Oxygen Nonstoichiometry and Defect Chemistry Modeling of Ce0.8Pr0.2O2-delta

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The oxygen nonstoichiometry ($\delta$) of Ce$_{0.8}$Pr$_{0.2}$O$_2$ has been measured as a function of $P_{O_2}$ at temperatures between 600 and 900°C by coulometric titration and thermogravimetry. An ideal solution defect model, a regular solution model, and a defect association model, taking into account the association of reduced dopant species and oxygen vacancies, were unable to reproduce the experimental results. However, excellent agreement with the experimentally determined oxygen nonstoichiometry could be achieved when using either a nonideal solution model with an excess enthalpic term linear in $\delta$ ($\Delta H^{\text{exc}}_\text{P} = a\delta$) and a completely random distribution of defects (referred to as “$\delta$-linear”), or a “generalized $\delta$-linear” solution model, where the excess Gibbs energy change in the reduction reaction of the dopant linearly varies with $\delta$ ($\Delta G^{\text{exc}}_\text{P} = a\delta$). A comparison of the partial molar enthalpy and entropy of oxidation, estimated from the defect models with those determined directly from the oxygen nonstoichiometry, suggests that the $\delta$-linear solution model is the most appropriate in accounting for the observed nonideal reduction behavior of Pr.

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Ceria-based mixed oxides have been extensively studied as their properties can be tailored by appropriate doping to match the requirements of a variety of applications, such as electrolytes and electrodes for solid oxide fuel cells,1 three-way catalysts,2 oxygen sensors,3 and oxygen permeation membranes.4

The oxygen nonstoichiometry ($\delta$) of pure and doped ceria increases with decreasing oxygen partial pressure and increasing temperature, accompanied by the partial reduction of Ce$^{4+}$ to Ce$^{3+}$. The partial molar enthalpy for oxygen incorporation ($\delta H^\text{P} = \delta H^\text{O} - \delta H^\text{Ce}$) of pure ceria strongly depends on $\delta$.5,6 The absolute value of $\delta H^\text{P} = 0$ rapidly decreases up to $\delta = 0.1$ and goes through a minimum at $\delta = 0.15$.5,6 The $\delta$ dependence of $\delta H^\text{P} = 0$ was less pronounced for $\delta$ between 0.1 and 0.25.5,6 The absolute value of $\delta H^\text{P} = 0$ in ceria doped with 10 atom % Gd also rapidly decreases up to a total oxygen vacancy concentration of $\sim 0.12$.7 For Ce$_{0.9}$Gd$_{0.1}$O$_2$, where the total oxygen vacancy concentration at oxidizing conditions is already 0.1, no clear $\delta$ dependence of $\delta H^\text{P} = 0$ could be observed.8 Despite the strong $\delta$ dependence of $\delta H^\text{P} = 0$, the absolute value of $\delta H^\text{P} = 0$ is only slightly affected by the degree of doping, when the values are compared at the same total oxygen vacancy concentration (or at the same concentration of trivalent cations).9

Examples of nonideal behavior with $\delta$-dependent $\delta H^\text{P} = 0$ have been reported for other oxide systems as well. A regular solution behavior (which for small $\delta$ corresponds to a linear dependence of $\delta H^\text{P} = 0$ on $\delta$) has been observed for the reduction of the perovskite compounds La$_{1-x}$Sr$_x$CoO$_{3-\delta}$.10 and La$_{1-x}$Sr$_x$CoO$_{3-\delta}$.11 In both cases, the partial molar enthalpy of oxidation linearly decreases with increasing oxygen nonstoichiometry. A generalization of the regular solution model has been proposed, allowing for a $\delta$ dependence of the excess free energy ($\Delta G^{\text{exc}}$) because an excess entropic term that varies with oxygen nonstoichiometry may also exist.12 Furthermore, for La$_{1-x}$Ca$_x$CrO$_{3-\delta}$, a linear relationship should be expected between the excess Gibbs energy change ($\Delta G^{\text{exc}}$) and $\delta$ based on the estimation of pair potential changes due to the defect-induced lattice expansion.12 A significant deviation from ideality has also been observed for the redox energetics of SrFeO$_{3-\delta}$.13 For this material, a regular solution model was unable to account for the deviation from ideality, which, however, could be successfully interpreted by a nonideal model with an excess term of a purely configurational entropic origin.13

When doping CeO$_2$ with Pr, a change in the oxygen nonstoichiometry with varying oxygen activity is expected in the high $P_{O_2}$ range, corresponding to the reduction of Pr, as well as at low $P_{O_2}$ values, corresponding to the reduction of Ce. Some discrepancies exist in the literature on the solubility limit of Pr in CeO$_2$. A solubility limit of 30 atom %, as determined by X-ray diffraction (XRD), was reported by Shuk and Greenblatt14 and Ftilos et al.15 In contradiction to this, single-phase fluorite samples with Pr doping up to 55 atom %16 or even 70 atom % were reported by Knauth and Tuller17 and Rajendran et al.18 The oxygen nonstoichiometry of 20 atom % Pr-doped ceria has been investigated in the high $P_{O_2}$ regime ($10^{-5}$–$10^{-3}$ atm) in the temperature ranges of 800–950°C (for microcrystalline powder)19 and 600–750°C (for nanocrystalline powder)20 by coulometric titration (CT). In both cases, a simple defect model treating the reduction of Pr$^{4+}$ to Pr$^{3+}$ in an ideal manner (i.e., not allowing $\delta H^\text{P} = 0$ to vary with the degree of nonstoichiometry) accounted well for the determined nonstoichiometry data.

Here, the oxygen nonstoichiometry of Ce$_{0.8}$Pr$_{0.2}$O$_2$ is reported (for microcrystalline powder, $D_{\text{particle}} = 1$ to 5 $\mu$m), as determined by CT and thermogravimetry (TG), over a wide $P_{O_2}$ range, covering both the reduction of Pr and Ce at temperatures between 600 and 900°C. Five defect models have been considered to analyze the experimental data.

1. Ideal model: ideal reduction of both Ce and Pr (i.e., constant standard enthalpy, $\Delta H^\text{P}$, and entropy, $\Delta S^\text{P}$, of the reduction reaction and random distribution of defects).

2. Association model: ideal reduction of Ce and Pr, but defect interaction among reduced Pr and oxygen vacancies is assumed, leading to the formation of singly charged defect associates.

3. Regular model: ideal reduction of Ce and regular solution behavior15 for Pr (i.e., $\Delta H^\text{P} = \Delta H^\text{P} = 0 = x_{\text{Ce}^{4+}} + x_{\text{Pr}^{4+}}$ for $x_{\text{Ce}^{3+}}$ and $x_{\text{Pr}^{3+}}$). $\Delta S^\text{P} = \Omega$, and randomly distributed defects; $\Omega$ represents the magnitude of the deviation from the ideal solution).

4. Nonideal ($\delta$-linear) model: ideal reduction of Ce and linear $\delta$ dependence of the excess enthalpic term for the reduction of Pr [i.e., $\Delta H^\text{P} = \Delta H^\text{P} = 0 + a\delta$, $\Delta S^\text{P} = \Omega$, and randomly distributed defects; $a\delta$ represents the rate of change in the partial molar enthalpy for the reduction of Pr with oxygen nonstoichiometry, $(d\Delta H^\text{P})/(d\delta)$].

5. Generalized $\delta$-linear model: ideal reduction of Ce and linear $\delta$ dependence of the excess free energy for the reduction of Pr (i.e.,
\[ \Delta G^\circ_p = \Delta G^\circ_o + \alpha_\text{G}_T \text{R} \; T \text{ln} \, \alpha_T \] (2)

α describes the deviation from the standard Gibbs energy change for the reduction of Pr.

The ability of each defect model to reproduce the experimental \( \delta(\log P_{O_2}) \) curves is examined, and the thermodynamic parameters estimated from the different defect models are compared to values directly determined from the oxygen nonstoichiometry data.

**Experimental**

Sample preparation and phase analysis.— Ce₀.₈Pr₀.₂O₂−₅ powder was synthesized by coprecipitation. Aqueous solutions of Ce(NO₃)₃.6H₂O (Alfa Aesar, 99.99%) and Pr(NO₃)₃.10H₂O (Alfa Aesar, 99.99%) were prepared. The solution concentrations were measured both gravimetrically and by titration. Appropriate amounts of each solution were mixed with ~400 mL distilled/filtered water in a glass container at 200°C. Oxalic acid (1 M) was added while stirring the solution to achieve coprecipitation of the metal oxalates. The solution was dried overnight at 300°C, and the resulting powder was calcined at 600°C. The powder was shaped into disks at a uniaxial pressure of 3 MPa and further isostatically compressed in an evacuated latex container suspended in water at a pressure of 325 MPa. The disks were sintered at 1500°C in air for 12 h and slowly cooled to room temperature. XRD was performed by room-temperature XRD using a Bruker D8 Advance diffractometer with Cu Kα radiation and a LynxEye position sensitive detector. The Ce₀.₈Pr₀.₂O₂−₅ powder was a single fluorite phase after sintering at 1500°C in air. A grain size in the range of 1−5 μm was determined by scanning electron microscopy (Zeiss Supra 35).

CT and TG.— Sintered disks were crushed into powders, examined for phase purity by XRD, and used for CT and TG. A schematic of the experimental setup used for CT is shown in Fig. 1. A small alumina cup with a known mass of sample powder (~2 g) was fitted in a Zr₀.₈Y₀.₁₅O₂ (YSZ) cup covered with a YSZ lid and sealed with glass (MgO/sodium aluminosilicate glass composite, 30/70 vol %) at 1000°C. The contact surfaces between the YSZ cup and the lid were polished using SiC paper and 3 μm diamond paste, and external uniaxial pressure was applied to the system to ensure a good seal. Four electrodes (two at the inner surface and two at the outer surface of the YSZ cup) served as working electrodes for electrochemical pumping of oxygen and reference electrodes for the determination of \( P_{O_2} \) inside the cup. Pt paste was used for the reference electrodes, whereas a thin (~10 μm) sprayed layer of YSZ/La₀.₈Sr₀.₂MnO₃ (LSM) cermet covered with Pt paste was used for the working electrodes. Integration of the current passing through the working electrodes over time yielded, by application of Faraday’s law, the total amount of oxygen removed from the sample powder.

The leak rate of oxygen through the glass seal and/or cup porosity, \( \langle d(\Delta\delta)/dt \rangle_{\text{leak}} \), was determined in three ways.

A. The \( \langle d(\Delta\delta)/dt \rangle_{\text{leak}} \) of an empty titration cell (no sample) was measured at various \( P_{O_2} \) values and temperatures over an extended period of time. The pressure was transformed to the number of moles of \( O_2 \) using the ideal gas law, \( \langle d(\Delta\delta)/dt \rangle_{\text{leak}} \), and finally expressed as \( \langle d(\Delta\delta)/dt \rangle_{\text{leak}} \). Assuming that the change in oxygen content originates from a change in \( \delta \) of an equal amount of sample as in the filled cup, the leak rate estimated in this way was of the order of \( 10^{-15}, 10^{-14}, \text{and } 10^{-9} \text{ s}^{-1} \) at 600, 700, and 800°C, respectively, indicative of a very good seal.

B. The difference in pumped charge over an isothermal CT cycle was determined, and a mean leak rate was obtained from

\[
\langle d(\Delta\delta)/dt \rangle_{\text{leak}} = \frac{Q_{\text{out}} - Q_{\text{in}}}{2 \pi t_{\text{cycle}}}
\]

where \( Q_{\text{out}} \) is the total charge needed to be pumped to reach the lowest \( P_{O_2} \) value of the titration cycle, \( Q_{\text{in}} \) is the total charge that has to be pumped to return to the starting point in \( P_{O_2} \), and \( t \) is the duration of the titration cycle. The advantage of this method is that the estimated leak rate is for the specific titration cell. Leak rates determined by this method were of the order of \( 10^{-10}–10^{-8} \text{ s}^{-1} \) between 600 and 900°C.

C. The \( \langle d(\Delta\delta)/dt \rangle_{\text{limited}} \) of the filled titration cell was measured at steady state, and the leak rates were estimated from

\[
\langle d(\Delta\delta)/dt \rangle_{\text{leak}} = \frac{\langle dP_{O_2}/dt \rangle_{\text{limited}} \langle d\Delta\delta/dP_{O_2} \rangle_{\text{limited}}}{\langle dP_{O_2}/dt \rangle_{\text{limited}}}
\]

This approach allows us to estimate the variation in the leak rate over time. The estimation of the leak rate with this method at 800°C at \( P_{O_2} \sim 5 \times 10^{-18} \text{ bar} \) is shown in Fig. 2. After equilibration, the rate of change in \( P_{O_2} \) in the titration cell is approximately constant and equal to \( 2.1 \times 10^{-23} \text{ bar/s} \). By differentiating the \( \Delta(\Delta\delta(P_{O_2})) \) plot, we find that the slope \( \langle d(\Delta\delta)/dP_{O_2} \rangle_{\text{limited}} \) at 800°C at \( P_{O_2} \sim 5 \times 10^{-18} \text{ bar} \) is \( 6 \times 10^{14} \text{ bar}^{-1} \). Therefore, the leak rate for this specific titration cell at 800°C at \( P_{O_2} \sim 5 \times 10^{-18} \text{ bar} \) is estimated to be \( 1.3 \times 10^{-8} \text{ s}^{-1} \). The leak rates determined in this manner were practically identical to the mean leak rates obtained by method B without a significant variation over time.

During the three estimates of the leak rates, it can be concluded that the relative error introduced in the measurement of \( \delta \) by gas leaks, \( \langle d(\Delta\delta)/dt \rangle_{\text{leak}} \), is of the order of \( 10^{-8} \text{ s}^{-1} \), and thus the titration cells could be allowed to equilibrate over extended periods of time without introduction of a significant error. The measured values of \( \delta \) have been corrected for the determined leakages, and the magnitude...
of the correction is taken as an uncertainty estimate (the experimental uncertainty of $\delta$, as indicated in the plots, is twice the magnitude of this correction). The variation in the oxygen nonstoichiometry of LSM and of the oxygen content of the gas present in the dead volume of the cup are estimated to contribute by less than 0.1% of the measured change in $\delta$ in the $P_{O_2}$ range related to the reduction of Pr and have therefore been neglected. However, in the $P_{O_2}$ range related to the reduction of Ce, decomposition of the inner LSM working electrode may be expected. This (assuming full decomposition of the LSM) gives rise to an error contribution of less than 10% of the change in oxygen nonstoichiometry $\Delta \delta$. This potential error has been taken into account in the depicted uncertainty estimates along with the error due to leakages.

TG was performed using both a Netzsch STA 409CD thermogravimeter and a Netzsch TG 439 thermobalance (TB). The $P_{O_2}$ of the sample atmosphere was varied by appropriate gas mixtures at constant temperature and measured with the help of a YSZ $P_{O_2}$ sensor connected downstream. An equilibration time of 2 h was allowed after each gas change. This duration is expected to be more than adequate because the time constant for the relaxation, which may be estimated from the relation $\tau \sim L^3/D_{\text{chem}}$, is $\tau \sim 10^{-3}$ s using a value of $D_{\text{chem}} \sim 1 \times 10^{-6}$ cm$^2$/s for the chemical diffusion coefficient$^{24,25}$ and $L \sim 1 \mu$m for the grain size. The weight loss, measured with the thermogravimeter, was corrected for buoyancy effects using an alumina powder reference sample of the same volume, measured under identical conditions.

Results and Discussion

The oxygen nonstoichiometry of Ce$_{0.8}$Pr$_{0.2}$O$_{2-\delta}$, measured by CT, TB, and TG as a function of $P_{O_2}$ at 600, 700, 800, and 900 °C, is shown in Fig. 3a-d, respectively. Literature values are shown for comparison.$^{5,20}$ In Ref. 19, the change in oxygen nonstoichiometry $\Delta \delta$ is reported as a function of oxygen partial pressure with respect to air. To compare with our data, the $\Delta \delta(P_{O_2})$ values of Ref. 19 have been transformed to $\delta(P_{O_2})$ by shifting them by a constant to match with the $\delta$ value of the present study at the highest $P_{O_2}$ reported in Ref. 19. Good agreement is observed between the oxygen nonstoichiometry data determined in the present study and those of Ref. 19 at 800 and 900 °C, the difference being within the scattering observed for the present data determined by CT, TB, and TG. The oxygen nonstoichiometry reported for Ce$_{0.8}$Pr$_{0.2}$O$_{2-\delta}$ at 700 and at 600 °C in the $P_{O_2}$ range $10^{-1}$–$10^{-3}$ bar is somewhat higher than the one determined in the current study. The discrepancy at 700 °C might be due to the errors in the determination of the absolute value of $\delta$. At 600 °C though, the reported $P_{O_2}$ dependence of $\delta$ shows a different behavior from our data, where oxidation appears sooner with increasing $P_{O_2}$. This difference is most likely because nanosized powder was used in Ref. 20, which facilitates the loss of oxygen.$^{26,27}$

Five defect models were considered in the analysis of the measured oxygen nonstoichiometry at varying $P_{O_2}$:

1. Ideal model: The reduction of Pr and Ce is treated in an ideal manner ($\Delta H^o$ and $\Delta S^o$ independent of $\delta$). The defect formation reactions and equilibrium constants read

$$\text{O}_2 + 2\text{Pr}^{4+} \rightarrow \frac{1}{2}\text{O}_2(g) + 2\text{Pr}^{3+}$$

$$K_{\text{Pr}} = \exp\left(-\frac{\Delta H^o_{\text{Pr}} - T\Delta S^o_{\text{Pr}}}{RT}\right)$$

$$\Delta H^o_{\text{Pr}} \Delta S^o_{\text{Pr}} = \text{const}$$

$$[\text{O}_2] + [\text{Pr}^{3+}] = [\text{O}^-] + [\text{Pr}^{4+}]$$

$$K_{\text{Ce}} = \exp\left(-\frac{\Delta H^o_{\text{Ce}} - T\Delta S^o_{\text{Ce}}}{RT}\right)$$

$$\Delta H^o_{\text{Ce}} \Delta S^o_{\text{Ce}} = \text{const}$$

The mass, site, and charge conservation conditions for this case are

$$[\text{Pr}^{4+}] + [\text{Pr}^{3+}] = 0.2$$

$$[\text{Ce}^{5+}] + [\text{Ce}^{4+}] + [\text{Pr}^{3+}] + [\text{Pr}^{4+}] = 1$$

$$[\text{O}^-] + [\text{V}^{0}] = 2$$

$$2[V^{0}] = [\text{Pr}^{4+}] + [\text{Ce}^{5+}]$$

$$\Delta G^o_{\text{Ce}} = \Delta H^o_{\text{Pr}} - T\Delta S^o_{\text{Pr}}$$

$$\Delta G^o_{\text{Pr}} = \Delta H^o_{\text{Ce}} - T\Delta S^o_{\text{Ce}}$$

$$\Delta G^o_{\text{Ce}} = \Delta H^o_{\text{Pr}} - T\Delta S^o_{\text{Pr}}$$

The mass, site, and charge conservation conditions for this case are

$$\text{Pr}^{4+} + \text{V}^{0} \equiv (\text{Pr}^{4+}\text{V}^{0})^+$$

$$K_{\text{assoc}} = \exp\left(-\frac{\Delta H^o_{\text{assoc}} - T\Delta S^o_{\text{assoc}}}{RT}\right)$$

$$\Delta H^o_{\text{assoc}} \Delta S^o_{\text{assoc}} = \text{const}$$

The mass, site, and charge conservation conditions for this case are

$$[\text{Pr}^{4+}] + [\text{Pr}^{3+}] + [\text{Pr}^{4+}\text{V}^{0}] = 0.2$$

$$[\text{O}^-] + [\text{V}^{0}] + [\text{Pr}^{4+}\text{V}^{0}] = 2$$

$$[\text{Pr}^{3+}] + [\text{Pr}^{4+}] + [\text{Pr}^{4+}\text{V}^{0}] = 1$$

$$[\text{Pr}^{4+}] + [\text{Pr}^{3+}] + [\text{Pr}^{4+}\text{V}^{0}] = 1$$

$$[\text{O}^-] + [\text{V}^{0}] + [\text{Pr}^{4+}\text{V}^{0}] = 2$$
Three parameters, $\Delta G^0_{PrO_2} \Delta S^0_{PrO_2}$, and $\Delta G^0_{\text{assoc}}$, are necessary in this case for the fitting of the experimental data at each temperature.

3. Regular solution model: This model includes the reduction reactions for Pr and Ce, Eqs. 3 and 4, with an excess enthalpic term contributing to the reduction of Pr

$$\Delta H^0_{Pr} = \Delta H^0_{\text{Pr}_0 + O} + \Delta H^\text{exc}_{Pr} = \Delta H^0_{\text{Pr}_0 + O} + 106(1 - 108)\Omega$$

where the parameter $\Omega$ represents the magnitude of the deviation from the ideal solution and accounts for all the interatomic interactions collectively. Ce$_{0.8}$Pr$_{0.2}$O$_{2-\delta}$ is considered to be the solid solution of CeO$_2$Pr$_2$O$_3$ and Ce$_{0.9}$Pr$_{0.1}$O$_{1.9}$ with $x_{\text{Ce}_0.8\text{Pr}_{0.2}O_2} = 1-106$ and $x_{\text{Ce}_{0.9}\text{Pr}_{0.1}O_{1.9}} = 106$. The equilibrium constant for the reduction of Pr becomes

$$K^\text{Pr}_0 = \exp \left( \frac{-\Delta H^0_{\text{Pr}_0 + O} + \Delta H^\text{exc}_{Pr} - T\Delta S^0_{Pr}}{RT} \right)$$

Five variables ($\Delta H^0_{\text{Pr}_0 + O}$, $\Omega$, $\Delta S^0_{Pr}$, $\Delta H^\text{exc}_{Ce}$, and $\Delta S^0_{Ce}$) were used for the fitting of the oxygen nonstoichiometry data with the regular solution model at all temperatures. The standard enthalpies and entropies used, as well as the parameter $\Omega$, are assumed to be temperature-independent.

4. Nonideal solution model: This model treats the reduction of Pr in a nonideal manner. It includes the reduction reactions for Pr and Ce, Eqs. 3 and 4, with an excess enthalpic term, linear with respect to $\delta$, contributing to the reduction of Pr

$$\Delta H^0_{Pr} = \Delta H^0_{\text{Pr}_0 + O} + \Delta H^\text{exc}_{Pr} = \Delta H^0_{\text{Pr}_0 + O} + a_{\text{Pr}}\delta$$

where the parameter $a_{\text{Pr}}$ represents the rate of change in the partial molar enthalpy for the reduction of Pr with oxygen nonstoichiometry ($\Delta H^0_{\text{Pr}_0 + O}$)/$\delta$. Five variables ($\Delta H^0_{\text{Pr}_0 + O}$, $a_{\text{Pr}}$, $\Delta S^0_{Pr}$, $\Delta H^\text{exc}_{Ce}$, and $\Delta S^0_{Ce}$) were used for the fitting of the oxygen nonstoichiometry data with the $\delta$-linear solution model at all temperatures. The standard enthalpies and entropies used, as well as the parameter $a_{\text{Pr}}$, are assumed to be temperature-independent.

5. Generalized $\delta$-linear solution model: The reduction of Pr is treated in a nonideal manner by the addition of an excess term to the standard Gibbs energy change

$$\Delta G^0_{Pr} = \Delta G^0_{\text{Pr}_0 + O} + \Delta G^\text{exc}_{Pr} = \Delta G^0_{\text{Pr}_0 + O} + a_{\text{Pr}}\delta$$

where the parameter $a_{\text{Pr}}$ is an indicator of the deviation from the standard Gibbs energy change for the reduction of Pr. This term allows for a linear $\delta$ dependency of both the standard enthalpy and entropy change in the reduction of Pr. Three fitting parameters, $\Delta G^0_{\text{Pr}_0 + O}$, $a_{\text{Pr}}$, and $\Delta G^0_{\text{Ce}}$, were used in this case at each temperature.

To clarify the behavior of each model, a schematic illustration of the $\delta$ dependence of $\Delta G^0_{Pr}$ for the ideal, regular, $\delta$-linear, and generalized $\delta$-linear solution models at two different temperatures, 600 and 1000°C, is shown in Fig. 4. In the ideal solution model, $\Delta G^0_{Pr}$ is constant with respect to $\delta$ at any temperature, whereas in the regular, $\delta$-linear, and generalized $\delta$-linear solution models, $\Delta G^0_{Pr}$ varies with $\delta$. In the regular solution model, $\Delta G^0_{Pr}$ has a temperature-independent, second-order-degree dependency on $\delta$ according to $\Delta G^0_{Pr} = \Delta H^0_{\text{Pr}_0 + O} - T\Delta S^0_{Pr} + 106(1 - 108)\Omega$. Its value is identical to that of the ideal model for $\delta = 0$ and $\delta = 0.1$. In the $\delta$-linear solution model, the slope ($d(\Delta G^0_{Pr})/d\delta$) has the same value at all temperatures, as the excess term is of purely enthalpic origin, whereas in the generalized $\delta$-linear solution model the slope ($d(\Delta G^0_{Pr})/d\delta$) can vary with temperature, as it comprises an enthalpic and an entropic contribution.
either has been neglected (δ-linear solution model) or has not been taken explicitly into account (generalized δ-linear solution model), could also play a role.

At temperatures above 750°C, the oxygen nonstoichiometry is well described by an ideal model consistent with the behavior reported in Ref. 19. Another study performed at lower temperatures, 600–750°C, at the \( P_{O_2} \) range between \( 10^{-5} \) and \( 10^{-1} \) atm suggested ideal behavior. However, the examined powder was nanosized, which significantly alters the reduction properties.\(^{26,27}\) The reduction of Ce was modeled in an ideal manner, i.e., \( \Delta \mu_C^{\text{Ce}} = \text{const} \), in all cases. Nevertheless, the \( \delta(\log P_{O_2}) \) behavior estimated from the various defect models, in the oxygen nonstoichiometry regime corresponding to the reduction of Ce \( (\delta > 0.1) \), overall, agrees well with the experimentally determined values.\(^{26,27}\) The measured \( \delta(\log P_{O_2}) \) values in this regime indicate a larger \( \delta P_{O_2} \) slope than that predicted from the ideal solution model for the reduction of Ce in all cases, especially at temperatures below 750°C. The limited oxygen nonstoichiometry range covered at low temperatures in this regime does not allow conclusion on whether the reduction of Ce takes place in a nonideal manner. The extended oxygen nonstoichiometry range covered for \( \delta > 0.1 \) at high temperatures indicates ideal behavior. However, this observation is not adequate to conclude ideal behavior because it could arise from a temperature dependence of the nonideality parameters. The reduction behavior of \( \text{Ce}_{0.8}\text{Pr}_{0.2}\text{O}_{2−δ} \) for \( \delta > 0.1 \) bears a close similarity to that of \( \text{CeO}_{2−δ} \) and \( \text{Ce}_{0.8}\text{Gd}_{0.2}\text{O}_{1.9−δ} \) at a total vacancy concentration above 0.1, namely, a small or almost negligible \( \delta \) dependence of the partial molar enthalpy for oxidation.

The change in the Gibbs energy of formation of \( \text{Ce}_{0.8}\text{Pr}_{0.2}\text{O}_{2−δ} \) relative to \( \text{Ce}_{0.8}\text{Pr}_{0.2}\text{O}_{1.9} \) can be estimated from

\[
\Delta G_f = \int_{δ_{1}}^{δ_{2}} [\mu(δ) - \mu(0)] dδ = \frac{RT}{2} \int_{0.1}^{0.5} \ln P_{O_2}(δ) dδ \quad [17]
\]

\( R \) being the universal gas constant. A comparison between the change in the Gibbs energy of formation directly obtained from the
Figure 6. Comparison of the change in the Gibbs energy of formation $\Delta G_f$ obtained directly from the nonstoichiometry data with those estimated from the various defect models at 600°C.

Figure 7. (a) Partial molar enthalpy $h_o - h_o^o$ and (b) entropy $s_o - s_o^o$ for oxygen incorporation as a function of $\delta$ for $\text{Ce}_0.8\text{Pr}_0.2\text{O}_2$.

\[
\begin{align*}
\Delta h_{\delta}(\delta) - h_o^o &= \frac{R}{2} \ln P_{O_2} \left. \frac{d \ln P_{O_2}}{d(1/T)} \right|_{\delta} \\
\Delta s_{\delta}(\delta) - s_o^o &= -\frac{R}{2} d(T \ln P_{O_2}) \left. \frac{d(1/T)}{d(1/T)} \right|_{\delta}
\end{align*}
\]

Values of $P_{O_2}$, corresponding to the same amount of $\delta$ at different temperatures, were obtained by a linear interpolation of the experimental data. The partial molar quantities were only estimated for values of $\delta$, where $P_{O_2}$ values for at least four temperatures could be used, as this is expected to increase the accuracy of the determined partial molar quantities from Eq. 18 and 19.

Values for $h_o - h_o^o$ and $s_o - s_o^o$ may also be estimated from the fitting parameters and defect concentrations obtained from the various defect models using expressions derived from a statistical thermodynamic analysis (see the Appendix).

\[
\begin{align*}
h_{\delta}(\delta) - h_o^o &= -\Delta H_{\text{Ce}}^{\text{exc}} + \frac{\partial \left[ \Delta H_{\text{Ce}}^{\text{exc}} \right]}{\partial \delta} \left( \frac{\Delta H_{\text{Ce}}^{\text{exc}} - \Delta H_{\text{Pr}}^{\text{exc}}}{2} \right)
\end{align*}
\]
The generalized δ-linear solution model yields larger values for the slopes \([d(\log P_{\text{O}_2})]/d\delta\) and \([d(\log P_{\text{O}_2})]/d\delta\) than the values directly determined from the nonstoichiometry data for \(\delta < 0.1\), whereas the δ dependency of \(h_0 - h_0^0\) and \(s_0 - s_0^0\) estimated from the δ-linear solution model agrees very well with the one directly determined from the oxygen nonstoichiometry data over the entire δ range examined. The observed nonideality in the reduction of Pr should therefore be attributed to a change in the enthalpy for oxygen incorporation with varying oxygen nonstoichiometry.

According to the definition of the ideal, regular, and δ-linear solution models, the value of \(h_0 - h_0^0\) should be identical at \(\delta = 0\) in all three cases. Also, the δ dependence of \(s_0 - s_0^0\) should be identical because there is no excess isotropic term present in any of these models, and a random distribution of defects is assumed in all cases. The estimated δ dependence of \(s_0 - s_0^0\) for the ideal and δ-linear solution models in Fig. 7a and b. This is because \(\Delta H_{\text{P},\text{O}_2}\) and \(\Delta S_{\text{P},\text{O}_2}\) were not constrained to have the same values in all defect models but instead were allowed to vary freely in each defect model. Because each defect model has a different number and type of fitting parameters, the best fit in each case is not expected to result in the same \(\Delta H_{\text{P},\text{O}_2}\) and \(\Delta S_{\text{P},\text{O}_2}\) values.

The negative sign of \(h_0 - h_0^0\) indicates that the incorporation of oxygen is an exothermic process. The value of \(h_0 - h_0^0\) for \(\delta < 0.1\) is related to the oxidation of Pr and varies between −103 and −93 kJ/mol in the nonstoichiometry range between 0.053 and 0.08. A value of \(-73 \pm 6\) kJ/mol has been reported15 for the partial molar enthalpy for the oxidation of Pr in \(\text{CeO}_2\) and \(\text{Ce}_0.8\text{Pr}_{0.2}\text{O}_2\) from the nonstoichiometry data obtained in the temperature range of 800–950°C after fitting with an ideal defect model. The value of \(h_0 - h_0^0\) increases with increasing δ, indicating a more facilitated expulsion of oxygen with increasing fractional concentration of trivalent Pr. A similar behavior has been observed for the reduction of Ce in \(\text{CeO}_2\) and \(\text{Ce}_0.8\text{Pr}_{0.2}\text{O}_2\) at a similar total oxygen vacancy concentration range. This is probably related to the decrease in the strength of the metal–oxygen bond with decreasing valency and increasing ionic radii of the cations. This possibility is discussed in more detail later on. A sharp decrease in the value of \(h_0 - h_0^0\) is observed in the nonstoichiometry range between 0.08 and 0.09. This is attributed to increased uncertainty in this region, arising from increased experimental errors, errors in the shifting of the \(\Delta H_{\text{P},\text{O}_2}\) curves to obtain absolute values of δ, as well as increased errors entering the analysis due to the sparser data points in this region.

In the oxygen nonstoichiometry range \(\delta > 0.1\), where the incorporation of oxygen is related to the oxidation of Ce, the values of \(h_0 - h_0^0\) and \(s_0 - s_0^0\) estimated from the various models are slightly underestimated relative to the values obtained directly from the oxygen nonstoichiometry data. A mean value of \(h_0 - h_0^0\) of \(-340 \pm 7\) kJ/mol is obtained from the experimental data in the δ range between 0.103 and 0.168, whereas values between \(-353\) and \(-359\) kJ/mol are estimated from various defect models. This indicates that a nonideal solution model might be more appropriate for the modeling of the reduction of Ce as well, but the limited amount of data in this region does not allow for a definite conclusion. The reduction of Ce was modeled in an ideal manner, i.e., \(\Delta H_{\text{P},\text{O}_2} = \Delta S_{\text{P},\text{O}_2} = 0\) const, in all cases. The small differences observed in the values of \(h_0 - h_0^0\) and \(s_0 - s_0^0\) estimated from the various models in this region are a result of the influence of the way the reduction of Pr was treated and of uncertainties arising from the fitting.

The values of the nonideality parameters, \(a_\delta\) and \(a_{\delta,\text{O}_2}\), obtained from the fit of the δ-linear and generalized δ-linear solution models to the \(\delta(\log P_{\text{O}_2})\) curves at temperatures between 600 and 850°C, are shown in Fig. 8. A constant value of \(-354 \pm 1\) kJ/mol was determined for \(a_\delta\) from the fitting of the \(\delta(\log P_{\text{O}_2})\) curves at each temperature show considerable scattering without indicating a certain temperature dependence. The mean value of \(a_\delta\) in this temperature range is \(-350 \pm 70\) kJ/mol, agreeing with the value determined for \(a_{\delta,\text{O}_2}\) from the δ-linear solution model within experimental uncertainty. The temperature dependency of \(a_\delta\) indicates the lack of an excess isotropic term for the nonideal reduction behavior of Pr

\[ U_{\text{i}} = A_{\text{i}} \exp \left(-\frac{r_{\text{i}}}{k T} - \frac{C_{\text{i}}}{r_{\text{i}}}\right) \]  

where \(A_{\text{i}}, r_{\text{i}}, C_{\text{i}}\) are constants characteristic of the pair \(i, j\) and \(r_{\text{i}}\) is the distance between ions \(i, j\), \(A_{\text{i}}, r_{\text{i}}, C_{\text{i}}\) values refined for ceria were used, and the interatomic distances \(r_{\text{i}}\) were determined by Rietveld refinement of the room-temperature XRD pattern of \(\text{Ce}_0.8\text{Pr}_{0.2}\text{O}_2\) and values of the deconvoluted thermal and chemical expansion coefficients,13 \(a_{\text{mol}}(30–600°C) = 11.3 \times 10^{-6} \text{ K}^{-1}\), \(a_{\text{mol}}(30–800°C) = 12.6 \times 10^{-6} \text{ K}^{-1}\), and \(A = 0.084 \text{ mol}^{-1}\), assuming isotropic expansion of all interatomic distances. The excess enthalpy may then be estimated from

\[ \Delta H_{\text{P},\text{O}_2}^{\text{ex}} = \sum_{i,j} \Delta U_{\text{i}} = \sum_{i,j} \left[U_{\text{i}}(r_{\text{i}}) + \Delta r_{\text{i}} - U_{\text{i}}(r_{\text{i}})\right] \]
nonideality parameter. This calculation is in support of the δ-linear
solution model that was able to describe the experimentally deter-
mined nonstoichiometry data, while at the same time it suggests a
decrease in the mean metal–oxygen bond strength to be the reason
for the observed nonideal behavior of the reduction of Pr in
Ce0.8Pr0.2O2−δ.

Conclusion

The oxygen nonstoichiometry of Ce0.8Pr0.2O2−δ was measured by
CT and TG over a wide P02 range, covering both the reduction of Pr
and Ce at temperatures between 600 and 900°C. Five defect models
were applied to the nonstoichiometry data. The reduction of Pr could
not be modeled in an ideal manner (constant ΔHf0 and ΔSf0) even
with an additional defect reaction taking into account the association
of reduced Pr species and oxygen vacancies. A regular solution
model was also unable to explain the P02 dependence of the oxygen
nonstoichiometry. A nonideal solution model with a linear δ depen-
dence of ΔHf0 (δ-linear) and a nonideal solution model with a linear δ
dependence of ΔGf0 (generalized δ-linear) for the reduction of Pr
could accurately reproduce the δ(log P02) curves in the entire non-
stoichiometry range examined and at all temperatures, at the same
time yielding a δ dependence of ΔG0, which agrees very well with the
the one directly determined from the nonstoichiometry data. The
partial molar enthalpy and entropy of oxidation estimated from the
ideal model were significantly underestimated relative to the values
obtained directly from the nonstoichiometry data, whereas the
δ-linear solution model yielded a very good agreement. This indica-
tes that the observed nonideal behavior of the reduction of Pr is
due to a δ-dependent h′0 – h′0 giving rise to a facilitated loss of
oxygen with increasing fractional concentration of trivalent Pr. This
is tentatively attributed to a decrease in the strength of the metal–
oxygen bond with decreasing oxidation state and increasing ionic
radii of Pr.

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Appendix

The standard Gibbs free energy change in Reactions 3 and 4 may be expressed as
a function of the standard chemical potential of the quasi-chemical species involved as

$$
\Delta G_f^0 = 2\mu_{Pr}^{ox} + \mu_{O}^{2-} + \frac{1}{2}\mu_{O_{2}} = 2\mu_{Pr}^{ox} - \mu_{O}^{2-}
$$  \[A-1\]

\[A-2\]

The combination and differentiation of the mass, site, and charge conservation condi-
tions, Eq. 5-8, yield

$$
\Delta G_f^0 = 2\mu_{Pr}^{ox} + \mu_{O}^{2-} + \frac{1}{2}\mu_{O_{2}} = 2\mu_{Pr}^{ox} - \mu_{O}^{2-}
$$

\[A-3\]

The chemical potential of oxygen of Ce0.8Pr0.2O2−δ relative to that of the standard state
can be estimated from

$$
\mu_{O}^{2-} - \mu_{O}^{2-} = \frac{\partial G}{\partial (2 - \delta)} \mu_{O}^{2-} = \frac{\partial \mu_{O}^{2-}}{\partial (2 - \delta)} \mu_{O}^{2-} = -RT \sum \mu_{O}^{2-} = \ln \gamma_{O}^{2-}
$$

\[A-4\]

The configurational entropy of Ce0.8Pr0.2O2−δ is

$$
S_{conf}^0 = k \left[ \ln \left( \frac{\Delta N_f^0}{\Delta N_f^0} \right) + \ln \left( \frac{[Pr_{Pr}^{0.2}]}{[Pr_{Pr}^{0.2}]} \right) + \ln \left( \frac{[Ce_{Ce}^{0.8}]}{[Ce_{Ce}^{0.8}]} \right) + \ln \left( \frac{[O_{O}^{2-}]}{[O_{O}^{2-}]} \right) \right]
$$

\[A-9\]

where $k$ is Boltzmann’s constant. The configurational part of the partial molar entropy
of oxidation of Ce0.8Pr0.2O2−δ is then estimated from

$$
S_{conf}^0 = \frac{\partial S_{conf}^0}{\partial (2 - \delta)} \mu_{O}^{2-} = \frac{\partial S_{conf}^0}{\partial \delta} \mu_{O}^{2-}
$$

\[A-10\]

Substituting $R = kN_A$ and Eq. A-5-A-9 in Eq. A-10 yields

$$
S_{conf}^0 = k \left[ \ln \left( \frac{\Delta N_f^0}{\Delta N_f^0} \right) + \ln \left( \frac{\Delta N_f^0}{\Delta N_f^0} \right) + \ln \left( \frac{\Delta N_f^0}{\Delta N_f^0} \right) + \ln \left( \frac{\Delta N_f^0}{\Delta N_f^0} \right) \right]
$$

\[A-11\]

Dividing Eq. 3 and 4, differentiating with respect to δ, and solving for the partial
derivative of the concentration of Pr, with respect to δ, we obtain

$$
\frac{\partial [Pr_{Pr}^{0.2}]}{\partial \delta} = 2K_{Pr}^{0.8}[Pr_{Pr}^{0.2}] + 4K_{Ce}^{0.8}[Ce_{Ce}^{0.8}] + 2K_{Pr}^{0.8}[Ce_{Ce}^{0.8}]
$$

\[A-12\]

from the entropic contribution, we obtain

$$
\Delta H_f^0 = -\Delta S_f^0 \gamma_{O}^{2-} + \gamma_{O}^{2-}
$$

\[A-13\]

$$
\Delta S_f^0 = \frac{\partial [Pr_{Pr}^{0.2}]}{\partial \delta} \left( \Delta S_{conf}^0 - \Delta S_{conf}^0 - \Delta S_{conf}^0 \right)
$$

\[A-14\]

References

3. O. Yamamoto, in Solid State electrochemistry, P. G. Bruce, Editor, p. 292, Cam-