wind farm.

As seen from figure 2, for the Horns Rev off-shore wind farm the actual wake deficit is about 50% of the boundary-layer-limited value. Hence Horns Rev wind farm is NOT “infinite” in the sense of the Sten Frandsen model.

Figure 2. Reduced speed ratio for an infinite wind farm Horns Rev. 

\( s_r = s_f \approx 7, h=80m, D_R = 60 m. \)

This also becomes apparent from the actual size compared to that required for heavy cross-wind wake interaction to become important, as well as from the actual power density compared to the power density set by the Frandsen boundary-layer limitation (see table 1).

<table>
<thead>
<tr>
<th>Horns Rev</th>
<th>Power density (W/m²) [5]</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Distance for severe wake interference</strong> (( k_{\text{wake}}=0.075 ))</td>
<td><strong>Actual extension</strong></td>
</tr>
<tr>
<td><strong>Distance for severe wake interference</strong> (( k_{\text{wake}}=0.075 ))</td>
<td><strong>Actual extension</strong></td>
</tr>
<tr>
<td>7.5 km</td>
<td>5 km</td>
</tr>
</tbody>
</table>

*Table 1.*

Wake Decay Constant for the Infinite Wind Turbine Array
3 WAsP Park Model Details.

The evolution of the individual wakes in the present wind farm "Park" wake model in WAsP [4] is illustrated in figure 3. It should be noted that the wake interaction with ground- or sea surface is represented by a reflected wake.

![Figure 3. Impact of direct wake (left) and reflected wake (right) on a downwind turbine.](image)

The speed deficit for a single wake (direct or reflected) as well as the resulting speed deficit at a downwind turbine are described by the following equations [6,7]. It should be noted that the combined effect of several wakes on a turbine rotor is described by an empirical quadratic summing rule:

\[
\delta V_{01}^{(\text{type})} = U_0 \left(1 - \sqrt{1 - C_i}\right) \left(\frac{D_0}{D_0 + 2kX_{01}}\right)^2 \frac{A_{\text{(type)overlap}}}{A_i^{(R)}}, \quad \text{(type) = "dir.", "ref."
}
\]

\[
\delta V_{\text{turb}}^2 = \sum_{i \in \text{upwind cables}} \left( (\delta V_{i,turb}^{(dir.)})^2 + (\delta V_{i,turb}^{(ref.)})^2 \right)
\]

4 Asymptotic speed deficit of the WAsP Park Model

For an infinite, regularly spaced wind farm, the reduced speed deficit, and thus also the turbine thrusts, is the same for all turbines. From the preceding section the asymptotic speed deficit for a certain turbine may thus be expressed as

\[
(\delta V)^2 = \left( U_{\text{upwind}} \varepsilon_0 \right)^2 \sum_{j=1}^{\infty} N(s_j) \varepsilon_u(x_j)^2; \quad \varepsilon_u(x) = \left(\frac{D_R}{D_R + 2kx}\right)^2; \quad \varepsilon_0 = \left(1 - \sqrt{1 - C_i}\right)
\]

Here \(N(s_j)\) indicates the number of turbines in upwind row \(j\) at upwind distance \(x_j\) that throw a wake which overlaps the rotor area of the turbine considered. Fortunately, the infinite sum has a definite value (is convergent), and it may be approximated by an infinite integral. This results in the following expression for the asymptotic speed deficit for the Park model, where the infinite integral is contained in a park-structure function \(G_{\text{Park}}\), which is easily calculable:

\[
\frac{\delta V}{U_{\text{upwind}} \varepsilon_0} = G_{\text{Park}}(k; s_r, s_f, h / D_R, C_i)
\]

The dependence of \(G_{\text{Park}}\) on the wake expansion coefficient \(k\) is illustrated in figure 4.

Wake Decay Constant for the Infinite Wind Turbine Array
Finally, since the upwind wind speed (=incident wind on the rotor) is in fact the reduced wind speed the following speed deficit relative to the free wind speed is obtained:

$$\frac{\delta V}{U_{free}} = \varepsilon_w = \frac{\varepsilon_w^{app}}{1 + \varepsilon_w^{app}}, \quad \varepsilon_w^{app} = \varepsilon_0 G_{park}^t(k, layout; C_t)$$

5 Adjustment of the WAsP Park Model

An attempt is made to adjust the WAsP Park-model to match the boundary-layer based asymptotic wind speed deficit for an infinite wind farm. For "deep" positions the wake expansion coefficient $k$ of the Park Model is modified to approach the $k$-value, $k_{inf}$, that corresponds to the asymptotic speed deficit of the boundary-layer model, defined implicitly by the following equation:

$$\delta V_{infin, park}(k_{inf}; [s_r, s_f, h, C_t]) = \delta V_{BL-based}(s_r, s_f, h, C_t)$$

Every time a wake overlaps with a downwind turbine rotor the wake expansion coefficient $k$ is made to relax towards $k_{inf}$ according to a relaxation expression, where the relaxation depends on the overlapping fraction and a relaxation constant $F_{relax}$ common to all turbines:

$$k_{adj}^{adj} = k_j + (k_{inf} - k_j^{adj}) \frac{A_{overlap}}{A_w} F_{relax}$$

The process of gradual adjustment of the wake expansion coefficient is illustrated in figure 5.

The following model parameters were used in the following (based on data from the Horns Rev wind farm):

$k_{initial} = 0.075$ (recommended value for onshore!)

$F_{relax} = 0.2$
6 Comparative wind farm predictions

Results from the adjusted WAsP Park model were compared to available data from two Danish off-shore wind farms: Horns Rev and Nysted.

6.1 Comparison with Horns Rev data.

The layout of the Horns Rev wind farm is shown in figure 6.

Figure 6. Horns Rev layout.
Wind directions and turbine lines used in the following are indicated.

Comparisons with data for wind speeds 8.5 m/s and 12 m/s at directions 270° and 222° are shown in figures 6 and 7.
Figure 6. Comparisons with data for wind direction 270°+/− 3° and speeds 8.5 m/s +/- 0.5 m/s (left); and 12.0 m/s +/- 0.5m/s (right).

Figure 7. Comparisons with data for wind direction 222°+/− 3° and speeds 8.5 m/s +/- 0.5 m/s (top); and 12.0 m/s +/- 0.5m/s (bottom).

6.2 Comparison with Nysted data.

The layout of the Nysted wind farm is shown in figure 8.
Figure 8. Nysted layout.
Wind directions and turbine lines used in the following are indicated.

Comparisons with relative turbine power data for a wind of about 10 m/s at directions 278° and 263° are shown in figure 9.

Figure 9. Comparisons with data for
wind direction 278° +/- 2.5° and speed 10.0 m/s +/- 0.5 m/s (upper part); and
wind direction 263° +/- 2.5° and speed 10.2 m/s +/- 0.5 m/s (lower part) are shown.
7 Conclusion.

- The adjustment of the wake expansion coefficient towards a value matching the BL-limited asymptotic speed deficit seems a valuable engineering approach;
- A value for the initial wake expansion coefficient close to that normally used for onshore – locations seems reasonable in this approach also for off-shore wind farms;
- The model – the relaxation factor - needs to be fine-tuned in order to avoid systematic overestimations.
- The model needs to be tested on situations with wake effects between neighboring wind farms.

8 References:


