Influence of the turbulent boundary layer pressure fluctuation on the sound intensity measurement in a mean flow

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Influence of the turbulent boundary layer pressure fluctuation on the sound intensity measurement in a mean flow

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Abstract: The influence of turbulent boundary layer pressure fluctuation on the sound intensity measurement in a flow is a subject of practical concern, because the sound intensity probe is generally exposed to the airflow and is sensed the turbulent boundary layer (TBL) pressure fluctuation which may even overwhelm the true source pressure in some cases. In this paper, the model of the sound intensity caused by the TBL pressure fluctuation is described firstly. Based upon the developed model, the sound intensity caused by the TBL pressure fluctuation is calculated using the available models of the wave-vector frequency spectra of the TBL pressure fluctuation. In order to validate the model and the numerical results, a series of measurements were carried out. It is shown that the calculated results of the TBL pressure fluctuation agree fairly well with the measured results which are corrected with the estimated spatial response function of the microphone. Also, the characteristics of the measured sound intensity are consistent with that of the calculated sound intensity.

Key words: sound; sound intensity; turbulent boundary layer; pressure fluctuation

1 INTRODUCTION

Sound intensity techniques have been widely applied in the relevant areas as a routine means, particularly in the noise control engineering. However, the sound intensity in a flow still deserves more studies. Moreover, this subject is of practical concern, since the sound intensity probe is generally exposed to the airflow during the measurements that take place in situ, such as the measurement on a moving vehicle, the measurement near an aircooled machinery and the measurement in heating, ventilation or air conditioning ducts or near the intakes or outlets of such ducts, and outdoor measurements, etc.

Some developments have been made on the subject of sound intensity measurement in the flow. Munro and Ingard\textsuperscript{[1]} had proved that the sound intensity measurement techniques using cross correlation between two closely spaced microphones can not be extended to general three dimensional situations in which there is a mean flow. However, it may be possible to measure sound intensity in a duct with mean flow using this technique, at frequencies below their cutoff frequency. A simplification with the expression derived by Munro and Ingard had been made by Jacobsen\textsuperscript{[2]}, and had been investigated numerically and experimentally. He concluded that the effect of flow on sound intensity measurement should be taken into account in the estimation of sound intensity, especially in the sound field with substantial reactive components. Chung and Blaster\textsuperscript{[3]} had proposed another technique for sound intensity measurement in flow ducts, which was based on the transfer function between
two adjacent microphones or transducers and was independent of the finite-difference approximation. Lauchle\textsuperscript{[4]} had analyzed the bias of usual sound intensity measurement made with two-sensor technique in a low Mach number flow. The effect of flow on the propagation of the sound source was not taken into account. In our previous paper\textsuperscript{[5]}, a model of sound intensity caused by the turbulent boundary layer (TBL) was developed. And the influence of some factors on the sound intensity measurement in a flow was investigated numerically.

The purpose of this work is to numerically investigate the sound intensity caused by the TBL pressure fluctuation based upon the developed model, using the available models of the wave-vector frequency spectra of the TBL pressure fluctuation, and to validate the developed model by some experiments. In this paper, the developed model of the sound intensity caused by the TBL pressure fluctuation is given firstly. Based upon the above model, the sound intensity caused by the TBL pressure fluctuation is calculated using the available models of wave-vector frequency spectra of the TBL pressure fluctuation. In order to validate the developed model and numerical results, some experimental measurements were made and the experimental results are presented. It is shown the experimental results of the TBL pressure fluctuation agree fairly well with the calculated results after the experimental results are corrected with the estimated spatial response function of the microphone. Also, the characteristics of the measured sound intensity are consistent with that of the calculated sound intensity.

2 MODELING OF THE SOUND INTENSITY IN A MEAN FLOW\textsuperscript{[5]}

A planar flow model is considered which is sketched in Fig. 1. The mean flow velocity is denoted by $u_0$ and it is assumed that $u_0/c_0 = M_0 < 0.1$, where $c_0$ is sound velocity in the flow medium, $M_0$ is Mach number.

![Fig.1 Sketch of the model for the sound intensity measurement in a duct with mean flow.](image)

Now the case of a uniform steady background flow is considered. That is, $\rho_0$, $c_0$ and $u_0$ are assumed to be constant in both space and time. The sound intensity in this case is as follows\textsuperscript{[5]}
\[
I = \frac{(1 - M_0^2)(1 + 3M_0^2)}{2\rho_o\omega} \text{Im}\left( \frac{\partial \bar{p}}{\partial x} \right) + \frac{M_0(M_0^2 + 1)}{\rho_o c_o} |\bar{p}|^2 + \frac{M_0 c_o (M_0^2 - 1)^2}{2\rho_o \omega^2} \frac{\partial \bar{p}}{\partial x}^2
\]

where the superscript denotes complex amplitude of variables, and * represents complex conjugation.

Based upon the two microphone method of sound intensity measurement, and taking into account the TBL pressure fluctuation, the sound intensity in the flow becomes

\[
I(\omega) = \frac{(1 - M_0^2)(1 + 3M_0^2)}{\rho_o \omega \Delta} \cdot \text{Im}(G_{i12} + G_{F,F_i}) + \frac{M_0(M_0^2 + 1)}{2\rho_o c_o} \cdot \left\{ G_{i1} + G_{22} + G_{F,F_i} + G_{F,F_2} + 2\text{Re}(G_{i12} + G_{F,F_2}) \right\} + \frac{M_0 c_o (M_0^2 - 1)^2}{\rho_o \omega^2 \Delta^2} \cdot \left\{ G_{i1} + G_{22} + G_{F,F_i} + G_{F,F_2} - 2\text{Re}(G_{i12} + G_{F,F_2}) \right\}
\]  

(2)

where \( G_{i1}, \ G_{22} \) are the auto spectra of the sound pressure at location 1 and 2 respectively, \( G_{i12} \) is the cross spectrum of the sound pressure at location 1 and 2, \( G_{F,F_i} \) and \( G_{F,F_2} \) are the auto spectra of the TBL pressure fluctuation at location 1 and 2 respectively, \( G_{F,F_2} \) is the cross spectrum of the TBL pressure fluctuation at location 1 and 2, \( \Delta \) is the distance between two microphones.

From the Eq. (2), the sound intensity caused by the pressure fluctuation beneath a TBL is given by

\[
I_\rho(\omega) = \frac{(1 - M_0^2)(1 + 3M_0^2)}{\rho_o \omega \Delta} \cdot \text{Im}(G_{F,F_2}) + \frac{M_0(M_0^2 + 1)}{2\rho_o c_o} \cdot \left\{ G_{F,F_i} + G_{F,F_2} + 2\text{Re}(G_{F,F_2}) \right\} + \frac{M_0 c_o (M_0^2 - 1)^2}{\rho_o \omega^2 \Delta^2} \cdot \left\{ G_{F,F_i} + G_{F,F_2} - 2\text{Re}(G_{F,F_2}) \right\}
\]

(3)

Based upon the Corcos model\[^7\], the cross spectrum of the TBL pressure fluctuation is given by

\[
G_{F,F_2}(\omega) = P_\rho(\omega) \exp(-\alpha_i k_\Delta) \exp(-ik_\Delta)
\]

(4)

where \( P_\rho(\omega) \) is the point power spectrum of the TBL pressure fluctuation. \( \alpha_i \) is a constant related to the convective characteristic of the TBL pressure fluctuation.

Substituting Eq. (4) into the Eq. (3) and using \( P_\rho(\omega) \) instead of \( G_{F,F_i} \) and \( G_{F,F_2} \), yields

\[
I_\rho(\omega) = -\frac{(1 - M_0^2)(1 + 3M_0^2)}{\rho_o \omega \Delta} \cdot P_\rho(\omega) \cdot \exp(-\alpha_i k_\Delta) \sin(k_\Delta) + \frac{M_0(M_0^2 + 1)}{\rho_o c_o} \cdot P_\rho(\omega) (1 + \exp(-\alpha_i k_\Delta) \cos(k_\Delta)) + \frac{2M_0 c_o (M_0^2 - 1)^2}{\rho_o \omega^2 \Delta^2} \cdot P_\rho(\omega) (1 - \exp(-\alpha_i k_\Delta) \cos(k_\Delta))
\]

(5)
3 CALCULATION OF THE SOUND INTENSITY CAUSED BY THE TBL PRESSURE FLUCTUATION

In order to calculate the sound intensity caused by the TBL pressure fluctuation, the point wall pressure spectrum should be calculated at first. In this paper, the point wall pressure spectra are calculated in the same way as that used by Capone and Lauchle\textsuperscript{[6]}.

In this paper, the Corcos\textsuperscript{[7]} and Chase\textsuperscript{[8-9]} models are chosen for the investigation of sound intensity caused the TBL pressure fluctuation, because much published work refers to Corcos model, and the Witting model\textsuperscript{[10]} yields results similar to those of the Corcos model, also the Chase models predict considerably different behavior in the low wave-number region using fewer empirical constants than the Ffowcs Williams model\textsuperscript{[11]}.

Figure 2 presents the calculated TBL point wall pressure spectra at different Mach numbers in air using Chase model in 1987. Note that the reference pressure is $20\mu Pa/\sqrt{Hz}$ and the baseline data used in the calculation are same as that used by Capone and Lauchle\textsuperscript{[6]}. It can be seen that the TBL wall pressure level increases with increasing Mach numbers and the level increased at low frequencies is less than that at high frequencies. Moreover, the frequency of the peak appeared increases with increasing Mach numbers.

![Figure 2](image)

**Fig.2** Calculated turbulent boundary layer point wall pressure spectra at different Mach numbers (Chase model 1987)

Figure 3 shows the calculated TBL point wall pressure spectra at different Mach numbers using the Corcos model. It can be seen that the pressure level increases dramatically as the Mach number increases, and the difference between two lines is a constant in the whole frequency range. It is due to that the Corcos model is proportional to $u_t^4$, while the friction velocity is proportional to the Mach number.
Figure 3 presents the calculated TBL point wall pressure spectra at different Mach numbers (Corcos model).

Figure 4 presents the calculated TBL point wall pressure spectra using two different Chase models in 1980 and 1987. It can be seen that the pressure level upon Chase 1987 model is a little higher than that upon Chase 1980 model, especially at low frequencies. The difference of pressure level between two models is diminished as the frequency increases.

Figure 5 shows the calculated sound intensity caused by the TBL wall pressure fluctuation modeled by Chase in 1980. Note that the reference intensity is $1pW/m^2/Hz$ in the calculation and the flow medium is air in this figure. Also the distance between two microphones is 50 mm. It can be seen that the sound intensity increases apparently as Mach number increases. And the spectra of the sound intensity fluctuate apparently at low frequencies. The frequencies of peaks and valleys are varied with different Mach numbers. However, the difference between valley frequencies is not a constant. Moreover, the increased level of the sound intensity at low Mach numbers, say from 0.01 to 0.02, is greater than that at high Mach numbers, say from 0.02 to 0.03. Besides, the sound intensity level falls with the increasing frequencies except at the valley frequencies.
Figure 6 and figure 7 show the calculated sound intensity caused by the TBL wall pressure fluctuation modeled by Chase in 1987 for the compressible flow and modeled by Corcos, respectively. The characteristics are similar to that of Fig. 5. However, the valley frequencies of the sound intensity caused by the TBL pressure fluctuation modeled by Corcos and by Chase are quite different.

Figure 8 shows the calculated sound intensity caused by the TBL pressure fluctuation for different distances between two microphones. The TBL pressure fluctuation was modeled by Chase in 1980. Note that the Mach number is 0.02 in this case. It is shown that the sound intensity falls as the distance increases. Also, the valley frequencies are quite different for the different distances between two microphones. Therefore, the valley frequencies are related to the distance between two microphones.
4 MEASUREMENT OF THE SOUND INTENSITY IN A DUCT WITH MEAN FLOW

4.1 Experimental set-up

In order to validate the developed model and numerical results above, an experimental set-up was built. The set-up consists of a centrifugal fan, a silence, a test section and auxiliary ducts etc. The air flow is driven by the centrifugal fan. The mean flow velocity in the test section is measured by a velocimeter. The duct section is square with the size of 100×100 mm². The controllable mean flow velocity ranges from 0 to 14 m/s. There are two well phase-matched condenser microphones (type of B&K 4135) flushed mounted on the duct wall of the test section. The outside diameter of the microphones is 1/8 inch. The distance between two microphones is changeable upon the requirement. The distances of 15 mm, 30 mm and 45 mm are used in the experiment. The B&K PULSE system is used to analyze the auto-power spectra, cross-power spectra and coherence function etc. Then the sound intensity is calculated upon the model described above.

4.2 Experimental results and discussion

Fig. 9 shows the auto-power spectral density of the TBL pressure fluctuation at different Mach numbers. As predicted in the numerical calculation, the pressure fluctuation level decreases as the frequency increases. And it increases obviously as the Mach number increases. Moreover, the increasing rate of the sound intensity at lower Mach numbers, say from 0.01 to 0.015, is greater than that at higher Mach numbers, say from 0.025 to 0.03.

![Fig. 9 Auto-power spectral density of the TBL pressure fluctuation at different Mach numbers.](image)

Fig. 10 presents the measured and calculated auto-power spectral density of the TBL pressure fluctuation. It can be seen that there is great discrepancy between them, except at low frequencies. The discrepancy could be caused by the spatial averaging effect of the microphone used. Based upon the geometrical parameters, the spatial response function of the microphone is calculated and shown in Fig. 11. It should be noted that the calculated frequency spectrum of the TBL pressure fluctuation is based upon the model of the
wave-vector frequency spectra of the TBL pressure fluctuation in a compressible flow, which was developed by Chase (1987)\(^9\).

Fig. 10  Measured and calculated auto-power spectral density of the TBL pressure fluctuation (Mach number Ma=0.01)

Fig. 11  Calculated spatial response function of the microphone.

The measured auto-power spectral density of the TBL pressure fluctuation was corrected with the spatial response function of the microphone used. Fig. 12 presents the corrected auto-power spectral density of the TBL pressure fluctuation (solid curve). It can be seen that the agreement between calculated and measured auto-power spectral density of the TBL pressure fluctuation is fairly well.

Fig. 12 Auto-power spectral density of the TBL pressure fluctuation (Mach number Ma=0.01).

Fig. 13 presents the measured sound intensity caused by the TBL pressure fluctuation at different Mach numbers. It can be seen that the sound intensity level increases as Mach number increases and it falls with increasing frequency. In addition, the increasing rate of the sound intensity at low Mach numbers is greater than that at high Mach numbers.
Figure 13 shows the sound intensity spectra measured with two differently spaced microphones at Mach number of 0.025. As can be seen in the figure, the measured sound intensity decreases as the distance between two microphones increases. However, the difference between sound intensity measured at 30 mm and at 45 mm is small.

Fig. 14 Measured sound intensity caused by the TBL pressure fluctuation (Ma=0.025).

Fig. 15 presents the measured and calculated sound intensity caused by the TBL pressure fluctuation at Mach number of 0.015. Note that the distance between two microphones is 30 mm and the pressure fluctuation measured was not corrected with the spatial response function of the microphone. In addition, the Chase 1987 model of the TBL pressure fluctuation was used in the calculation. Due to the background noise of the set-up is high at low frequencies, the sound intensity measured at low frequencies are higher than that calculated. Also, the measured sound intensity at high frequencies are much lower than that calculated, because the pressure fluctuation measured was not corrected with the spatial response of the microphone, i.e. the true pressure fluctuation was attenuated due to the spatial averaging effect of the microphone. It can be predicted that the agreement between calculated
and measured sound intensity could be well after the spatial response of the sensor is corrected.

5 CONCLUSION

This work has attempted to investigate the influence of TBL pressure fluctuation on the sound intensity measurement in a flow. Actually this subject is of practical concern, because the sound intensity probe is often exposed to airflow and is sensed the TBL pressure fluctuation which may even overwhelm the true source pressure in some cases.

In this paper, the model of the sound intensity caused by the TBL pressure fluctuation is presented firstly. Based upon the model, the sound intensity caused by the TBL pressure fluctuation is calculated using the available models of the wave-vector frequency spectra of the TBL pressure fluctuation. In order to validate the developed model and the numerical results, an experimental set-up was built and a series of measurements was carried out. In the course of this investigation, some conclusions have been reached.

It is shown from the calculated and measured results, the TBL pressure fluctuation increases apparently as the Mach number increases. The characteristic of the calculated results is almost the same as the measured results. After corrected with the spatial response function of the microphone, the agreement between calculated and measured power spectra of the TBL pressure fluctuation is fairly well.

The sound intensity caused by the TBL pressure fluctuation increases obviously with the increasing Mach number. As the distance between two closely spaced microphones increases, the sound intensity induced by the TBL pressure fluctuation decreases apparently. In addition, the spectra of the sound intensity fluctuates greatly at some frequencies which are related to the convective wave-number and the distance between two microphones.

The measured spectra of the sound intensity caused by the TBL pressure fluctuation agree well with the calculated results at low frequencies. The discrepancy occurred at high frequencies could be resulted from the spatial averaging effect of the microphones.

References

湍流边界层脉动压力对声强测量的影响研究

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摘要：实际声强测量时常常存在风流或水流，如在飞机或船舶上进行测量，声强探头将受到湍流边界层脉动压力的影响。如何评估该影响以及如何修正测量结果是人们十分关心的工程实用问题。本文首先简要介绍了建立的由湍流边界层脉动压力诱发声强的理论模型。接着利用现有的湍流边界层脉动压力频率-波数谱模型，对湍流边界层脉动压力及其诱发的声强进行了数值分析。为了验证理论模型及数值结果，设计制作了一套实验装置，对湍流边界层脉动压力及其产生的声强进行了具体测量分析。结果表明，湍流边界层脉动压力的测量结果与数值结果吻合良好，测量得到的边界层脉动压力诱发的声强特性与计算结果也十分一致，但必须注意对测量传感器的空间响应进行修正。

关键词：声学；声强；湍流边界层；脉动压力

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