Thulium distributed-feedback fiber lasers

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Abstract

The subject of this thesis is experimental and theoretical studies of thulium doped Distributed-FeedBack (DFB) fiber lasers. The work covers a broad line of results obtained in fundamental spectroscopic properties of thulium doped silica fibers through a detailed theoretical study of DFB fiber lasers ending up with the actual demonstration and maturing of thulium doped DFB fiber lasers.

In the study of rare-earth doped devices, the most fundamental parameters are the stimulated emission and absorption cross sections and the dopant concentrations. This work presents for the first time a systematic approach to estimating these parameter values for thulium doped silica fibers. The methods used to determine the spectroscopic parameters have been chosen to avoid problems and uncertainties of both calibration of equipment and theoretical predictions. Emission and absorption cross sections are determined and rare-earth concentrations are estimated to form the basis for theoretical simulations.

An analytical model of DFB fiber lasers have been carefully studied and improvements to the model have been developed. The validity of the model has been verified by comparison to a numerical model and agreement is shown to be good in situations of negligible pump power absorption. The analytical model is used to open new insight into parameter dependencies and cavity design of DFB fiber laser in general. The numerical model is also used to simulate thulium DFB fiber lasers using the spectroscopic data recovered through the study. The results are valuable tools in predicting the performance of future thulium doped devices over the very broad emission band of thulium in silica.

The first demonstration of a thulium doped DFB fiber lasers is presented with a laser emitting single-frequency radiation at a wavelength of 1.74 $\mu$m. The record is extended with the demonstration of two other single-frequency lasers at 1.98 $\mu$m and 2.09 $\mu$m thereby enabling single-frequency lasing throughout the emission spectrum of thulium in silica. The laser designs are matured and made more efficient and system design considerations are presented.
Finally, the application of DFB fiber lasers are demonstrated in an experimental study of a coherent optical anemometry system. A near infrared coherent anemometry system is reviewed in terms of optics and system design enabling it to produce aerosol speed indication with a repetition rate of about 5 Hz. An improved monostatic system design is suggested which may have the potential to overcome limitations of leaking transmit power in a high-power system.
Resume

Emnet for dette Ph.D. projekt med titlen *Thulium Distribueret-Feedback fiber lasere* er en teoretisk og eksperimentiel gennemgang af Distribueret FeedBack (DFB) fiber lasere doteret med thulium. Studierne dækker en bred vifte af resultater, bl.a. indenfor grundforskning af spektroskopiske parametre for thulium doteret silica glas fibre og anvendt forskning i forbindelse med udviklingen og realiseringen af en thulium doteret DFB fiber laser.

En systematisk eksperimentiel gennemgang af de fundamentale spektroskopiske værdier for thulium doteret silica fibre er præsenteret. Den eksperimentielle fremgang er valgt ud fra nøje overvejelser om at undgå usikkerheder i forbindelse med teoretiske beregninger og kalibrering af eksperimentielt udstyr. Emissions- og absorptionstætheder for thulium doteret silica er fundet sammen med doteringsniveauer og levetider for de eksisterede tilstande i thulium ionen. Parametrene er værdifulde værktøjer i simuleringen af optiske komponenter baseret på thulium fibre.

En analytisk model for DFB fiber lasere er nøje gennemgået, og der er bidraget med forbedringer til modellen. Ud fra den analytiske model er nye betragtninger omkring kvalitetsdesign og støjegenskaber gennemgået. En numerisk model er opbygget med henblik på at simulere DFB fiber lasere i grænser, hvor den analytiske model har begrænsninger, hvilket er vist som tilfældet hvor pumpe absorptionen bliver signifikant. Begge teoretiske modeller anvendes sammen med resultaterne fra spektroskopien til at simulere thulium DFB fiber lasere.

Det primære resultat af afhandlingen er realiseringen af den første enkeltfrekvens DFB laser i thulium doteret silica fiber ved en bølgelængde på 1.74 μm. Resultatet er fulgt op af enkeltfrekvens lasere produceret efterfølgende ved 1.98 μm og 2.09 μm, hvilket demonstrerer potentielt for at opnå høj-kohærent laser virkning over hele thulium ionernes emissionsbånd.

Den sidste del af afhandlingen omhandler en forholdsvis ny og spændende
anvendelse af DFB fiber lasere som kikler i koherente anemometer systemer. Eksperimentielle undersøgelser af et optisk koherent system er undersøgt mht. optik og system design, og systemet er bragt til at kunne detektere aerosol hastigheder med en opdaterings rate på 5 Hz. Et nyt design af et monostatisk system er præsenteret, hvilket kan minimere problemer med at operere systemet ved endnu højere optiske effekter.
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Chapter 1

Introduction

Thulium is an element in the lanthanide group also referred to as a rare-earth. The name is said to originate from Thule, an old indication of Scandinavia [1]. The study of thulium doped silica fiber lasers began in the late 80's by Dave Hanna et al. [2, 3], who demonstrated continuous singlemode output from a thulium doped silica fiber laser at wavelengths from 1.88 – 1.94 μm. Since then developments have gradually continued with demonstration of high power CW operation [4], pulsed operation [5, 6] and now also, with the present work, single-frequency operation [A1].

Using Distributed-FeedBack (DFB) cavity designs in achieving single-frequency lasing in silica fibers is by far the most popular method today. The DFB fiber laser has the advantage of being relatively simple to fabricate and very stable compared to competing fiber technologies, such as ring lasers. Compared to conventional single-frequency sources it has the obvious advantage of being all-fiber, eliminating the stability issues related to free-space optics which also offers the opportunity of laser wavelengths in a wide continuous bands.

The first DFB fiber laser was demonstrated using Er/Yb codoped fiber in the early 90’s [8] and was believed to be a promising candidate for a signal source in the rapidly developing field of wavelength division multiplexed optical communication systems. However, the narrow linewidth inherent of the relatively long-cavity DFB fiber laser and the tough competition of semiconductor DFB diode lasers buried the dream of the optical communication market. Instead, applications of DFB fiber lasers in the field of sensing should turn out to be an interesting area, where the lasers good optical properties could be used with advantage. Today, DFB fiber lasers are used in a variety of sensor systems...
which rely on the lasers sensitivity to thermal and mechanical vibrations, the low noise and the narrow-linewidth [9, 10, 11, 12]. Recently, a new exciting area of interest is coherent optical radars, also known as coherent lidars. The high degree of coherence of DFB fiber lasers and the possibility of power scaling combined with modern fiber-optic technology has opened a gate to efficient fiber-based lidar systems. Coherent lidars offers the unique possibility of remote sensing of wind speed and direction in for example turbulence and gust warnings in airplanes. Fiber based lidar systems are suitable for operation in aircrafts and satellites because of the small size, low weight and low power consumption.

The emission wavelength of DFB fiber lasers has for a long time been limited to the gain bandwidths of erbium 1.52 – 1.61 μm and ytterbium 0.98 – 1.1 μm. Thulium opens a new window of potential single-frequency fiber lasering all the way from 1.65 μm to 2.1 μm, i.e. covering a bandwidth up to 450 nm. A single-frequency laser in this wavelength range could have applications in high precision near-infrared spectroscopy of a number of gases, for example CO$_2$, NO, HCl, H$_2$O [13, 14, 15, 16]. Also as a source to reach new wavelengths with frequency mixing and optical parametric amplification is apparent. And finally in the field of eye-safe coherent lidars as the master oscillator around 2.0 μm and in general as a Tm:YLF/YAG replace. Even though thulium doped fiber lasers are a step behind their cousin Er/Yb fiber lasers in terms of output power and functionality, they still provide an important step towards a complete coverage of wavelengths in the near-infrared wavelength region.

The opportunities are already present and with the availability of the technology, new areas of applications are likely to emerge. The perspectives are manifold, specifically the advantage of power scaling the DFB fiber laser output in Double-Clad amplifier modules to reach high output powers, is attractive. High power single-laser CW output has already been demonstrated with up to 85 W at a wavelength of 2.04 μm using a diode pumped Double-Clad fiber laser [4]. With a Tm:DFB fiber laser in a Master-Oscillator-Power-Amplifier configuration it should be possible to reach high output powers limited only by nonlinearities such as stimulated Brillouin scattering.

The purpose of the present thesis is to present a study of the possibilities with thulium as the active medium for DFB silica fiber lasers. This implies basic research of thulium in silica fibers for a better understanding of the intrinsic parameters used to describe a laser or amplifier model. It also involves general theoretical considerations of the parameters of DFB fiber laser performance. And finally, it involves the actual demonstration of single-frequency lasing using a thulium doped silica fibers along with practical considerations in future system design.

2
The thesis is divided into 4 main chapters.

Chapter 2 describes a study of the spectroscopic parameters of thulium doped silica fibers. Basic considerations and limitations of methods to determine cross sections and lifetimes are given with the emphasis on the transition in thulium. A line of thought for accurate determination of the important emission and absorption cross sections of thulium is derived and used in an experimental investigation. The cross sections of thulium doped silica fibers are for the first time measured accurately using a minimum of assumptions. Also upconversion and cross relaxations processes in thulium are reviewed.

Chapter 3 is devoted to a novel analytical model for DFB fiber lasers proposed by Scott Foster [17, 18]. The basic theory is presented with improvements involving asymmetric cavities and comparison to numerical simulations. The application and limitations of the model are discussed, while finally looking at noise properties of the DFB fiber laser.

Chapter 4 is an extension of the two previous chapters. The analytical and numerical model of the DFB fiber laser are combined with the spectroscopic results of the thulium doped fibers in order to model threshold condition, output power and noise properties of thulium DFB lasers.

Chapter 5 presents the most important achievement of the study by the first real demonstration of single-frequency lasing at wavelengths of 1.74, 1.98 μm and 2.09 μm using the DFB laser design in thulium doped fibers. The achievements are followed by maturing of the design, leading to an efficient and reliable laser for single-frequency emission in the wavelength range of 1.65 – 2.1 μm.

Chapter 6 is a presentation of a practical application of single-frequency DFB fiber lasers as a component in a future airborne lidar system at EADS GmbH. Basic considerations regarding a CW lidar system are studied in theory and practice, while improving the setup for future flight testings.

Finally the thesis is concluded in chapter 7.
Chapter 2

Thulium spectroscopy

2.1 Introduction

As mentioned, thulium is a rare-earth metal in the lanthanide group similar to the more common rare-earths erbium and ytterbium. When incorporated into an optical fiber it appears in ion form as Tm$^{3+}$, where the optical transitions originates from within the shielded 4f electron shell. The energy level of the 4f ground configuration is split into several manifolds by atomic forces in the host material. Each manifold is further split into several multiplets or Stark-levels by the crystal fields of the host. In an amorphous host as silica, there is no local symmetry and a strong electron-phonon coupling results in the Stark-level splitting being overlapping within each manifold, to form a continuous energy band as opposed to discrete lines.

The continuous energy band of the rare-earth ion can be homogeneously or inhomogenously broadened. In silica, the homogeneous broadening is large because the strong electron-phonon coupling effectively links each Stark-level effectively together, to form one homogeneous band. Inhomogenously broadened bands are also present in silica. These are collections of homogenously broadened lines that have different characteristic as a result of site variations and the linking between these lines is not present. Usually, in fiber optic amplifiers and lasers, one prefers completely homogenously broadened lines, since it uniformly distributes its gain characteristics as opposed to an inhomogenously broadened line, where the gain can drop completely in a specific wavelength region, because isolated ions are depleted.
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Figure 2.1: (a): Thulium ion energy level diagram. Center absorption band wavelengths are indicated in red and center emission wavelengths in blue. Approximate lifetimes are shown to the right. (b): Ground-state-absorption spectrum for a Al/La-thulium doped silica fiber (Tm1).

The most important energy levels of the thulium ion are sketched in figure 2.1 along with an optical absorption spectrum with indications of the different transitions. As there are relatively many energy levels within reach of visible to near-infrared photon energies, there exists a lot of potential ways of pumping the thulium ions. However, three pump levels attract special attention, since the availability of pump lasers are limited. The first is the \(^3\text{H}_4\) band with a peak absorption around 790 nm. In this wavelength range, high-power multimode AlGaAs lasers are readily available at 808 nm, which have a history of pumping Nd:YAG lasers. Also singlemode AlGaAs lasers at 790 nm are being developed in these years, which could be a future potential pump source. Otherwise, the versatile Ti:Sapphire laser is used, but this is more a research tool because of the complexity of such a system.

The second pump band is the \(^3\text{H}_5\) band, which has a peak absorption at 1210 nm. It is mostly pumped in the low-wavelength tail using high-power Nd:YAG lasers at 1064 nm, but unfortunately this pump wavelength suffers from quite strong pump-excited state absorption, which in a 3-step absorption
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process excites the ions all the way up to the \(^1\text{G}_4\) level. Therefore, pumping into this band is rarely seen in practice unless specific interest is within upconversion.

The third absorption band is the first excited state of the ion, \(^3\text{F}_4\). This pump band is subject to increasing attention, since high power Erbium/Ytterbium fiber lasers and amplifiers are readily available at wavelengths from 1565 nm to 1610 nm. The pump to signal photon energy defect is halved as with respect to the \(^3\text{H}_4\) band and the 1550 nm optical technology is well developed as a consequence of the optical communications industry and the success of the erbium amplifier. There is no doubt that future single-clad Tm-doped lasers and amplifiers will be pumped in this \(L\)-band region, a term borrowed from optical communications, where \(L\) stands for the long-wavelength end of the erbium gain profile.

The application of the thulium doped fiber is of course an important issue in choosing the pump wavelength. In silica fibers there are only few applications that involve upconversion lasers, which are much more effective in low-energy phonon host as fluoride or ZBLAN fibers. Amplifiers for optical communications in the S-band from 1460-1530 nm have also been demonstrated in silica between the states \(^3\text{H}_6\) and \(^3\text{F}_{3}\), but they usually require a double-pumping at 1064 nm and 790 nm or 1565 nm in order to quickly remove population from the \(^3\text{F}_6\) level[19].

To date, the primary application of Tm:silica fibers is laser operation in the \((^3\text{F}_4, ^3\text{H}_6)\) band and the work presented here also specifically treats this transition. This means that a fundamental characterization and description of this transition is important for understanding the basics of optical transition. The starting point of all models of rare-earth fiber lasers or amplifiers are the spectroscopic parameters. They may be obtained through direct measurements, theoretical calculations or a combination hereof. The parameters are the emission- and absorption cross sections, the excited-state lifetime, the rare-earth concentration and the confinement factor of spatial overlap between the optical fields and the dopant distribution. In the following, a study of these parameters of the \((^3\text{F}_4, ^3\text{H}_6)\) is presented.

2.2 Rare-earth basics

Consider a photon flux interacting with a rare-earth ion. The probability that the photon flux stimulates an absorption or emission event of the ion is called the ion cross section, denoted \(\sigma_a\) and \(\sigma_e\), respectively. It has the units of area and can be thought of as an effective area the ion fills in space. This important
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parameter is fundamental in understanding and modeling rare-earth doped devices and hence a short review of the possible ways to extract the cross sections is now given.

Consider a two level system with energy separation $\Delta E = E_2 - E_1$ and two stimulated transitions strengths, namely the stimulated emission strength $B_{21}$ and stimulated absorption strength $B_{12}$ and a spontaneous transition strength $A_{21}$. In a situation were the two energy levels are discrete energy levels, Einstein has shown that both stimulated strengths are equal, i.e. $B_{21} = B_{12}$. However, as explained above, this is not the case for rare-earths in silica where each energy level is split into a number of stark levels. If the degeneracy of the upper level is $g_2$ and for the lower level $g_1$ and it is assumed that all degeneracies are equally populated then the transition strengths can be shown to be given by[20]

$$g_2 B_{21} = g_1 B_{12}$$

$$A_{21} = \frac{8\pi\hbar}{\lambda^3} B_{21} = \frac{1}{\tau_{21}}$$

where $\hbar$ is Planck’s constant and $\lambda$ is the transition wavelength and $\tau_{21}$ is the radiative lifetime of the $2 \to 1$ transition. The transitions can then be thought of as originating from an average energy level. Furthermore, if the energy degeneracy are characterized by a line-shape function, $g_{12}(\nu)$ and $g_{21}(\nu)$ which depends on the frequency and if the definitions of cross sections and Einstein coefficients are compared in an equation for the rate of change of the photon population then relations are established to give

$$\sigma_a(\nu) = \frac{\hbar n c}{\lambda} B_{12} g_{12}(\nu)$$

$$\sigma_e(\nu) = \frac{\hbar n c}{\lambda} B_{21} g_{21}(\nu)$$

where $n$ is the material index of refraction. Equations 2.3 and 2.4 are widely known as the Fuchtbauer-Landenburg equations[21]. The upper level may spontaneously de-excite through emission of a photon with a characteristic time constant known as the radiative lifetime, $\tau_r$, which is linked to the cross section as

$$\frac{1}{\tau_r} = 8\pi n^2 c \int \frac{\sigma_e(\lambda)}{\lambda^4} d\lambda = \frac{g_2}{g_1} \frac{8\pi n^2 c}{\lambda^4} \int \frac{\sigma_a(\lambda)}{\lambda^4} d\lambda$$

The leftmost equation between the radiative lifetime and the emission cross section is a fundamental character of lifetime broadening, whereas the last term is derived from the Fuchtbauer-Landenburg equations and hence assumes equal populations in the Stark-levels.
2.2. RARE-EARTH BASICS

The Fuchtbauer-Ladenburg equations offers the possibility of determining the emission cross sections by either measuring the radiative lifetime and a relative emission spectrum or measuring the absorption cross section from an accurate attenuation measurement and scaling the emission spectrum by integration the absorption spectrum. However, the assumption of equal populations in the stark-levels is not a good assumption for rare-earth doped silica since the energy separation between stark levels is well above the thermal energy $k_B T$ and the Fuchtbauer-Ladenburg equations have previously been shown to significantly overestimate the ratio of cross sections for erbium doped fibers [22].

Furthermore, the scaling of the emission cross section from the radiative lifetime is only possible if all de-excitations to the ground level are in fact radiative. De-excitations may also be non-radiative where an excited state gives up its energy to the surrounding material by the creation of one or more phonons. The non-radiative decay is exponentially dependent on the number of phonons required to bridge the energy gap and for silicate glasses the non-radiative decay rate becomes significant around 5 phonons corresponding to an energy separation of $\approx 5500 \text{ cm}^{-1}$ [23]. Unfortunately, as figure 2.1 reveals, the mean energy separation of the ($^3\text{F}_4,^3\text{H}_6$) levels of thulium is also around 5500 cm$^{-1}$ thus indicating possible phonon induced decay of this level. Hence, the emission cross section cannot be scaled from a measurement of the lifetime, since the observed lifetime cannot be assumed equal to the radiative lifetime.

An alternative and very popular way of scaling the cross sections is by using a relation derived by McCumber [24]. McCumber assumed the populations in the manifolds to be Boltzmann distributed and that this distribution was reached on a timescale much faster than the overall lifetime of that level. The ratio of cross sections would then follow a simple exponential relation

$$
\frac{\sigma_e}{\sigma_a} = \frac{Z_1}{Z_2} e^{\frac{\Delta E - h\nu}{k_B T}} = e^{\epsilon - \frac{h\nu}{k_B T}},
$$

where $Z_i = \sum_{m_1, m_2} e^{-\frac{E_{m_2}}{k_B T}}$ is the sum of the $m^{th}$-levels partition functions. As seen, one fundamental parameter, the cross-over energy $\epsilon$ in equation 2.6 relates the cross sections and if either cross section is known, the other is easily obtained if $\epsilon$ is known. Equation 2.6 has been used with great success in erbium doped fibers, however it has been pointed out that the relation has shortcomings for broad linewidths that are mainly homogeneously broadened [25]. Nevertheless, the accuracy of the McCumber-relation has proven to be good and much better than the Fuchtbauer-Ladenburg equations for erbium doped silica, proving that the assumption of Boltzmann distributed populations is a good assumption.
2.3. MEASUREMENT PROCEDURES

A final method to scale the cross sections to be mentioned is the so-called Judd-Ofelt theory\cite{26}. It assumes a much smaller energy spread of the excited state with respect to the average energy separation of the excited and ground state, which is a good approximation for a crystal host. Judd-Ofelt parameters have been used with some success even though errors up to 50\% have been reported in estimating the radiative transition rates. The advantage of the method is that it only requires an absorption spectrum of the rare-earth ion considered in order to calculate the radiative lifetime. However, because of the large uncertainty involved in the method, it was avoided.

The most common ways of obtaining the emission and absorption cross sections of rare-earth doped silica fibers have been reviewed above. In the following, a procedure to determine the cross sections of thulium doped silica fibers will be given, but the methods used are different from those described above. The reason being that the assumptions and limitations of the above procedures were found to be problematic with thulium in silica. Instead, a simple procedure combining well-known procedures was used to scale the cross sections and estimate the rare-earth dopant concentration. The procedure is outlined as

- Estimate the fluorescent lifetime from curve-fitting of fluorescent decay experiments
- Find Ground-State-Absorption (GSA) from cutback measurements
- Scale the absorption cross section using the method of saturated fluorescence
- Scale the emission cross section using the ratio of maximum gain to Ground-state-absorption
- Obtain the emission cross section spectrum using a relative measurement procedure of gain tilt in order to avoid spectral distortion by equipment.

2.3 Measurement procedures

All measurements concerning spectroscopy are done in-fiber using pump sources at wavelengths of 786 nm and 1600 nm and signal sources of either a broadband thermal lamp or single-frequency thulium doped fiber lasers at wavelengths from 1735 nm to 1740 nm. Measurements were carried out several times to check reproducibility and to refine the measurement procedures.
Figure 2.2: Experimental setup used for gain/absorption measurements. A lock-in technique was employed to avoid ASE contributions and reduce noise.

A sketch of a typical experimental setup used to measure gain/absorption spectra on the thulium doped fibers is shown on figure 2.2. All measurements were carried out using a lock-in amplifier referenced to a mechanical chopper at a frequency around 270 Hz. The monochromator was a standard 1.5 μm blazed single-grating type with fiber optic input and output to a long-wavelength InGaAs detector TE-cooled to −10 °C. It was experienced that the monochromator had to be constantly purged with Nitrogen, in order to avoid ripples in spectral transmission measurements due to water absorption in the humid laboratory air. Water vapor has an absorption band located in the wavelength range from 1800 nm – 1950 nm. The majority of measurements were carried out with a tungsten lamp as the signal source, which provided good stability and low spectral density, such that the thulium ions could be considered as being unpumped by the signal power. Unfortunately, the spectral density was roughly an order of magnitude below the optimum value and the setup had to be adjusted to capture very weak signals, which limited the maximum achievable SNR to approximately 25 dB because of limitations in the AD-converter in the data capturing equipment. An improved SNR of gain measurements could have been performed with a selfmade single-frequency laser if it was not for the grating in
the monochromator which was found to be polarization dependent. As it is very
difficult to maintain polarization in cutback experiments of optical fibers, where
successive fiber lengths are cut from the fiber, polarized signal sources were out
of the question.

Two fibers were of primary interest and are frequently referred to in this
report. The first is a thulium doped Al/La fiber from OFS Fitel Denmark, from
now on termed as \( \text{Tm1} \). The other is an Al/Ge codoped fiber, named \( \text{Tm2} \).
Other fibers were also available but they showed poor performance in terms of
short excited state lifetime or too low dopant levels. It must be emphasized
that at the time of the project, thulium doped silica fibers were not easily ac-
cessible and particularly not when special co-dopants and concentrations were
wished for. In particular, the fiber had to be photo-sensitive which in general
requires it to be co-doped with germanium and furthermore the rare-earth con-
centration needed to be high enough to ensure adequate gain for the realization
of short length fiber lasers. Hence in the early stages of the project a dialog
with OFS Fitel Denmark was established to make a preform and draw a fiber
using their knowledge and skills in rare-earth doped fibers. The dopant level
was estimated from results obtained with the \( \text{Tm2} \) fiber and from early laser
fabrication. A particularly important parameter is a high Al-codoping to avoid
clustering effects and enhance solubility of the rare-earth and improve efficiency
of lasers.

A fundamental problem with the fibers at hand was splice losses. Especially
the \( \text{Tm2} \) fiber had a splice loss using a standard splicing program of 1.5 dB
per splice. By optimization it was possible to reduce this to around 0.7 dB
per splice, but only through the use of a certain splice recipe. The \( \text{Tm1} \) fiber
was clearly better at this point with a splice loss around 0.2 dB per splice.
The difficulties in splicing the fibers together affect measurements in two ways.
First, the splicings are difficult to reproduce, which causes problems in cutback
measurements. Second, the coupling (modal overlap) between the fiber modes
in a bad splice causes cladding modes to appear which gives large modulations
of the signal level as a function of wavelength in transmission measurements.
Cladding modes may be stripped for example by coiling the fiber, but this makes
fiber handling difficult, especially for the particular fibers where the dopant
levels are so high, that only small lengths of about a dozen of centimeters are
possible in order to have a reasonable signal-to-noise ratio in a transmitted
signal.

The optimal fiber lengths and coiling techniques were found by trial and
error and optimized measurement methods were developed through experience
gathered over many measurement series. For example, a method to greatly
reduce cladding modes with the Tm1 fiber, was to use a fiber with identical fiber parameters but without rare-earths. This fiber was most kindly borrowed from OFS Denmark and it also provided the opportunity to perform background attenuation measurements, see section 2.10.

2.4 $^3F_4$ lifetimes

The fluorescent (or observed) lifetime, $\tau_{fl}$ is in general given by the radiative, $\tau_r$, and the non-radiative, $\tau_{nr}$, lifetimes as

$$\frac{1}{\tau_{fl}} = \frac{1}{\tau_r} + \frac{1}{\tau_{nr}}.$$  \hspace{1cm} (2.7)

It is very important to separate the lifetime in a radiative and a non-radiative lifetime, because as opposed to the $^3I_{13/2}$ level of erbium which is 100% radiative, thulium has a significant part of the excited states decaying non-radiatively. This means that the observed lifetime is much shorter than the radiative lifetime.

The fluorescent lifetime can be measured from a simple fluorescence decay experiment, in which the $^3H_4$ level of thulium is excited by a pump diode laser at a wavelength of 786 nm. A short piece of fiber (\(<\ 2\ mm\)) is on/off pumped by the diode and the fluorescence is focused through a microscope objective to a 1 MHz-bandwidth long-wavelength InGaAs detector. The fluorescence should ideally have been collected transversally to the fiber in order to avoid stimulated events disturbing the decay process and this was also tried with several different setups, but without success. The fluorescence collected transversally was simply not strong enough to obtain a reasonable signal-to-noise ratio. Therefore it was decided to splice a short piece of the thulium doped fiber in between two standard fibers which guided the fluorescence to the microscope objective. Residual pump power was blocked using an IR transmitting glass filter.

A schematic representation of the levels involved in pumping the thulium ions at 790 nm is shown below. The ions excited to the $^3H_4$ level quickly relaxes to the $^3F_4$ level from where it again relaxes partly radiatively to the ground state $^3H_6$. The fluorescence collected is proportional to the population of the $^3F_4$ level, which decays exponentially with a lifetime of $\tau_{fl}$. 

13
In the figure to the right, a simplified diagram of the energy levels involved in pumping the $^3H_4$ level is shown. Assuming that the pump is constant and then turned off very fast, the populations decay exponentially with the lifetimes indicated in the figure. The rate equations for the system are found to be

$$\frac{dN_2}{dt} = -\frac{1}{\tau_2}N_2$$  \hspace{1cm} (2.8)

$$\frac{dN_1}{dt} = \frac{1}{\tau_2}N_2 - \frac{1}{\tau_1}N_1$$  \hspace{1cm} (2.9)

$$P_{sp} \propto \frac{1}{\tau_r}N_1,$$  \hspace{1cm} (2.10)

where $P_{sp}$ is the spontaneously generated power, $\tau_r$ is the radiative lifetime of the ground state and $\tau_{1,2}$ are the fluorescent lifetimes. When the pump is abruptly turned off at $t_0$, equation 2.8 gives $N_2(t) = N_{20}e^{-\frac{t-t_0}{\tau_2}}$ and inserting this into equation 2.9 can be shown to give

$$N_1(t) = N_{20}\frac{\tau_1}{\tau_2 - \tau_1} \left( e^{-\frac{t-t_0}{\tau_2}} - \frac{\tau_1}{\tau_2} e^{-\frac{t-t_0}{\tau_1}} \right).$$  \hspace{1cm} (2.11)

Since the fluorescent power is proportional to $N_1$ from equation 2.10, it is possible to fit equation 2.11 to the spontaneous emission decay from the experiment described above. Thus, the fitting parameters extracted will be the fluorescent lifetimes $\tau_1$ and $\tau_2$. In order for the 3-level system to be effective as a laser system, it is necessary that $\tau_2 << \tau_1$ such that inversion can easily be built up. This also means that the fast term has most significance in a short time just after the pump is turned off. This leaves a single-exponential with a slower decay, which is linear in a logarithmic plot.

Experimental curves of the fluorescence are seen in Fig. 2.3 along with fitted double exponential lines. The decay curves are least squares fitted to a double-exponential to recover a lifetime of 650 $\mu$s of Tm1 and 560 $\mu$s for Tm2 for the $^3F_4$ level. The short lifetime, $\tau_2$ is uncertain mostly because of the detectors 3 dB bandwidth being only 1 MHz. The lifetimes found are in good
2.5. CROSS-SECTION RATIO

Figure 2.3: (a): Fluorescence decay from a 2 mm long fiber (Tm1) excited by a step pump power at \( \lambda_p = 786 \) nm. (b): Decay curves in logarithmic scale for all four fibers available. Lifetimes found from fitting algorithm indicated in parenthesis. The cyan curve corresponds to the decay seen in the left figure.

agreement with previous reported lifetimes which range from 300 to 600 \( \mu s \). Notice also the linearity of the decay curves in the logarithmic plot is best for the Tm2 and Tm1 fiber. Those fibers have been co-doped with high amounts of aluminum and hence signifies good agreement of the decay processes with the simple model derived above. For the two other fibers, the linearity is worse and could suggest other mechanisms of relaxations such as ion upconversion or pair-induced quenching. Ion upconversion can be a detrimental processes in which two excited nearby ions exchange energy such that one excited ion is effectively lost from the upper level [27]. The process is proportional to \( N_2^2 \) and therefore the decay is no longer exponential.

2.5 Cross-section ratio

The fluorescent lifetime is used to find the absolute value of the absorption cross section. Scaling of the absorption cross section is possible with knowledge of the concentration of rare-earths and the ground-state-absorption per unit length \( GSA = 2\pi\sigma_a \int \rho\phi_s^2 rdr \), where \( \rho \) is the concentration and \( \phi_s \) is the nor-
mized mode profile of the fiber and \( r \) is the cylindrical coordinate. However, the concentration of the particular fibers used is unknown. Nevertheless, once the excited state lifetimes is known, it is possible to scale the absorption cross section, even without knowledge of the rare-earth concentration. This is possible, since the inversion is independent of the concentration. The inversion of a 2-level system in a short piece of fiber where the pump power is much higher than any signal power is given by

\[
x(r) = \frac{N_1(r)}{N_1(r) + N_0(r)} = \frac{\sigma_{ap} n_p \phi_p(r)^2}{\sigma_{ap} n_p \phi_p(r)^2 \eta_p + 1} + \frac{1}{\tau_{ii}},
\]

(2.12)

where \( \sigma_{ap} \) is the pump absorption cross section, \( \phi_p \) is the optical mode of the pump, \( n_p \) is the pump photon current and \( \eta_p = \frac{\sigma_{em}}{\sigma_{ap}} \) is the ratio of emission-to-absorption cross section at the pump wavelength. The ion populations of the upper and lower laser levels are \( N_1 \) and \( N_0 \), respectively. The \( z \)-dependency of the light propagating in the fibers positive direction is given by the small signal gain and fluorescence by

\[
\frac{dn_s}{dz} = 2\pi n_s \int_0^{\infty} \rho(r) \left[(x(r)\sigma_{ss} - (1 - x(r))\sigma_{as})\phi_s(r)\right] dr.
\]

(2.13)

In the following, the rare-earth distribution is assumed proportional to the refractive index profile, \( \Delta n(r) \), as \( \rho(r) = \rho_0 \frac{\Delta n(r)}{\max(\Delta n)} \), \( \rho_0 \) being the peak concentration. The refractive index profile is known from measurements on both fibers and the mode profile is found numerically using a cylindrical 1D Finite Element Modesolver, see Appendix B for details of the modesolver. This defines a confinement factor \( \Gamma \) as

\[
\Gamma = 2\pi \int_0^{\infty} \frac{\Delta n}{\max(\Delta n)} \phi_n^2 r dr,
\]

(2.14)

where \( \Delta n \) is the refractive index step of the fiber core relative to the cladding. The justification of doing so is found in that the index raising dopants are added along with the rare-earth dopants and that the diffusion processes are somewhat similar for the modifiers and rare-earths added[28]. The confinement factor for the Tm1 fiber is seen in figure 2.4 along with an inset of the mode-profile and the index distribution. Notice that the confinement factor fits well to a straight line.
2.5. CROSS-SECTION RATIO

Figure 2.4: Calculated confinement factor, $\Gamma$, using the refractive index distribution shown in the inset figure (blue curve) and the mode-profiles (red dashed curve) found from a cylindrical FEM modesolver. The inset figure is for a wavelength of $2.0 \, \mu m$

Using the confinement factor and assuming that the inversion is transversally independent, then equation (2.13) can be written as

$$\frac{dn_s}{dz} = \rho_0 \Gamma n_s [(x \sigma_{es} - (1-x)\sigma_{as}) + 2x\sigma_{cs}],$$

(2.15)

where the first term is the gain term and the second term is the spontaneous emission term. The gain term in equation (2.15) can be neglected if the fiber length is short. The spontaneous emission term generates fluorescence inside the fiber. The fluorescence can be used to determine $\eta_p$ for an in-band wavelength $\lambda_p$ through measurement of the ratio of generated fluorescence to the fluorescence level generated by pumping at a wavelength where full inversion can be reached[29]. Thus pumping at $\lambda_p = 786 \, \text{nm}$ and $\lambda_p = 1600 \, \text{nm}$ and observing the fluorescence at an optimum wavelength results in the curve shown in Fig. 2.5. First, the cross section ratio $\eta_p$ is found by the ratio of the extrapolated fluorescence levels at infinite pump power for the pump wavelengths 786 nm and 1600 nm. Then, the fluorescence levels are scaled and along with values of the launched pump power fitted to equation 2.15 using the inversion, where the fitting parameter is $\sigma_{ap}$.

At first, the ratio of cross sections at $\lambda_p = 1600 \, \text{nm}$ is found to be $\eta_p = 0.10 \pm 0.02 \, (\text{Tm1})$ and $\eta_p = 0.15 \pm 0.02 \, (\text{Tm2})$ from the extrapolated level at
infinite pump power. This number is then used in (2.12) along with values of the
launched pump powers and fluorescent lifetime to fit the generated saturated
fluorescence to equation (2.13) and thereby obtain a value of \( \sigma_a = 3.5 \pm 0.1 \cdot 10^{-25} \text{ m}^2 \) for Tm1 and \( \sigma_a = 4.3 \pm 0.1 \cdot 10^{-25} \text{ m}^2 \) for Tm2 at a wavelength of
\( \lambda = 1600 \text{ nm} \).

As the absorption cross section is now known at \( \lambda' \), then the absorption
cross section for all other wavelengths in the band can be found from a GSA
measurement. The GSA is simplified using the confinement factor as \( GSA(\lambda) = \rho_0 \sigma_a(\lambda) \Gamma(\lambda) \). This relation gives

\[
\sigma_a(\lambda) = \sigma_a(\lambda') \frac{GSA(\lambda) \Gamma(\lambda')}{GSA(\lambda') \Gamma(\lambda)},
\]

(2.16)

from which the absorption spectra are shown in Fig. 2.6

2.5.1 Scaling of the emission cross-section

With the knowledge of the absorption cross section and \( \eta(\lambda') \) at one wavelength,
it is possible to scale the emission cross section, if the emission spectrum can be
2.5. CROSS-SECTION RATIO

![Graph showing absorption cross section for Tm1 and Tm2 fibers.]

Figure 2.6: Absorption cross section of the Tm1 and Tm2 (Tm2) fiber.

measured. However, this is not an easy task, since the measurement equipment has to be calibrated from 1.6 $\mu$m to 2.1 $\mu$m in order for the real emission spectrum to appear from a fluorescence measurement. Alternatively, one may use $\sigma_a e^{-\frac{x}{kT}}$ from the McCumber relation to get the emission spectral shape, but as $\sigma_a$ is very small for wavelengths beyond 1.9 $\mu$m and the short-comings of the McCumber theory mentioned before, this is also problematic. To overcome this, a relative measurement of the emission spectrum is necessary and this is accomplished by measuring the gain tilt.[30]

The difference in gain of a small signal passing through a rare-earth doped fiber for two different mean inversions is found from equation (2.15) to be (ignoring contribution from ASE)

$$
\Delta g = \ln \frac{n_a(L)}{n_a(0)} <x_1> - \ln \frac{n_a(L)}{n_a(0)} <x_2> = GSA(\eta_s + 1) <\Delta x>,
$$

(2.17)

where $\eta_s = \frac{\sigma_a}{x}$ is the cross section ratio and $<x>$ is some mean inversion, $<\Delta x> = <x_1> - <x_2>$. If the ratio of cross sections, $\eta_s$, is known for one wavelength $\lambda'$, it is possible to take the ratio of equation (2.17) to obtain the
emission cross section as

$$\sigma_e(\lambda) = \sigma_a(\lambda')(\eta(\lambda') + 1) \frac{\sigma_a(\lambda)}{\sigma_a(\lambda')} - \sigma_a(\lambda)$$  \hfill (2.18)

Since the ratio of emission to absorption cross section is only known at 1600 nm, which is in the low-end of the emission spectrum, it provides poor accuracy for the entire spectrum. High power pumps were not readily available at higher wavelengths, so instead of measuring the fluorescence ratios as above, it was decided to make a small signal gain measurement instead. To improve the accuracy, the gain measurement was performed at $\lambda = 1740$ nm by pumping at 786 nm and extrapolating the ratio of max. gain to the GSA for a length L to give the ratio of cross sections [22]. A number of accurate measurements were performed at different lengths to provide a result of $\eta(1740 \text{ nm}) = 1.20 \pm 0.02$ for Tm1 and $1.25 \pm 0.02$ for Tm2. The scaled emission cross sections then follows from equation (2.18) and are shown in Fig. 2.7 and 2.8 together with their respective absorption cross sections.

The disadvantage of the gain tilt method is that it cannot give the correct emission spectrum where there is signal excited state absorption (ESA). Excited-state absorption is present for the $(^3\text{F}_4, ^3\text{H}_6)$ band in thulium for wave-
2.5. CROSS-SECTION RATIO

Figure 2.8: Absorption- and emission cross section of the Tm2 fiber.

lengths below 1550 nm and above wavelengths of 2065 nm. Nevertheless, in the important central region of the emission spectrum, the gain tilt method works well.

The peak absorption cross section has been reported in [31] to be $4.5 \cdot 10^{-25}$ m$^2$, which is in good agreement with the value found here of 4.2 and $4.5 \cdot 10^{-25}$ m$^2$. On the other hand, peak emission cross sections reported in [31],[32],[33] for thulium in silica have values of $6.0 \cdot 10^{-25}$ m$^2$, $6.1 \cdot 10^{-25}$ m$^2$ and $4.6 \cdot 10^{-25}$ m$^2$, respectively. This is in contrast to the value obtained here of only 3.5 and $3.9 \cdot 10^{-25}$ m$^2$, which agrees very well with the value of $4.0 \cdot 10^{-25}$ m$^2$ in [34]. The emission cross sections in [32] are found by applying the McCumber relation to the absorption cross section and scaling by comparison to a measured emission spectrum lineshape of a bulk sample. In [33], the emission spectrum was measured in a preform sample and scaled by the Judd-Olfelt calculated radiative lifetime. The large discrepancy is unclear at the moment, but could be caused by measurements obtained in different samples.

The scaled emission cross section allows the calculation of the radiative lifetime through the relation in equation 2.5, which gives a radiative lifetime of 6.0 ms and 6.6 ms for Tm1 and Tm2 respectively. Radiative lifetimes reported in [32] and [33] are 6.3 ms and 4.56 ms, which agrees fairly well with the values
2.5. CROSS-SECTION RATIO

![Graph showing emission cross section versus wavelength]

Figure 2.9: Emission cross section obtained by using the gain-tilt method and the McCumber relation assuming a cross-over energy corresponding to a wavelength of 1725 nm.

obtained here. Their approach was to use Judd-Ofelt theory and the absorption spectrum to scale the radiative lifetime. The radiative quantum efficiency of this laser level in thulium-doped silica fibers is therefore only around 10%. The low efficiency is a well-known problem associated with the high phonon energy of silica and the relatively low energy difference between the ground- and first excited-state of thulium.

2.5.2 Rare-earth concentration

The last parameter to be determined is the rare-earth average concentration, $\rho_0$, which is now easily obtained by the relation

$$\rho_0 = \frac{GSA}{\sigma_{\alpha}T}$$  \hspace{1cm} (2.19)

The concentration is hereby estimated to be approximately $8.4 \cdot 10^{25} \text{ 1/m}^3$ for fiber Tm1 and $6.1 \cdot 10^{25} \text{ 1/m}^3$ for fiber Tm2. The difference in concentration primarily originates from the definition of the confinement factor and that the concentration profile is assumed to follow the index distribution. The ratio of confinement factors for the two fibers is 0.7.
2.6. DISCUSSION

<table>
<thead>
<tr>
<th></th>
<th>Tm1</th>
<th>Tm2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho_0 \times 10^{20} \text{ m}^{-2} )</td>
<td>8.4</td>
<td>6.1</td>
</tr>
<tr>
<td>( A_c \text{ (\mu m}^2)</td>
<td>14.0</td>
<td>15.9</td>
</tr>
<tr>
<td>( R_c \text{ (\mu m)}</td>
<td>2.1</td>
<td>2.3</td>
</tr>
<tr>
<td>( \Gamma_s \text{ (1.75 \mu m)}</td>
<td>0.51</td>
<td>0.75</td>
</tr>
<tr>
<td>( \Gamma_s \text{ (1.95 \mu m)}</td>
<td>0.44</td>
<td>0.70</td>
</tr>
</tbody>
</table>

Table 2.1: Dopant parameters and values used for modeling. Confinement factors are given for two wavelengths and may be approximated at other wavelengths by a straight line. Definitions of \( A_c \) and \( R_c \) are given in Appendix B.

The order of magnitude of the estimated concentration in the fiber was also verified by comparing it to that of an erbium-doped fiber. All fiber parameters of the erbium-doped fiber, including co-doping, was the same as those for the Tm1-fiber, the only difference being that the erbium concentration was known to be approximately 11 times lower than the thulium concentration. The exact same procedure and measurements were carried out on the erbium-doped fiber to estimate its concentration. The absorption cross section of the erbium fiber at a wavelength of 1470 nm was estimated to \( 1.24 \times 10^{-20} \text{ m}^2 \) using a cross section ratio at this wavelength of 0.22. Since the GSA was measured to 0.63 \( 1/m \), the concentration of the erbium doped fiber was found to be around \( 8.7 \times 10^{14} 1/m^2 \), which is 9.5 times lower than in the thulium concentration in fiber Tm1. This result verifies the methods used to obtain all the necessary information in order the calculate the rare-earth concentration of the fibers.

2.6 Discussion

The most important underlying assumption in the results obtained above is that it is possible to reach an inversion of 1, when pumping the thulium doped fiber into the \(^3\text{H}_6\)-excited state, i.e. at a pump wavelength around 790 nm. This means that the cross section ratio, \( \eta_0 \), is assumed equal to 0 as in the case of erbium pumped at 980 nm. Moreover, the assumption is also that the \(^3\text{H}_6\) lifetime is comparatively shorter than the lifetime of the \(^3\text{F}_4\)-level, such that the system is effectively a two-level system. The lifetime of the \(^3\text{H}_6\)-level has not been measured directly at this stage and the only information is from the double exponential fit as shown in Fig. 2.3, which is not very accurate. However, the
value is close to the reported lifetimes which range from 14 to 20 $\mu$s. From the relation \( \frac{1}{1 + \frac{1}{\tau}} \), a maximum obtainable inversion is around 0.97 in the thulium doped fibers and the approximation is believed to be reasonably good.

Another issue is ion clustering. Ion clusters are 2 or more ions occurring in physical close vicinity which provides a possibility of a very fast energy exchange between the ions through Energy-Transfer-Upconversion (ETU). Effectively, this means that a fraction of the ions, \( k_{cl} \), cannot be excited with practical pumping rates leading to a maximum attainable inversion of \( x = 1 - k_{cl} \) for a pump wavelength with \( \sigma_{ep} = 0 \) [35]. However, it is believed that clustering in the two thulium doped fibers used is moderate. It is seen from the decay curves of figure 2.3 that the decays are following the assumed exponential behavior of equation 2.11 quite well, especially in the beginning of the decay. If significant ion clustering was present, it would result in a fast non-exponential decay following immediately after the pump shut off, but this was never observed to be the case.

However, pump excited state absorption at 790 nm is present, which is observed by a faint blue glow from the pumped fiber. The pump photons may excite the ions into the \( ^1G_4 \)-level from where they may decay radiatively by emission of 470 nm light. It is therefore, in principle, possible to excite all ions to the \( ^1G_4 \) level using a very high power pump, and thereby reduce the inversion between the ground-state and the first excited-state. However, the fluorescence observed from a short piece of fiber, which is directly proportional to the inversion level, was never seen to decrease with increasing pump power, this being the case even at pump powers above 500 mW. This justifies a low influence from pump ESA at 786 nm.

Even though it is not correct to assume full inversion in any rare-earth doped system, it is still believed to be a good approximation, which will fall within the general uncertainty range of the measured values.

### 2.7 Gain Cross section

Once all the basic parameters have been determined there is a lot of information to be extracted from general considerations and modeling. The issue of positive and negative gain is of course of very high importance and this can be formulated in an inversion cross section (or gain cross section) which is defined as

\[
\sigma_x = x(\sigma_c - \sigma_{ESA}) - (1 - x)\sigma_a.
\] (2.20)
2.7. GAIN CROSS SECTION

Figure 2.10: Inversion cross section of the Tm1 fiber. Notice wavelengths below 1525 nm and above 2065 nm where amplification, in principle, is not possible due to excessive ESA.

where $x$ is the inversion defined in equation 2.12. This very important parameter is easily calculated from the cross sections found above. However, the cross sections above are only shown where they are positive, since the gain tilt method cannot give the emission cross section were ESA is present because of the mixing in equation 2.20. A possible way to extract the ESA (or emission) cross section would be to measure a calibrated fluorescence spectrum, but this was not possible with the equipment at hand. Nevertheless the gain cross section of equation 2.17 is a measure of the inversion cross section in equation 2.20 including ESA. It actually provides additional information concerning the issue of ESA from a practical point of view. The inversion cross sections of fiber Tm1 and Tm2 are shown in figure 2.10 and 2.11. Clearly, both fibers have two nodes in the inversion cross section at a wavelength of 1525 nm and 2065 nm. At these wavelengths, the relation $\sigma_{ESA} = \sigma_e + \sigma_a$ holds. Below 1525 nm absorption from the $^5F_4 \rightarrow ^3H_4$ causes negative small signal gain and above 2065 nm the $^3H_5 \rightarrow ^3H_4$ causes negative gain. This means that in theory no lasing should be possible in those regions, however a 2.1 $\mu$m laser in a thulium doped fiber is reported in [36] and lasing has also been achieved as a part of the present
work at 2.095 $\mu$m from a 5 cm DFB laser. This issue will be further addressed in chapter 5.

Another interesting feature is that the Al/Ge-codoped fiber (Tm2) has a bigger inversion cross section below 1730 nm with respect to the Al/Ia-codoped fiber and the opposite is true for wavelengths above 1750 nm. This means that Ge-codoping effectively shifts the gain peak toward lower wavelengths with respect to Al-codoping for thulium. This has been confirmed in [37]. Two other interesting outcomes of the inversion cross section is the transparency wavelength, i.e. the wavelength of zero gain/loss, and the peak gain wavelength relation with the inversion which are both shown in figure 2.12. The jumpy character of the curves is a consequence of the noise in the measured spectrums, but the general trend is easily followed. The usual property of rare-earths of gain peak and transparency wavelength moving toward lower wavelengths for increasing inversion is also found here. An amplifier for wavelengths above 1900 nm should thus operate at a low inversion level.
2.8 Pump excited state absorption

Pump excited-state-absorption (ESA) is the process of absorption of a pump photon from excited states. Intuitively, this is not a process which is desired for lasers or especially not for amplifiers which operate at high inversion levels. Pump ESA excites ions into higher excited states and thus lowers the inversion between the primary upper and lower laser level and may in principle completely dry-out the upper laser level population at high pump rates. Since the energy level diagram reveals several energy levels above the \(^4\)F\(_4\) laser level, this is an important issue in choosing the most optimal pumping wavelength. A measurement of the ESA spectrum is shown in figure 2.13. Besides pump ESA, there are many factors involved in choosing the correct pump wavelength such as diode availability, cost, complexity, stability and efficiency. A pump band ideally suited for one factor may fall for other requirements. At the moment, three main pump wavelengths are of practical interest for pumping thulium doped fiber lasers. These wavelengths are 800, 1064 and 1600 nm.

Pumping around 800 nm into the \(^3\)H\(_4\) level reveals a faint blue glow from the fiber. The blue emission originates from the \(^1\)G\(_4\) level at around 470 nm. The
2.8. PUMP EXCITED STATE ABSORPTION

Figure 2.13: Indication of ESA bands measured from the $^3\!F_4$ level by transmitting a counterpropagating probe beam under pumping with 1600 nm. The two primary ESA bands are indicated in the figure.

energy of a 800 nm photon is $12500 \text{ cm}^{-1}$ and the energy level difference between the $^1G_4$ and the $^3\!F_4$ level is approximately $20500 - 5400 = 15100 \text{ cm}^{-1}$, which means that pump ESA is not solely responsible for the blue emission observed, but is also related to energy transfer processes, see section 2.9.

Another possibility is to pump into the $^3\!H_3$ band which has an absorption peak at 1210 nm. However, pumping this band at a wavelength of 1064 nm attracts the most attention because of the available sources. Unfortunately, strong pump ESA occurs exactly at this wavelength and it is in fact resonant in energy all the way up to the $^1G_4$ level, which means that three pump photons contribute to the upconversion. Again, a strong blue glow from the fiber is evidence of this process, which raises the threshold of lasers and degrades the amplifier performance. The only reason for pumping thulium at 1064 nm is the availability of very high power sources [38, 39, 40, 41, 42]. Low phonon glass host such as fluoride and ZBLAN fibers can use this upconversion process to produce lasing from the $^1G_4$ directly to the ground state. Such blue upconversion fiber lasers are quite efficient and can provide relatively high output powers up to around 100 mW [43].

The last practical pump wavelength is an erbium MOPA L-band source
from 1565 nm to 1620 nm. As seen from figure 2.13 and the figures 2.10, 2.11 there is negative gain below 1525 nm for all inversion corresponding to strong ESA to the $^3H_4$ band. This is in fact the absorption band used to facilitate S-band amplifiers. The tail of this band reaches into the C- and L-band wavelength regions, but the exact spectrum is not easily recovered as it overlaps with the absorption of the ($^3H_6, ^3F_4$) band. However, by observing the fluorescence generated at 800 nm as a function of pump wavelength gives an estimate of the relative ESA cross section and this is shown in figure 2.14a. Clearly it is most favorable to go for wavelengths deep into the L-band to avoid ESA, which also provides more absorption to the $^3F_4$ band. Nevertheless, a compromise often has to be made, since erbium amplifiers have a sharp drop in gain as the wavelengths go beyond 1610 nm which is treated in detail in chapter 5.5

2.9 Resonant upconversion and energy transfer

Upconversion and energy transfer processes are well known in rare-earth doped glasses. The processes are associated with the exchange of energy between closely spaced ions in an interaction with the host material. Through emission or absorption of one or more phonons, energy may transfer between ions, where the process requiring least phonons are the most probable. Ion upconversion has not been studied in detail with these fibers, despite that energy transfer processes are well known in high concentration thulium doped fibers [44, 45]. However, some quantitative experiments have been performed.

The nomenclature used here is that in the case the energy transfer promotes an ion from the ground state to an excited state is called a cross-relaxation (CR). The inverse process, where two excited ions exchange energy to promote one of them to a higher excited state is called Energy-transfer-upconversion (ETU). Relevant CR and ETU processes for thulium are sketched at the figure to the right.

As seen from the figure, cross relaxation processes are advantageous if amplification between the lower levels is desired whereas ETU processes are detrimental.
2.9. RESONANT UPCONVERSION AND ENERGY TRANSFER

<table>
<thead>
<tr>
<th>Crystal</th>
<th>( \Delta E_{\text{CR1}}^{\text{ub}} )</th>
<th>( \Delta E_{\text{CR2}}^{\text{ub}} )</th>
<th>( \Delta E_{\text{ETU1}}^{\text{ub}} )</th>
<th>( \Delta E_{\text{ETU2}}^{\text{ub}} )</th>
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<tr>
<td>Y(_3)Al(_5)O(_12)</td>
<td>+141</td>
<td>-1288</td>
<td>+1500</td>
<td>-1495</td>
</tr>
<tr>
<td>YVO(_4)</td>
<td>+765</td>
<td>-1231</td>
<td>+2277</td>
<td>-1423</td>
</tr>
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</table>

Table 2.2: Energy mismatch in \( cm^{-1} \) of the 4 most significant energy transfer processes in thulium doped crystals, from [45].

The rate of each process is dependent on the energy-imbalance between the initial and final state, \( \Delta E_{\text{ub}} \), and the populations of the initial levels. The energy-imbalance has been measured experimentally in crystals at low temperature and a few important results hereof are shown in table 2.2

The important thing to note from table 2.2 is that the energy mismatch is smallest for the CR1 process and that it is exothermic, i.e. it delivers energy to its surroundings. The CR1 process is widely known as the two-for-one process in thulium because it effectively produces two excited-state ions for one \(^{3}\text{H}_4\) pump photon, thus enhancing the overall efficiency of the pumping process. This process has been used to explain slope-efficiencies of heavily thulium-doped double-clad fiber lasers above the theoretical efficiency limit of \( \frac{\Delta \lambda}{\Delta \lambda} = \frac{\lambda_0}{2\Delta \lambda_0} \approx 0.4 \) [44]. However, the efficiency enhancement was first seen when the thulium concentration reached more than 1.3 wt.% \( \approx 8.5 \times 10^{26} \text{ ions/cm}^3 \) and at high concentrations of Al-codoping. Nevertheless, the CR1 cross relaxation process favours pumping into the \(^{3}\text{H}_4\) band.

The inverse process of CR1 is ETU2, but as the ETU2 process has a much higher endothermic energy difference, the rate of ETU2 is believed to be smaller than CR1, even in silica. Nevertheless, evidence of the ETU2 process has been found by pumping into the long wavelength tail of the \(^{3}\text{F}_4\) band and fluorescence from the \(^{3}\text{H}_4 \rightarrow ^{3}\text{F}_4\) band around 1460 nm has been observed [45]. The two other processes, CR2 and ETU1, are believed to have the smallest rates compared to the competing processes CR1 and ETU2.

Finally, a third ETU process not mentioned is the \((^{3}\text{H}_4,^{3}\text{H}_4) \rightarrow (^{1}\text{G}_4,^{3}\text{F}_4)\) process which has an energy mismatch of approximately 20500 - (12500 + 7100) \( cm^{-1} \approx -900 \text{ cm}^{-1} \). It is observed by blue fluorescence when pumping at 800 nm. As the process is endothermic, it requires one phonon to contribute to the process and this is experimentally verified by heating the fiber under pumping and recording the blue emission level, see figure 2.14b. The blue emission is clearly enhanced by the heating process and even changes in the emission spectrum of the \(^{3}\text{F}_4 \rightarrow ^{3}\text{H}_6\) band are seen. The cause of this is not certain, but it does not look like an inversion drop, since the change is not constant.
2.9. RESONANT UPCONVERSION AND ENERGY TRANSFER

Figure 2.14: (a): Fluorescence at 800 nm generated by pump ESA as a function of pump wavelength. Constant pump power of 50 mW. (b): Third ETU process shown for a cold and a hot fiber pumped at 790 nm. Inset shows the strong blue fluorescence at 470 nm, which has a more than 15 dB increase over a temperature range of 300 K.
throughout the spectrum.

Resonant transitions in thulium doped silica are a very interesting and comprehensive subject. Studies devoted to this subject are of key importance from the view of applications, both in terms of detrimental effects but certainly also for effects that can be turned into improved performance.

2.10 Background attenuation

Background attenuation is believed to be a limiting factor for DFB fiber lasers with high Q-cavity values as the wavelength of the emitted laser light is shifted into the 1.8 - 2.1 μm range because of increasing IR losses. It is well-known that modern telecom silica fibers have a very low attenuation of only 0.2 dB/km at a wavelength around 1.55 μm. Such low losses are not expected in rare-earth doped fibers as a consequence of fiber design and co-dopants, and the attenuation is expected to rise for increasing wavelength as a consequence of IR losses in silica and bending losses.

Measuring the background attenuation of doped fibers is rendered difficult by the presence of the rare-earths absorption. Fortunately, another fiber with the same design and parameters as those of the Tm1 fiber was available, without rare-earth dopants. Background attenuation measurements were performed on 980 m of this fiber with the result shown in figure 2.15. This fiber has a very low loss, despite the high concentration of Al and La in the core. Also notice, that the lowest loss of 0.5 dB/km appears at a wavelength of 1750 nm which is different from a standard transmission fiber which has a minimum loss at 1550 nm. This is due to a smaller spotsize of the Tm1 fiber as compared to a standard telecom fiber, which has the influence of making the fiber more immune to bending losses for higher wavelengths, because of a more confined modefield. The cutback measurement was performed on a spooled fiber for practical reasons and possible bending loss mechanisms have not been compensated for. The lamp source used for the measurements had a very low spectral power density beyond 2 μm and hence extrapolation of the attenuation was necessary to achieve values above this wavelength. IR losses can in a simple model be described as[46]

\[ \alpha_{IR} = \alpha_0 \alpha_1 \lambda^{-n} \]

(2.21)

where \( \alpha_0 \) and \( \alpha_1 \) are constants found by least square fitting the equation to the last part of the measured attenuation spectrum. The resulting extrapolation is seen in figure 2.15 as a dashed line.
2.10. BACKGROUND ATTENUATION

Figure 2.15: Background attenuation as a function of wavelength. Measured by a cutback of 980 m of a fiber with same configuration and co-dopants as the Tm1 fiber, but without thulium. Red curve shows extrapolation of IR-losses.

Figure 2.16: Blue fluorescence generated by pumping a thulium doped fiber at a wavelength of 790 nm.
2.11 Summary

All spectroscopic parameters of the \( ^3\text{F}_4, ^3\text{H}_6 \) transition of a thulium doped Al/La-silica and a Al/Ge-silica fiber have been obtained using simple and well-known techniques in analyzing rare-earth doped fibers. Absorption cross sections have been obtained using the method of saturated fluorescence and emission cross sections have been found by the gain tilt method. Finally, the rare-earth concentration has been estimated from the knowledge of the absorption cross section and the found concentration has been verified with that of a similar fiber doped with erbium instead. For both fibers it was found that the peak emission to peak absorption ratio was significantly below 1 and that the crossover wavelength of the absorption- and emission cross sections was in the range from 1725 – 1730 nm. Radiative lifetimes for the two fibers were estimated from integrated emission cross section spectra to be 6.0 and 6.6 ns, which reveals a radiative efficiency of the \(^3\text{F}_4 \) level of only 10\%, primarily due to the phonon quenching in silica. The parameters obtained may be used in further analysis of two-level amplifier and laser models for thulium doped silica fibers.
Chapter 3

Modeling DFB fiber lasers

3.1 Bragg gratings

Bragg gratings are well known devices in modern optical fiber technology. They appear in various devices in fiber laser technology, optical communication and signal processing. The fabrication technology is mature and the gratings are reliable and simple and the reflectivity can be controlled accurately over many orders of magnitude. The most profound advantage is however that the grating is formed inside the fiber core which avoids problems as coupling losses, handling and aligning problems. This is especially interesting for laser fabrication, since one can avoid internal cavity losses and hence lower the lasing threshold pump power or even obtain lasing where it by other means is impossible.

In its most basic form, the Bragg grating is nothing more than a narrow-stopband filter. It works by distributed reflections along its length and the peak reflectivity is at the wavelength where the resonance condition is fulfilled. The resonance condition is that the individual reflections are all in-phase. This means that \( \Delta \phi = n_e \frac{2 \pi}{\lambda} = 2 \pi \) \( \Rightarrow \lambda_b = n_e \lambda \). The reflectivity and transmission of an ideal Bragg grating of length \( L \) is given by [47]

\[
T = e^{i \beta_b L} \frac{\gamma}{\gamma \cosh (\gamma L) - i \delta \sinh (\gamma L)} \tag{3.1}
\]

\[
R^{\pm} = i e^{ \mp i \phi} \frac{\kappa \sinh (\gamma L)}{\gamma \cosh (\gamma L) - i \delta \sinh (\gamma L)} \tag{3.2}
\]

where \( \gamma = \sqrt{\kappa^2 - \delta^2} \) is an eigenvalue, \( \beta_B = n_e k_b = n_e \frac{\pi}{\lambda} \) is the Bragg wavenum-
Figure 3.1: (a): Transmission spectrum for a 5 cm long symmetrically discrete phase shifted grating. \( \kappa = 200 \frac{\text{pm}}{m} \). (b): Same as in figure a, but with negative distributed optical phase shifts instead. \( \kappa = 0 \) in the phase shifted region, which lengths are varied according to \( \Delta \phi = 2k\Delta n_{DC}L \).

The grating has a start phase of \( \phi \). When \( \gamma \) is real valued (\( \delta < \kappa \)), the grating is highly reflecting, i.e. \( R \to 1 \) as \( L \to \infty \) and if \( \gamma \) becomes imaginary (\( \delta >> \kappa \)), it is highly transparent, i.e. \( T \to 1 \). Thus the grating acts as an efficient stopband filter. It can be turned into a passband filter by coupling it to another device such as a coupler or a circulator.

To create a laser cavity from a Bragg grating, one has to introduce a phase shift, since the \( 2\pi \)-phase shift condition of a laser cavity is not fulfilled by the grating as seen from equation 3.2. A phase-difference \( \Delta \phi \) of \( \pi \) is needed, since \( R^+R^- \) has an intrinsic phaseshift of \( \pi (i \cdot e^{-i\pi}) \). A phase shifted grating has an ideal transmission spectrum as shown in figure 3.1, where it is seen that a sharp transmission peak appears inside the highly reflecting stopband. The phase shift is essentially a defect created within a highly periodic medium and this opens up a spectral band-gap inside the highly reflective spectrum. The phase shift can in practice be realized either as a discrete grating phase shift or distributed optical phase shift. In the case of an distributed optical phase shift, the phase shift has to be only \( \pi \) since the optical field traverses the phase shifted
region twice in a cavity round trip. Furthermore, an optical phase shift is also characterized as being either positive or negative depending on the sign change of the optical path length induced by the phase shift. Positive phase shifts allow a transmission peak originating from the negative wavelength detuning side to enter the stopband, whereas a negative optical phase shift allow a transmission peak to enter from the positive detuning side. A discrete grating phase shift can also be ±π, which is seen from equation 3.2, and more, each half-grating can also be π/2 phase shifted positively and negatively with respect to each other, but this is mere academic. In all, a grating phase shift of π or an optical phase shift of π/2 is necessary for lasing to occur in the center of the reflection band.

Fiber laser cavities based on Bragg gratings are often referred to as being either Fabry-Perot (FP), Distributed-Bragg Reflectors (DBR) or Distributed Feedback Reflectors (DFB). In a FP cavity, the grating lengths constitute a very small fraction of the total cavity length and as this fraction increases, the cavity is referred to as a DBR cavity. If the grating occupies a very large fraction of the total cavity length it is referred to as a DFB cavity. In the present study, focus has been on DBR/DFB cavities as these have the potential to provide lasing in a single longitudinal and transversal mode in a single polarization. This property is conveniently named single-frequency lasing.

3.2 Theory of a DFB fiber laser

The theory of a DFB fiber laser presented in the following section is based on recent work of Scott Foster [17, 18, 48] with a few corrections and modifications to emphasize issues related to DFB fiber lasers in the practical work [A3]. Until recently, all models and simulations of DFB fiber lasers have been made numerically with the inherent drawbacks of every numerical model, but as the work of Scott Foster has been published a new insight into the fundamental mechanisms of DFB fiber lasers are revealed. It is the intention with this section to review these fundamentals and use the results in modeling and comparison to a more comprehensive numerical model.

The Bragg grating is mathematically described by a sinusoidal variation in the susceptibility, χ(z), as

\[
\frac{k \chi(z)}{2} = k_0 + 2 \kappa \cos(2k_b z + \phi(z)) = \kappa_0 + \kappa(z) \left( e^{i 2(k_b z + \phi/2)} + e^{-i 2(k_b z + \phi/2)} \right),
\]

where \( \phi \) is the grating phase. The AC grating strength, \( \kappa \), is assumed constant in the following, which is often the case in practice. However, modulation of \( \kappa \)
3.2. THEORY OF A DFB FIBER LASER

as a function of $z$, which is known as apodization, can also be treated with this model [17], but has been left out for the sake of simplicity and overview.

The treatment of a Bragg grating in a singlemode fiber takes offset in the slowly varying amplitudes of the scalar electrical field propagating forward ($A$) and backward ($B$) in an infinite Bragg grating

$$e(z) = A(z)e^{-i(k_bz+\frac{\delta}{2})} + B(z)e^{i(k_bz+\frac{\delta}{2})}$$

(3.4)

$$\equiv [A(z), B(z)]$$

Using the Helmholtz equation and a susceptibility as in equation 3.3 gives the well-known coupled mode equations

$$\begin{bmatrix}
\frac{dA}{dz} \\
\frac{dB}{dz}
\end{bmatrix} =
\begin{bmatrix}
-i(\delta + \kappa_0 - \frac{1}{2} \frac{d\phi}{dz}) & -i\kappa \\
\kappa & -i(\delta + \kappa_0 - \frac{1}{2} \frac{d\phi}{dz})
\end{bmatrix}
\begin{bmatrix}
A \\
B
\end{bmatrix},$$

(3.5)

where $\delta = \frac{\omega}{c} - k_b$ is the detuning from the Bragg wavenumber.

If the grating parameters are constant and it is assumed that $A = e_f e^{i\beta z}$ and $B = e_b e^{i\beta z}$, then the matrix equation reduces to a simple eigenvalue problem by insertion in equation 3.5 to give

$$\begin{bmatrix}
\delta + \kappa_0 - \frac{1}{2} \frac{d\phi}{dz} & \kappa \\
-\kappa & -\delta - \kappa_0 - \frac{1}{2} \frac{d\phi}{dz}
\end{bmatrix}
\begin{bmatrix}
e_f \\
e_b
\end{bmatrix} =
-\beta
\begin{bmatrix}
e_f \\
e_b
\end{bmatrix}$$

(3.6)

The eigenvalues of the 2x2 matrix are found to be

$$\beta = \pm \sqrt{\delta^2 - \kappa^2},$$

(3.7)

which are inserted in equation 3.4 to give the fundamental mode of an infinite Bragg grating

$$+\beta : e^+(z) = e^{-\kappa \sigma z} [1, -(i\sigma + \theta)]$$

(3.8)

$$-\beta : e^-(z) = e^{\kappa \sigma z} [-\theta, i],$$

(3.9)

where $\theta = \frac{\delta}{\kappa}$ and $\sigma = \sqrt{1-\theta^2}$. The fundamental mode profiles are sketched in figure 3.2.

If for example, light is launched from $z = -\infty$ into an infinite grating then the mode-profile is given by equation 3.8 depicted as the green curve in figure 3.2. Each eigenmode consist of a forward and a backward propagating field, which at the resonance frequency, i.e. at $\delta = 0$, is of equal amplitude and in quadrature. This means that there is no net flow of energy anywhere in the grating at the resonance frequency.

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3.3. SYMMETRIC PHASE SHIFTED GRATING

![Diagram of symmetric phase shifted grating]

Figure 3.2: Finite Bragg grating with sketched mode profiles for the two fundamental eigenmodes $e^+$ and $e^-$ of an infinite grating. At the resonance wavelength $\lambda_0$ of an infinite grating there is no net flow of energy anywhere in the grating.

### 3.3 Symmetric Phase shifted grating

Now, suppose that two symmetric finite gratings of length $\frac{L}{2}$ are shifted in phase with respect to each other with a magnitude of $\Psi$ and assume for simplicity that the DC-grating strength $\kappa_0 = 0$. For an infinite grating there can be only one pure eigenmode in each half-grating, since the other diverges as $z \to \pm \infty$. However, for a finite grating there must exist a superposition of the two eigenmodes, since there cannot be any incoming field at the edges of the grating, i.e. $A(-\frac{L}{2}) = B(\frac{L}{2}) = 0$. Also because of symmetry there must exist a constant $C$ which relates the fields in the left and right gratings as $e_l = [A(z), B(z)]$ and $e_t = C[B(-z), A(-z)]$. As the total mode has to be continuous across the phase shift, this gives for $z = 0$

$$A(0) = \pm B(0)e^{i\frac{\Psi}{2}} \quad (3.10)$$

If the solution in the left half grating is assumed to be of the form $e_l = e^+ + re^-$, where the reflection coefficient, $r$, is small and that $e^+(0) = [1, \zeta]$ and
3.3. SYMMETRIC PHASE SHIFTED GRATING

e^- (0) = [ζ, 1], equation 3.6 gives

\[ \delta = -\frac{1}{2} \kappa (\zeta + \frac{1}{\zeta}) \]  \hspace{1cm} (3.11)  
\[ \beta = -\frac{1}{2} \kappa (\zeta - \frac{1}{\zeta}) \]  \hspace{1cm} (3.12)

and the boundary condition equation 3.10 gives at \( z = 0 \)

\[ \zeta = \frac{1 - re^{\frac{i\psi}{2}}}{e^{\frac{i\psi}{2}} - r} \]  \hspace{1cm} (3.13)

The small reflection coefficient has to be found from the boundary condition of no incoming field at \( z = -\frac{L}{2} \)

\[ 0 = e^{-\kappa \sigma \frac{L}{2}} ((-\theta + i\sigma)) + re^{\kappa \sigma \frac{L}{2}} \Rightarrow \]  \hspace{1cm} (3.14)
\[ r = (i + \theta)e^{-\kappa L} \]  \hspace{1cm} (3.15)

and substituting the reflection coefficient back into 3.13 while assuming that \( \theta = \frac{\pi}{2} \ll 1 \), which is a good approximation for the DFB fiber laser, and again inserting \( \zeta \) into equation 3.11 gives

\[ \delta = -\kappa \left( \cos \frac{\Psi}{2} - i2e^{-\kappa L} \sin^2 \frac{\Psi}{2} \right) \]  \hspace{1cm} (3.16)
\[ \beta = i\kappa \sin \frac{\Psi}{2} \]  \hspace{1cm} (3.17)

which corresponds to a decaying mode because of the detuning now being imaginary.

The spatial mode-profile is found by substitution back into equation 3.4 to give (to a leading order in \( \theta = \frac{\pi}{2} \))

\[ e(z) = e^{-\kappa \sin \frac{\Psi}{2} |z|} e^{-i \frac{\phi}{2}} \left[ \cos \left( k_{b} z - \frac{\pi}{4} \right) \right] \]  \hspace{1cm} (3.18)

The mode intensity profile is sketched in figure 3.3. Notice the very strong confinement of the laser field at the cavity phase shift. The distributed cavity has an effective length of \( L_{c} = \frac{\int e^{-2k|\zeta|} d\zeta}{\zeta} \approx \frac{1}{\zeta} \) which corresponds to the \( \frac{1}{2} \)-intensity points.

In the ideal case of \( \Psi = \pi \), the frequency of the light field is found from equation 3.16 to be

\[ \omega = \omega_{b} + i2\kappa e^{-\kappa L} \]  \hspace{1cm} (3.19)
3.3. SYMMETRIC PHASE SHIFTED GRATINGS

\[ I(z) \]
\[ \exp(-\kappa L) \]
\[ \exp(-1) \]
\[ \exp(-1) \]
\[ \exp(-\kappa L) \]
\[ I_\kappa = 1/\kappa \]

Phase shift \( \Psi \)

\[ z = -L/2 \quad z = 0 \quad z = L/2 \]

Figure 3.3: Longitudinal intensity profile of the fundamental mode in a DFB fiber laser. The cavity has a non-distributed equivalent length of \( L_c \).

From which a \( \frac{1}{e} \)-cavity intensity lifetime is defined as

\[ \tau_{1/e} = \frac{\tau_c}{2} = \frac{1}{2c\gamma} = \frac{e^{\kappa L}}{4c\kappa}. \]  
(3.20)

where \( \gamma = 2ke^{-\kappa L} \) is the cavity loss coefficient. To put things in perspective, a typical grating strength of \( \kappa = 200 \frac{1}{m} \) gives an effective cavity length of 5 mm and for a 5 cm long cavity the photon intensity cavity lifetime becomes approximately 0.1 \( \mu s \). It is noted here, that the cavity Q-value is given as \( Q = \frac{1}{L_c(\gamma + \alpha)} \), i.e. the number of photon round-trips before death. It is also interesting to see that if \( \Psi = \pi + 2\epsilon \) a fundamental laser equation can be formulated as

\[ \delta = \frac{\epsilon}{L_c} + i\frac{1}{c\tau_c}. \]  
(3.21)

where \( \epsilon \) is a small phase error. This laser equation is similar to a normal laser cavity equation which has been perturbed by a phase \( \frac{\epsilon}{L_c} \) and has a loss of \( \frac{1}{c\tau_c} \).
3.4 Active $\pi$-shifted grating

In a DFB fiber laser, the cavity is formed within the core of a fiber containing rare-earth dopants. This means that the cavity is distributed along the length of an active media and this has a significant influence on the properties of the laser. As seen above, the spatial mode is not uniform along the length of the cavity and the optical intensity is also varying fast along the fiber length. The gain at points along the cavity with a high optical intensity are depleted with respect to points of low optical intensity. This effectively sets up a so-called gain-grating or gain-modulation along the laser cavity, since the optical intensity is proportional to the electrical field as $I(z) \propto |e(z)|^2 \propto 1 + \sin(2k_b z + \Psi)$ and thus for an active medium, the susceptibility of equation 3.3 can be considered to have the form of

$$\frac{k\chi(z)}{2} = \kappa_0 + 2\kappa(z) \cos(2k_b z + \phi(z)) + i\gamma_0 + i2\gamma_1 \sin(2k_b z + \phi(z)), \quad (3.22)$$

where $\gamma_0$ is the DC-gain coefficient and $\gamma_1$ is the gain-grating coefficient coming from the intensity modulated gain. Using the form of the susceptibility from equation 3.22, a procedure like the one for the passive grating can be performed to find a new fundamental laser equation in the form of equation 3.21 as

$$\delta = \frac{\epsilon}{L_c} - i(\gamma_{avg} - (\gamma_c + \alpha)), \quad (3.23)$$

where the average gain coefficient $\gamma_{avg}$ has been introduced and the fiber background losses have been included through $\alpha$. Also now the effective detuning $\epsilon$ is more general taking into account phase and DC-index changes as

$$\epsilon = \int_{-L_c/2}^{L_c/2} \left( \frac{d\phi}{dz} - \kappa_0 \right) e^{-2\kappa|z|}dz \quad (3.24)$$

The average gain is the gain coefficients weighed by the spatial mode-profile as

$$\gamma_{avg} = \frac{\int_{-L_c/2}^{L_c/2} (\gamma_0 + \gamma_1)e^{-2\kappa|z|}dz}{L_c} = \int_{-L_c/2}^{L_c/2} \gamma|e_0(z)|^2dz, \quad (3.25)$$

where $\gamma$ is the local gain coefficient and $e_0(z)$ a normalized spatial mode-profile found from equation 3.18 as

$$e_0(z) = \sqrt{\frac{2}{L_c}} e^{-\kappa|z|} \cos(k_b z - \frac{\pi}{4}) \quad (3.26)$$

$$(e_0(z))^2 = \frac{1}{L_c} e^{-2\kappa|z|} (1 + \sin(2k_b z)), \quad (3.27)$$
3.5. RARE EARTH GAIN PROPERTIES

where \( c_0(z)^2 \) has also been given, since it is proportional to the optical intensity and will be used later on.

It is seen from equation 3.23 that the real part of the fundamental laser equation is not changed by the inclusion of the active silica doped fiber because of the very weak dependency of the refractive index on population modulations. The imaginary part is off course changed such that the optical field is now increasing with time if \( \gamma_{avg} > \gamma_c + \alpha \) and decreasing if \( \gamma_{avg} < \gamma_c + \alpha \), i.e. light amplification and lasing action is now possible. Thus, the equations describing an active DFB fiber laser has been briefly presented and the model now requires a link of these cavity equations to the equations describing the amplification in the fiber involving the rare-earth dopants.

3.5 Rare earth gain properties

The rare-earth system is modeled using the usual formalism of a two-level (or quasi three level) system of energy levels of an optically active medium. The basic parameters for thulium has already been discussed in chapter 2 and more is added to the general theory as the DFB laser model is applicable to all rare-earths having the basic assumptions of a two-level system fulfilled. The two-level system treated here has the general energy level structure shown in figure 3.4 where the relevant emission and absorption rates are indicated.

Refering to figure 3.4, a steady state inversion between levels 1 and 2 is found from the rates of transitions to be

\[
\frac{N_1}{N_0 + N_1} = x = \frac{P_a + S_a}{P_a + P_s + S_a + S_c + \frac{\tau}{\tau}},
\]

(3.28)

where \( N_{1,0} \) are the ions in the excited- and ground-state respectively and \( P \) and \( S \) are the pump and signal stimulated event rates, see figure 3.4. \( \tau \) is the fluorescence lifetime as defined in equation 2.7. The signal amplitude gain is given by an expression equivalent to equation 2.12 where amplified-spontaneous emission (ASE) has been neglected

\[
\gamma = \frac{1}{2} \left( x\sigma_{es} + (1 - x)\sigma_{as} \right) \Gamma_s \rho, \quad (3.29)
\]

where \( \rho \) is the rare-earth concentration and \( \Gamma_s \) is the signal confinement factor defined in equation 2.14. Neglecting ASE is an approximation, but the laser cavities are often short and ASE has low influence once the laser is above threshold. In situations where the gain is very limited, for example at the band edges of the
gain cross sections, there is a significant influence from ASE, which may deplete the available gain and thus hinder the laser to reach threshold. Therefore, the results derived from this laser model should always be evaluated with the influence of ASE in mind and of course other factors which are not included in the simple two-level model.

Considering a steady-state situation, the inversion of equation 3.28 is inserted into equation 3.29 to give

$$\dot{\gamma} = 2 \left( \frac{P_a + S_a}{P_a + P_e + S_a + S_e + \frac{1}{\tau}} \right) \sigma_{cs} - \left( \frac{1}{P_a + P_e + S_a + S_e + \frac{1}{\tau}} \right) \sigma_{as} \rho \Gamma_s, \quad \text{(3.30)}$$

where the $\gamma$-sign signifies steady-state conditions. One should pay attention to the factor 1/2, since all gain/loss coefficients until now are defined as amplitude coefficients, the gain derived from above has to be converted likewise in order not to mix up amplitude and intensity coefficients [A3]. Now, $S_a \sigma_{cs} = \sigma_{as} I \Gamma_s$, $\sigma_{cs}$ cancels $S_e \sigma_{as}$ and the gain equation reduces to

$$\dot{\gamma} = 2 \frac{P_a \sigma_{cs} - P_e \sigma_{as} - \frac{1}{2} \sigma_{as}}{P_a + P_e + \frac{1}{\tau}} \frac{(\sigma_{as} + \sigma_{cs}) \rho \Gamma_s}{1 + \frac{E^2}{P_s}} \quad \text{(3.31)}$$
3.5. RARE EARTH GAIN PROPERTIES

where \(|E|^2\) is the signal power in units \([W/m^2]\) and the saturation power \(P_s\) and the small signal gain (unsaturated) \(\gamma_t\) are given by

\[
P_s = \left( \frac{(\sigma_{ap} + \sigma_{ep}) P_p \Gamma_p + \frac{1}{\tau}}{\sigma_{es} \sigma_{as} \Gamma_s} \right) \frac{\hbar \omega_s}{\sigma_{es} \sigma_{as} \Gamma_s} \left( \frac{1 + \frac{P_p}{P_{sp}}}{(\sigma_{es} + \sigma_{as}) \Gamma_s} \right)
\]

(3.32)

\[
P_{sp} = \frac{\hbar \omega_p}{\tau}, \quad P_{p,s} = \frac{\hbar \omega_p}{(\sigma_{ap,s} + \sigma_{ep,s}) \Gamma_p \tau}
\]

(3.33)

\[
\gamma_t = \frac{1}{2} \frac{P_p \sigma_{es} - P_p \sigma_{ep} + \sigma_{as}}{P_p \Gamma_p + 1} \rho \Gamma_s = \frac{1}{2} \frac{P_p}{\rho \Gamma_p} \Gamma_p \sigma_{es} - \sigma_{ep} \rho \sigma_{as} - \frac{1}{\gamma} \sigma_{as} + \frac{P_p}{\rho \Gamma_p} \Gamma_p + \frac{1}{\gamma}
\]

(3.34)

where \(P_p\) is the pump power density in units \([W/m^2]\). The pump power is assumed constant throughout the laser cavity, which is a necessary assumption in order to complete the analytical model. This assumption is not fulfilled in practice, even though the laser cavity is only a few centimeters long, as will be demonstrated by a numerical model. Nevertheless, the assumption greatly simplifies the modelling and will still provide valuable information and understanding of the basic processes in a DFB fiber laser. Notice, that if the pump power is much higher than the threshold power, \(P_{th}\) and pump emission is negligible, then the small signal gain becomes \(\gamma_t \approx \sigma_{es} \rho \Gamma_s\), which is called the high pump power limit. However, this is only realistic with an erbium gain medium and not for a thulium gain medium because of the factor of 20 in the lifetime ratios.

A rate equation for the gain is also needed and following the derivation in appendix A.1, one may show that the time derivative of the gain is given by

\[
\frac{d\gamma}{dt} = \frac{1}{\tau_g} \left( -\gamma \left( 1 + \frac{|E|^2}{P_s} \right) + \gamma_t \right),
\]

(3.35)

\[
\tau_g = \frac{\tau}{1 + \frac{P_p}{P_{sp}}}
\]

(3.36)

where a new gain lifetime \(\tau_g\) is defined. It is seen that the steady-state gain of equation 3.31 is recovered from equation 3.35 if \(\frac{d\gamma}{dt}\) is set equal to zero.

This concludes the simple model of a two-level system which is the gain medium in the fiber laser. Relevant parameters for the particular rare-earth may then be inserted where appropriate. All the relevant parameters for thulium doped silica fibers were extracted from measurements in chapter 2 and they can now be used to model the behaviour of a DFB fiber laser. This is done in chapter
4. For now, the analysis continues with a general discussion of the outcome of the equations derived until now.

### 3.6 Laser rate-equations

The goal of the model is to be able to predict dynamical and steady-state conditions for a DFB fiber laser. Thus, rate-equations which involve time-derivatives of the amplitude and phase of the optical field are necessary and are derived in the following. Assume now an electric field as

\[ E(z, t) = e_0(z)A(t)e^{i(\omega t + \psi(t))}, \]  

(3.37)

where \( e_0(z) \) from equation 3.26 is a pure spatial function and \( A(t) \) and \( \psi(t) \) are purely time-dependent slowly varying functions. This separability of the electrical field in a spatial and a time varying function is a first-order approximation in \( \frac{d}{z} \) and is the basis of the dynamical model [18]. The time varying terms are collected in the phase of equation 3.37 as \( e^{i(\omega t + \psi(t) - i \ln A)} \) and as the time-derivative of the phase is the instantaneous frequency \( \omega \), the detuning \( c\delta = \omega - \omega_0 \) becomes

\[ c\delta = \frac{d\psi}{dt} - i \frac{1}{A} \frac{dA}{dt} \]  

(3.38)

and equating this to equation 3.23 and separating the real and imaginary parts gives

\[
\begin{align*}
\frac{d\psi}{dt} &= \frac{ce}{L_c} \\
\frac{dA}{dt} &= i c A (\gamma_{avg} - \gamma_c - \alpha). 
\end{align*}
\]  

(3.39, 3.40)

Together with equation 3.35, equation 3.39 and 3.40 form three first order differential equations describing the dynamics of DFB fiber lasers.

### 3.7 Steady-state

In a steady-state, all time derivatives are zero. From equation 3.40 this equates to the intuitive relation \( \gamma_{avg} = \gamma_c + \alpha \) which states that the average gain must equal the total losses. The average gain can be calculated from equation 3.31,
3.7. STEADY-STATE

3.37 and 3.25

\[
\gamma_{avg} = \int_{-\frac{L}{2}}^{\frac{L}{2}} \gamma |e_0|^2 dz
\]

\[
= \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{\gamma t}{1 + \frac{|E|^2}{P_c}} |e_0|^2 dz
\]

\[
= \gamma t \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{e^{-2\epsilon |z|} \frac{1}{L} (1 + \sin(2k_bz + \phi))}{1 + \frac{E^2}{P_c} e^{-2\epsilon |z|} \frac{1}{L} (1 + \sin(2k_bz + \phi))} dz
\]

\[
= 4\gamma t \frac{\ln \left( \frac{1 + \sqrt{1 + X}}{2} \right)}{X},
\]

where \(\hat{X} = \frac{2J^2}{\frac{1}{P_c}}\) is a normalized steady-state intensity. The calculations involved in reaching the final equation can be seen in appendix A.2. The normalized intensity \(X\) is an important parameter of a DFB fiber laser and can be viewed as a working point of the laser. Looking at the expression for \(\hat{X}\), it is seen that it is in fact just a ratio of inter-cavity laser power to the pump power. The expression for the average gain can be set equal to the losses, which introduces a gain threshold parameter \(\sigma_t\) as

\[
\sigma_t = \frac{\gamma_t}{\gamma_c + \alpha} = \frac{\hat{X}}{4 \ln \left( \frac{1 + \sqrt{1 + X}}{2} \right)},
\]

which is essentially the ratio of small signal gain to the total losses (not to be confused with a cross section). Equation 3.45 is very important, since it dictates the operating point, \(\hat{X}\), of the laser by locking it to the excess gain. However, \(\gamma_t\) is an increasing function of the pump power above threshold and likewise is \(\hat{X}\). Increasing the pump power above threshold results in an increase in the cavity laser power in order to keep the average gain fixed, as seen from equation 3.44. This corresponds to the situation in every laser, that the inversion is clamped to a certain value above threshold, determined by the signal laser power in the cavity. The expression in equation 3.45 can be expanded for small \(X\) to give (using a 2nd order expansion of \(\ln(1 + x)\) and \(\sqrt{1 + x}\))

\[
\hat{X} = \frac{8}{3} (\sigma_t - 1),
\]

47
which means that the threshold condition for a DFB fiber laser is that $\sigma_t > 1$, i.e. the small signal gain must equal the total losses. The expression for the pump threshold is then found to be

$$P_{p,th} = \frac{\hbar \omega_p}{\sigma_{as} \Gamma_p} \frac{\sigma_{as} \rho \Gamma_s + 2(\gamma_c + \alpha)}{\sigma_{as} \Gamma_p \sigma_{as} (\eta_s - \eta_p) \rho \Gamma_s - 2(1 + \eta_p)(\gamma_c + \alpha)}$$

(3.47)

$$= \frac{\hbar \omega_p}{\sigma_{ap} \Gamma_p} \frac{\sigma_{as} \rho \Gamma_s + 2(\gamma_c + \alpha)}{\sigma_{as} \rho \Gamma_s - 2(\gamma_c + \alpha)}, \quad (\eta_p = 0)$$

(3.48)

which is in fact a general threshold condition for rare-earth doped fiber lasers. Different lasers can have different thresholds which are reflected in the cavity losses $\gamma_c$. Intuitively, the equation makes sense, since the pump threshold increases with $\sigma_{as}$ and decreases with $\sigma_{cs}$. It also decreases with increasing pump absorption cross section and fluorescent lifetime. Yet, one must be aware that the contribution from ASE has been neglected in deriving the threshold pump power and that this will tend to raise the pump threshold in general.

In the high signal intensity regime, i.e. for $X >> 1$, it can be shown by recursive substitution in equation 3.45 that

$$X = 2\sigma_t \ln(2\sigma_t)$$

(3.49)

Substitution of $X$ and $\sigma_t$ and using that $e^{(X)} = \frac{2}{\ln(2)} e^{-\kappa L} = \gamma_s$ together with the fact that the total output power is given by $P_{out} = P_{out}(-L/2) + P_{out}(L/2) = |E(-L/2)|^2 + |E(L/2)|^2 = 2\gamma_c A^2$, the single-sided output power becomes

$$P_{out} = \frac{2}{L_c} e^{-\kappa L} \left[ L_c \gamma_s P_s \ln \left( \frac{2 \gamma_t}{\gamma_c + \alpha} \right) \right]$$

(3.50)

which is written in a way such that the parenthesis is the peak power in the cavity. In the remainder, $P_{out}$ always refers to the right-hand side output power. To understand the equation intuitively, $\frac{\gamma_t}{\gamma_c + \alpha}$ is the net gain and $P_s$ is a measure of how saturable the gain medium is. Insertion of parameters into equation 3.50 gives

$$P_{out} = \hbar \omega_p e^{-\kappa L} \left( \frac{\eta_s \sigma_{ap} \rho \Gamma_p \eta_s - \frac{1}{2}}{\eta_s + 1(\gamma_c + \alpha)} \right) \ln \left( \frac{2 \gamma_t}{\gamma_c + \alpha} \right),$$

(3.51)

which may be greatly simplified under strong pumping conditions ($P_p >> P_t$) to

$$P_{out} = \frac{\eta_s}{\eta_s + 1(\gamma_c + \alpha)} \frac{\omega_p e^{-\kappa L} \sigma_{ap} \rho \Gamma_p}{P_p} \ln \left( \frac{\sigma_{as} \rho \Gamma_s}{\gamma_c + \alpha} \right) P_p, \quad P_p >> P_t$$

(3.52)
3.7. STEADY-STATE

from which it is seen that the output power has a linear dependency on the pump power as is the usual case for a laser above threshold. It can be shown numerically, that this is also the case in the general situation. The linear slope is called the laser slope-efficiency. Once a laser is above threshold, the inversion and thereby the gain is clamped to a fixed value which is set by ratio of the pump and signal powers. If the pump power is increased above threshold, every absorbed pump photon is converted into an output signal photon, such that the inversion remains fixed. This also means that the rate of spontaneously generated photons is always the same for a laser above threshold.

Some interesting conclusions can be drawn from equation 3.52. First, the slope efficiency with respect to launched pump power above threshold is independent on the fluorescent lifetime, which is so because the easier saturated medium is counteracted by a higher net gain for an increasing lifetime. Second, the efficiency becomes increasingly insensitive to \( \eta_s \) as this ratio increases and the slope efficiency becomes more dominated by the absolute value of the emission cross section. And last, the slope-efficiency is nearly linear in the pump absorption \( \sigma_{ap} \Gamma_p \rho \), favoring a high concentration of dopants. However, the high pump power approximation should be used with caution, since it assumes \( \gamma_t \approx \sigma_{es} \Gamma_s \rho \), which has limitations as discussed before.

In the situation of zero background attenuation, the optimum grating strength can be shown to be infinite, since

\[
\alpha = 0 : \quad P_{\text{out}} = \frac{1}{2} \frac{\eta}{\eta + 1} \frac{\omega_s}{\omega_p} \left( \sigma_{ap} \Gamma_p \rho \right) \left( \sigma_{es} \rho \Gamma_s \right) \frac{1}{\kappa} \ln \left( \frac{\sigma_{es} \rho \Gamma_s}{2 \kappa e^{-\kappa L}} \right) P_p \quad (3.53)
\]

\[
\lim_{\kappa \to \infty} P_{\text{out}} = \frac{1}{2} \frac{\eta}{\eta + 1} \frac{\omega_s}{\omega_p} \left( \sigma_{ap} \Gamma_p \rho L \right) P_p \quad (3.54)
\]

However, equation 3.54 is never reached in reality and a quick conclusion would be that the slope efficiency may exceed 1, which is of course not practically possible. The reason is that the high power expansion \( X = 2 \tau_i \ln(2 \tau_i) \) is inaccurate for high \( X \). Also, when drawing conclusion from the equations, one should be cautious not to violate the model, as for example: \( \lim_{\kappa \to 0} \gamma_t \to 0 \), which is unphysical. But this is the same as having a grating strength such that the effective cavity length is longer than the physical length of the cavity, which is nonsense. This sets the limit that \( \kappa L > 1 \).

One could differentiate equation 3.52 with respect to \( \kappa \) to maximize the output power in the situation of finite background losses, but there is no analytical solution to this problem. The only general thing to conclude is that the dependency is like \( \kappa_{\text{opt}} = \frac{1}{\tau_c} \ln \left( \frac{\sigma_{ap}}{\sigma_{es} \rho \Gamma_s} \right) \), which is seen to be an im-
3.7. STEADY-STATE

... explicit equation in $\kappa$. The expression in the inner parenthesis typically has values around 0.5 for cavities and gain media in practice, which gives an estimate of the optimal grating strength of

$$\kappa_{opt} \approx -\frac{1}{L} \ln(\alpha L),$$

which is near the optimum value in many practical cases of interest. For the special case of $\alpha^* = 4 \text{ dB/km}$ and $L = 5 \text{ cm}$ gives a grating strength of $\kappa \approx 214 \text{ /m}$. One should here remember that the background attenuation coefficient, $\alpha$ used in the model is an amplitude coefficient and is thus related to the intensity background attenuation coefficient $\alpha = \alpha_i/2$. The amplitude background attenuation, $\alpha_i$ in the model is calculated from the more commonly referenced attenuation, $\alpha^*$, in units of dB/km as $\alpha = \frac{1}{2} \frac{\alpha^*}{10^{10 \log_{10}[e]}}$.

Optimization of the output power with respect to different parameters is not straightforward. The problem is that the output power is dependent on two many parameters and it it not possible to formulate the equation in a general way. However, it is still interesting to see some typical trends of the output power under the influence of certain important parameters. For this purpose, let's introduce a standard gain medium with the spectroscopic parameters, $\sigma_{sp} = 1.0 \times 10^{-25} \text{ m}^2$, $\sigma_{ep} = 0 \text{ m}^2$, $\sigma_{as} = 1.0 \times 10^{-25} \text{ m}^2$, $\sigma_{es} = 1.0 \times 10^{-25} \text{ m}^2$ and lifetime $\tau = 1 \text{ ms}$ and unit confinement factors and unit ratios for the frequencies for the signal and pump. Also the effective dopant area is set to $A_e = 1 \mu \text{m}^2$.

The laser with these parameters will be referred to as the unit laser. The parameters, $\kappa$, $L$, $\rho$ and $\alpha$ may then be varied and the single-sided output power $P_{out}$ may be plotted, typically for an input pump power of 100 mW. The standard gain medium parameters are chosen in a way to reflect laser parameters that are close to often encountered parameters for rare-earth doped fibers without reflecting any particular rare-earth. The standard medium is only introduced to draw some general conclusions about DFB fiber lasers which have parameter values of nearly the same magnitude as the standard medium. For the remainder of this chapter, all conclusions drawn are on the basis of the above introduced unit laser. When dealing with a particular rare-earth, the actual behaviour has to be found by numerically solving equation 3.51 with the correct parameters inserted, but simple rules of thumb can be also estimated by simple parameter scaling with equation 3.52 and the following results for the unit laser.

In the following, equation 3.51 is be solved for the standard medium to reveal some general trends for a DFB fiber laser, starting with a situation were the concentration is doubled from 1 to $2 \times 10^{25} \text{ m}^{-3}$, which is shown in figure 3.5 for
3.7. STEADY-STATE

(a) Output power for a standard gain medium DFB fiber laser as a function of grating strength for different background attenuations. $Pp = 100 \text{ mW}, \rho = 1 \times 10^{25} \frac{1}{\text{m}^3}$.

(b) Same as in left figure but with a dopant concentration of $\rho = 2 \times 10^{25} \frac{1}{\text{m}^3}$.

Figure 3.5: (a) Output power for a standard gain medium DFB fiber laser as a function of grating strength for different background attenuations. $Pp = 100 \text{ mW}, \rho = 1 \times 10^{25} \frac{1}{\text{m}^3}$.

(b) Same as in left figure but with a dopant concentration of $\rho = 2 \times 10^{25} \frac{1}{\text{m}^3}$.

A cavity length of 5 and 10 cm for zero and 4 dB/km background attenuation.

The low value of background attenuation of a standard optical fiber of 0.2 dB/km is not reached in a rare-earth doped fiber and a more realistic value of 4 dB/km is adopted. Looking at figure 3.5, stronger gratings are needed for smaller background cavity losses, however increasing the grating strength beyond a certain critical value causes the output power to decrease. In this situation, more and more of the radiation is trapped inside the cavity and cannot escape out of the ends of the cavity. Increasing background attenuation lowers this critical grating strength, since the cavity Q-value drops with increasing background attenuation. In practice, there exists a maximum in the possible inducable grating strength as a consequence of finite UV-absorption in a fiber, which means that the critical grating strength may not be seen in reality.

As also seen from figure 3.5, the slope efficiency of the unit laser more than doubles for double rare-earth concentration, otherwise leaving the optimal grating strength unchanged. One should also notice, that the simple relation for the approximate optimum grating strength in equation 3.55 estimate values of $\kappa_{opt} = 241 \frac{1}{\text{m}}$ for a 5 cm cavity and $\kappa_{opt} = 114 \frac{1}{\text{m}}$ for a 10 cm cavity with a 4 dB/km background attenuation, which agrees very well with the values read of
3.8 Asymmetric design

The preceding sections dealt with the symmetrically $\pi$-phase shifted grating. However, in practice asymmetries and distributed phase shifts are more common. The asymmetric DFB fiber laser can to a first order be treated with the above equations using a coordinate transformation. This is so, since the condition that the left and right going field components continuity across the phase shifted region is independent on the coordinate system. The boundary conditions that no fields are allowed to enter the cavity are also found by transformation and this provides a new expression for the cavity losses

$$\gamma_c = \frac{1}{2} (e (-L/2)^2 + e (L/2)^2) = \kappa e^{-\kappa (\frac{L}{2} + x)} + \kappa e^{-\kappa (\frac{L}{2} - x)}$$  

(3.56)

which is easily shown to give a cavity loss coefficient of

$$\gamma_c = 2\kappa e^{-\kappa L} \cosh(2\kappa x),$$  

(3.57)

where $-\frac{L}{2} < x < \frac{L}{2}$ is the displacement of the phase shift position relative to the grating center. That is $x = 0$ for a symmetric grating. It immediately follows that the threshold condition changes according to equation 3.48. The average gain will drop as $x$ increases, finally being halved for $x = \frac{L}{2}$, see equation A.22 in appendix A.2, but as the $x$-values considered in practice are relatively small this has minor significance. The first primary result of an asymmetry is the higher cavity losses and therefore increasing threshold pump power. But on the other hand, the output power has a local maximum, since as the cavity losses increase, so does the intensity at the side of the grating to which the phase shift is displaced. If the phase-shift position is shifted towards the right end, then the righthand side output power becomes

$$P_{\text{out}}(\frac{L}{2}) = |E(\frac{L}{2})|^2 = e^{-2\kappa (\frac{x}{2} - x)} P_0 \tilde{X},$$  

(3.58)

which at first seems to be an increasing function of the phase shift position $x$, but one must remember that $\tilde{X}$ depends on the cavity losses. This effect can be seen in figure 3.6, where $x$ is varied for two situations of cavity losses. It is seen that the slope efficiency has a local maximum in the $x$-direction. It looks as if the optimum phase shifts relative position is around 0.54 to 0.56. A situation in
3.8. ASYMMETRIC DESIGN

![Graphs showing output power and normalized slope efficiency for different cavity lengths and phase shift positions]

Figure 3.6: (a): Output power for a standard gain medium DFB fiber laser as a function of phase shift position relative to the cavity length. $P_p = 100$ mW, $\rho = 1 \times 10^{25}$ m$^3$. (b): Same as in left figure but with 4 dB/km background attenuation.

which both $\kappa$ and $x$ are varied at the same time is seen in figure 3.7 for different background attenuations. It is again seen that as the cavity Q-value increases, the role of the phase shift position becomes increasingly important.

However, finding the exact optimum phase shift position analytically is not possible. The distance $x$, which maximizes the right-end output power, is found by differentiating equation 3.52 with respect to $x$. As shown in appendix A.3 this leads to the implicit equation

$$e^{-2\kappa x} + \frac{\alpha e^{\kappa L}}{4\kappa} \ln \left( \frac{2\gamma_t}{\gamma_c + \alpha} \right) = \sinh(2\kappa x), \quad (3.59)$$

which must be solved numerically. This is done for a standard gain-medium with $\rho = 1 \times 10^{25}$ m$^3$ in figure 3.8 along with the improvement in the single-end output power relative to the case of a symmetrically $\pi$-phase shifted grating. Here it is first noticed that for relatively small background attenuations compared to the cavity losses, the optimum position is in fact relatively constant at about 0.55 to 0.57. For negligible background attenuation, one may find the optimum
Figure 3.7: Output power in mW for the standard gain medium DFB fiber laser for different lengths and background attenuations, $P_p = 100$ mW. A relative length $(\frac{x-L/2}{L})$ of 0.5 corresponds to a symmetrical $\pi$-phase shifted cavity. The graphs are the 2D version of the graphs shown in figure 3.6, but notice that the background attenuations should read 4 dB/km!
Figure 3.8: Blue curves: Optimum relative position of the phase shift for a standard gain medium DFB fiber laser, $p = 1 \times 10^{25}$ for zero background attenuation (Dark) and $\alpha^* = 4 \text{ dB/km}$ (light). Green curves: Single-end output improvement for optimal placement of the phase shift relative to a symmetrical positioned phase shift. Zero (Dark curve) and 4 dB/km (light curve) background attenuation.
phase shift position by neglecting the cosh-term in the cavity losses. This gives

\[ x_{\text{opt}} \approx \frac{1}{4\kappa} \ln \left( 2 \ln \left[ \frac{\gamma t}{\kappa e - \kappa L} \right] \right) \quad \alpha = 0, \quad (3.60) \]

which gives 0.55 and 0.54 for \( \kappa = 200 \) km for lengths 5 cm and 10 cm, respectively. Secondly, it is also seen from the green curves in figure 3.8 that the relative improvement is an increasing function of the grating constant, which increases drastically if the background attenuation becomes dominating. However, one should remember, that this does not signify higher output for increasing background attenuation, just that the improvement is higher in a high background loss fiber.

What remains, is to optimize the slope efficiency by choosing the best grating strength and phase shift position for a given cavity length. This is seen in figure 3.9, where the situation is shown for a background attenuation of 4 dB/km and slopes for the case of zero background attenuation, in which the curves are straight lines. Optimizing for the phase shift position becomes increasingly

Figure 3.9: Output power against cavity length. Blue curves show design optimized for grating strength \( \kappa \) and phase shift position \( x \), whereas red curves only are optimized for grating strength while keeping a symmetric cavity. Dotted lines indicate the slopes for zero background attenuation.
3.9. ALTERNATIVE PHASE SHIFTS

important as the length increases, which is best illustrated by the lower right figure of figure 3.7. This shows the importance of a low-loss cavity for a DFB fiber laser and using careful design, one may to some extent compensate more lossy fibers. The improvement in output power as the length is increased can also be read off figure 3.9, taking into account that the absolute values are for the unit laser but the trend being general. In general, the improvement in the slope efficiency when the length is changed from $L_1$ to $L_2$ under conditions of negligible background attenuation is given by

$$\frac{P_{\text{out}}(L_2)}{P_{\text{out}}(L_1)} = 1 + \frac{\kappa}{\ln \sigma_{t,1}} L_2,$$  \hspace{1cm} (3.61)

where $\sigma_{t,1}$ is the gain excess per unit length for a cavity length of $L_1$. The improvement is seen to be a strong function of the gain excess parameter, but no general conclusion can be drawn even in this situation with zero background attenuation. In theory, a longer laser would be better, but in practice the phase masks used to write the gratings become increasingly difficult to manufacture for longer lengths.

3.9 Alternative phase shifts

One may also have a distributed phase shift, in which the phase shift is optical (rather than a discrete phase shift in the grating) and has a finite extension. Typically, this is achieved by lowering or increasing the grating strength over a region from $z_1$ to $z_2$ causing an either negative or positive optical phase shift. In general, it can be shown that the intensity in the phase shifted region is proportional to [48]

$$|e(z)|^2 \propto \sin \left( \Delta \phi(z) - 2 \int_{z_1}^{z} \kappa \sin(\phi(z)) dz \right), \quad z_1 < z < z_2$$  \hspace{1cm} (3.62)

which in the special case of $\kappa = 0$ gives a constant intensity in the phase shifted region. The intensity in the rest of the cavity follows the usual $e^{-2\kappa |z|}$ dependency. Since the optical field has to be continuous at $z_1$ and $z_2$, the total phase shift has the constraint

$$\Delta \phi(z_2) - 2 \int_{z_1}^{z_2} \kappa \sin(\Delta \phi(z)) dz = \pi,$$  \hspace{1cm} (3.63)
3.9. ALTERNATIVE PHASE SHIFTS

![Graphs showing normalized intensity and inversion inside the cavity for different phase shifts](image)

Figure 3.10: (a): Normalized intensity inside the cavity for three different ways of realizing the phase shift. $\Delta n = 10^{-4}$ outside the phaseshifted region. Output powers are nearly identical. (b): Inversion inside the cavity for three different realizations of the phase shift as in figure a.

which for a symmetrically distributed optical phase shift of the form $\Delta \phi(z) = 2k\Delta n_{DC}(z - z_1)$ with constant grating strength is found to be

$$\Delta \phi(z_2) - \frac{2K}{k\Delta n_{DC}}[1 - \cos(2k\Delta n_{DC}z_2)] = \pi.$$  (3.64)

It is seen that if $\kappa \neq 0$ in the phase shifted region, then the optical phase shift $\Delta \phi(z_2)$ must be bigger than $\pi$ to fulfill the condition. This situation arises because the actual phase shift is weighted by the mode profile of the cavity.

More advanced designs are also possible, particularly in which the grating strength is non-constant throughout the cavity. This has been used to successfully demonstrate improved efficiency with up to a 57% increase in the single-sided output power as compared to an optimized constant-grating strength DFB fiber laser [49, 50]. The optimized cavity is designed such that the effective cavity length is increased beyond the usual $1/\kappa$ by forming a short length left grating section with a very high grating constant and a longer right grating section with a much smaller grating constant. Despite the proven success, this cavity design has problems associated with it. First, each half-grating has to be effective index matched in order to have the same resonance frequency. Sec-
3.10. NUMERICAL RESOLUTION

ond, the bandwidth of each half-grating has to be matched while at the same time keeping each half-grating $\kappa L$-product at a constant optimum value. The FWHM bandwidth of an ideal bragg grating is given by

$$\Delta\lambda_{dB} = \frac{\lambda_0^2}{\pi n_e L} \sqrt{(\kappa L)^2 + \pi^2}$$

(3.65)

All of these restrictions require a UV photosensitivity of the fiber that is linear and has a high saturation, i.e. linearity to be able to induce a DC-index in the low-grating strength half-grating and high saturation such that very high $\kappa$-values can be obtained. The bandwidth matching is not of concern for the actual lasing condition, but stable single-frequency operation may be jeopardized by the sidelobes of the smaller bandwidth grating which may couple to the high bandwidth grating. Despite these evident problems there has been nothing mentioned about them.

Yet, the idea of increasing the effective cavity length is of course very interesting and the correct way of achieving a better efficiency, since the DFB fiber laser only effectively uses the gain medium near the phase shift position. Using a longer distributed phase shift to increase the effective cavity length does not improve efficiency, since the signal mode envelope still follows the same $e^{-|z|}$ dependency, see figure 3.10a. So the idea of apodization of the grating strength is certainly interesting, but the efficiency improvements have to be weighed out by a more complex and difficult fabrication of the gratings.

3.10 Numerical resolution

The analytical model presented above of a DFB fiber laser is only a few years old. Previously, all modeling was performed with numerical computations based on the coupled mode equations and field propagation and with fairly good success. Analytical and numerical models each have pros and cons, often complementing each other. The analytical model provides basic understanding and quick conclusions of the behavior to simple parameter dependencies, whereas the numerical model show trends, however having the possibility of more complex and detailed investigations.

A very important issue regarding computational calculations is the verification of the computer code as misprints and errors are easily overlooked in the many lines of code. Usually, a simple analytical model is used for this purpose, but the exact opposite procedure will be adopted here. This may seem weird at first, but since the numerical programme for simulating DFB fiber lasers is
well-known and extensively tested and the fact that the programme is relatively simple offers the possibility to verify the analytical model. Fortunately, a numerical code used to simulate DFB fiber lasers including spatial hole-burning was programmed in the early stages of the project and was readily available at the publication of the analytical model.

The numerical programme is based on propagation of the electrical field using coupled mode equations [51]. The longitudinal advancement of the forward and backward propagating electrical field amplitudes are given by the matrix transfer as

$$
\begin{bmatrix}
A(z) \\
B(z)
\end{bmatrix} = T
\begin{bmatrix}
A(0) \\
B(0)
\end{bmatrix}
$$

(3.66)

where

$$
T =
\begin{bmatrix}
e^{-i\beta_0 z} \left( \cosh(\gamma_n z) + \frac{b_0 - i\Delta\beta_n}{\gamma_n} \sinh(\gamma_n z) \right) & \frac{b_1 - i\beta_n}{\gamma_n} e^{i(\beta_n z + \phi)} \sinh(\gamma_n z) \\
b_{-1} + ik e^{-i(\beta_n z + \phi)} \sinh(\gamma_n z) & e^{-i\beta_0 z} \left( \cosh(\gamma_n z) - \frac{i\Delta\beta_n - b_0}{\gamma_n} \sinh(\gamma_n z) \right)\end{bmatrix}.
$$

(3.67)

and

$$
\gamma_n^2 = \kappa^2 - \Delta\beta^2 + b_1 b_{-1} + b_0^2 - i(2\Delta\beta_n b_0 + \kappa(e^{-i\phi} b_{-1} - e^{i\phi} b_1)),
$$

(3.68)

where $\Delta\beta_n = \beta - \beta_n$ is the detuning from the Bragg wavenumber and $\kappa = \frac{\pi A_H}{k}$ is the usual grating strength. The spatial hole-burning (and also the Kerr effect) is introduced through the parameters $b_0, b_1$ and $b_{-1}$, which are derived in detail in [47, 51].

The routine starts by assuming an initial condition of $(A, B)_{L/2} = (0, B)$ and advances the amplitudes through the grating while updating the signal and pump intensities along the advancement. At the right-side end, the boundary condition states that $(A, B)_L = (A, 0)$, which the propagated field amplitudes must obey. If this is not the case, a new iteration starts with another initial condition chosen on the basis of multidimensional downhill simplex method [52]. This corresponds to zero-finding in the $(\lambda, B)$-plane with the function being transfer matrix equations of equation 3.67. Unfortunately, this zero can be highly localized and numerically difficult to find, which has to do with the fact that the routine actually tries to match two very steep exponential functions at the origin, remembering that the spatial mode profile behaves as $e^{-\kappa |z|}$. This may cause the routine to fail to converge for certain conditions.

The primary advantage of the numerical routine is that it may handle arbitrary grating designs like asymmetries, apodized- and chirped gratings. Furthermore, as the inversion is followed along the laser cavity, the pump intensity
3.10. NUMERICAL RESOLUTION

![Graphs showing output power vs relative length for different fiber parameters.](image)

**Figure 3.11:** (a): Comparison of analytical (solid blue) and numerical (red squares) results for the output power of an asymmetric standard DFB fiber laser. \( P_p = 300 \text{ mW, } L = 5 \text{ cm} \). (b): Comparison as in figure a), but as a function of grating strength for a symmetric cavity.

is correctly treated as opposed to the analytical model, where the pump power is assumed constant. These are the primary differences between the two models.

The purpose with the numerical routine is to compare it to the analytical model. This is done for the standard DFB fiber laser, which has negligible pump absorption, in figure 3.11 for a pump power of 300 mW. It is seen from the figure that the agreement is nearly perfect, both for the case of an asymmetric cavity and a cavity of varying strength. It is also noticed in figure 3.11a that the routine fails to converge as the asymmetry becomes too big for the reasons explained above. The similarity in the results prove that the main assumptions in the analytical model are in fact very good, i.e. the separability of the electrical field and ingoring terms of higher orders in \( \frac{1}{d} \).

It is of course expected that the analytical model fails to predict the output power in the case of significant pump power absorption and this has been illustrated in figure 3.12. It is seen that as the rare-earth concentration is increased, the pump absorption rises and the transmitted pump decreases, while the deviation between the two models become increasingly evident. A future improvement of the model could be to include a dependency of the pump power through the cavity, which would approximate the real situation. However, it
Figure 3.12: Output power and pump transmission as a function of rare-earth density, $\rho$ for a standard DFB fiber laser as obtained by the analytical model (solid blue) and the numerical model (squares). $\sigma_{ap} = 2 \times 10^{-25}$ m$^2$ and $P_p = 100$ mW. The similarity decreases as the pump absorption increases.
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must be emphasized that this has not been considered in further detail and difficulties are certainly expected to arise in the derivations, not least in calculating the average integral of equation 3.42.

Despite the high pump absorption problem of the analytical model, it still has great potential in simulating DFB fiber lasers. This is primarily due to the ease-of-use of an analytical model as compared to a time-consuming and in-flexible numerical routine and the ability to quickly predict simple results just by looking at the equations. It holds in almost all practical cases that the simplest method prevails and this will probably also be the case for simulations of DFB fiber lasers.

The simulations of thulium doped DFB fiber lasers in this project are primarily performed with the numerical model and only in certain cases the analytical model will be used. This is because of a non-negligible pump absorption of the thulium doped lasers.

3.11 Relaxation oscillations

The laser dynamics are also interesting to analyze as they determine the laser noise properties. In particular, the dynamics of the gain medium and of the laser field amplitude are of interest and their mutual coupling. If either variable is disturbed, the laser will exhibit relaxation oscillations with a characteristic frequency \( \nu_0 \), which will now be derived. In order to treat this, the steady-state variables are allowed to vary in time through small disturbances as

\[
A(t) = \hat{A}(1 + \varepsilon(t)) \\
\gamma = \hat{\gamma} + \eta_n(t) \\
\gamma_{avg}(t) = \hat{\gamma}_{avg}(1 + h(t))
\]

where \( h(t) = \frac{1}{\gamma_{avg}} \int \eta_n|e|^2 dz \) and all steady-state variables cancel. The differential equation for the amplitude \( A \) is then found from equation 3.40

\[
\frac{1}{A} \frac{dA}{dt} = \frac{de}{dt} = c(1 + \varepsilon)(\hat{\gamma}_{avg} + \hat{\gamma}_{avg} h - (\alpha + \gamma_c)) \quad (3.72)
\]

\[
= c(1 + \varepsilon) \hat{\gamma}_{avg} h \quad (3.73)
\]

\[
\approx \frac{h}{\tau_e} \quad (3.74)
\]

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where \( \tau_c = \frac{1}{c(\alpha + \gamma_c)} = \frac{1}{c\gamma_{avg}} \). The gain differential equation is found from 3.35

\[
\frac{d\gamma}{dt} = \frac{1}{\tau_g} \left[ -(\gamma + \eta) \left( 1 + \frac{\hat{A}^2(1 + 2\varepsilon)|\varepsilon|^2}{P_s} \right) + \gamma \right]
\]

(3.75)

\[
= \frac{1}{\tau_g} \left[ -\eta \left( 1 + \frac{\hat{A}^2|\varepsilon|^2}{P_s} \right) - \frac{2\hat{A}^2\varepsilon^2}{P_s} \right],
\]

(3.76)

where 2nd order contributions have been neglected. Fourier-transforming both sides of the above equations and using that \( \mathcal{F}(\frac{d}{dt}) = iv \) where \( \nu \) is the radial frequency and inserting for \( \hat{\gamma} \) gives

\[
iv \varepsilon(\nu) = \frac{1}{\tau_c} h(\nu)
\]

(3.77)

\[
\eta_0(\nu, z) = -\frac{2\hat{A}^2\gamma_0|\varepsilon|^2(\nu)}{P_s} \left( \frac{1}{i\nu \tau_g + 1 + \frac{\hat{A}^2|\varepsilon|^2}{P_s}} \right) \left( 1 + \frac{\hat{A}^2|\varepsilon|^2}{P_s} \right)
\]

(3.78)

Through partial decomposition, the last equation can be written as

\[
\eta_0(\nu, z) = -\frac{2\gamma_0 \varepsilon}{i\nu \tau_g} \left( \frac{1}{1 + \frac{1}{2} \frac{1}{\nu \tau_g + 1} L_c |\varepsilon|^2} \right) \left( 1 + \frac{1}{2} x L_c |\varepsilon|^2 \right),
\]

(3.79)

and is now written in a form which can be integrated to eliminate the \( z \)-dependency by multiplication with \( |e(z)|^2 \) and integrating as was done in equation 3.42. It is helpful to introduce a new monotonically decreasing function as

\[
G(x) = \int_{-\frac{x}{2}}^{\frac{x}{2}} \frac{|e(z)|^2}{1 + \frac{1}{2} x L_c |e(z)|^2} \, dz = 1 - \frac{3}{8} x, \quad |x| < 1,
\]

(3.80)

where the small argument expansion is to 2nd order. The definition of \( h(t) \) is then used with equation 3.79 to give

\[
h(\nu) = -\frac{2\gamma_0 \varepsilon}{i\nu \tau_g \gamma_{avg}} \left( \frac{\hat{X}}{i\nu \tau_g + 1} \right) G \left( \frac{\hat{X}}{i\nu \tau_g + 1} \right),
\]

(3.81)

where \( \nu_0^2 = \frac{2\gamma_0 \varepsilon}{\tau_c} \) is a small signal relaxation oscillation frequency, as will be seen shortly. The two equations 3.79 and 3.81 in the two unknowns \( \eta(\nu) \) and \( h(\nu) \) have non trivial solutions if the determinant vanishes, i.e.

\[
\nu^2 - \nu_0^2 G \left( \frac{\hat{X}}{i\nu \tau_g + 1} \right) + \nu_0^2 G(\hat{X}) = 0.
\]

(3.82)
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The solutions to this polynomium can be shown to be, see appendix A.4

\[ \nu_\pm = \pm \nu_0 + i \frac{3X}{16\tau_g}, \] (3.83)

where the double inequality \(1 << X << \nu_0\tau_g\) must be valid.

If the solutions to the above equations in general are \(\nu_\pm = \pm \nu_r + i \nu_i\), then it can be shown that the time-dependency of the gain and amplitude disturbances are given by

\[ \epsilon(t) = A_0 \cos(\nu_r t + \phi_0) e^{-\nu_i t}, \] (3.84)

\[ h(t) = A_0 \tau_c \cos(\nu_r t + \phi_0 - \tan^{-1}(\frac{\nu_r}{\nu_i})) e^{-\nu_i t}, \] (3.85)

which means that the amplitude and gain disturbances are exponentially damped oscillations which are out of phase by \(\Delta\phi = -\tan^{-1}(\frac{\nu_r}{\nu_i})\). It is now clear why \(\nu_0\) is called the small signal relaxation oscillation frequency. The system behaves as an underdamped harmonic oscillator!

An underdamped harmonic oscillator is characterized by a damping coefficient, \(\zeta\), which has a value between 0 and 1. When \(\nu_r\) approaches zero, the system behaves like a critically damped system for which \(\zeta = 1\). The overdamped situation where \(\zeta > 1\) is never reached since this requires two different complex poles, which is seen never to be the case. The damping coefficient and the damped resonance frequency is related to the angle and magnitude of the poles. Converting to the Laplace domain, where the poles are \(s_\pm = i\nu_\pm = -\zeta\omega_n \pm i\omega_n\sqrt{1 - \zeta^2}\), the damping ratio and damped resonance frequency becomes

\[ \zeta = \sin\left(\frac{\angle s_\pm - \frac{\pi}{2}}{2}\right) \] (3.86)

\[ \nu_r = |s_\pm| \sqrt{1 - \zeta^2}, \] (3.87)

which means that they can be visualized in a figure showing the poles location in the negative complex plane. This is seen in figure 3.13 for a varying grating strength for the unit DFB fiber laser. The poles have a loop-like behavior in the complex plane starting at the origin. Increasing the grating strength increases the angle to the imaginary axis and likewise does the damping coefficient, which leads to the conclusion that a low-noise laser should be fabricated with as high as possible grating strength. In practice, however, a very high Q-cavity may cause higher order modes to start lasing. It is also seen that both the resonance

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Figure 3.13: (a) Real and imaginary part of the complex roots of the transfer function as a function of grating strength, 100 – 300 km. The pump power is 100 mW (-) and 40 mW (...). The arrows indicate the direction for increasing grating strength. The symmetry about the real axis is left out for clarity. (b) Resonance frequency $\nu_r$ and damping coefficient $\zeta$ as a function of grating strength. Both figures are for the unit laser.
3.12. NOISE SOURCES

frequency and the damping coefficient rises with increasing pump power. These trends have been showed previously and are also found in practice[53].

In a more general situation, where the amplitude or gain are disturbed (or driven) by functions $F_z(t)$ or $F_h(t)$, the transfer functions for the amplitude variable in the Laplace domain is

$$
\varepsilon(s) = \frac{s}{s^2 + \nu_0^2 \Gamma(s)} F_z(s) + \frac{1}{s^2 + \nu_0^2 \Gamma(s)} F_h(s),
$$

(3.88)

where $s = \nu$ is the Laplace variable and $\Gamma(s) = G \left( \frac{\bar{X}}{1 + \tau_{eq}} \right) - \bar{G}(\bar{X})$. Here it is also seen that the solutions to the eigenvalue equation 3.82 are the poles of the transfer function. The transfer functions of the DFB fiber laser can be used to improve a control-loop design, for example the pump power, in order to decrease the influence this has on the intensity noise properties of the laser. This is actually already used with success in practice, where a simple PID regulator is coupled to the pump laser drive current to form a closed-loop design[54].

3.12 Noise sources

Noise sources can be roughly classified into four main groups. Pump induced noise, spontaneous emission noise, thermal and acoustic noise. The two first noise terms are actually the above introduced disturbance functions $F_z$ and $F_h$ respectively, which both induce noise to the laser amplitude and frequency. The thermal and acoustic noise term only effects the laser frequency to a first order, where they together dominate other noise mechanisms. Acoustic vibrations causes frequency noise which is a complete study of its own and carefully acoustic damped designs can make the difference in achieving low-noise lasers[55]. Acoustic noise is very complex and depends to a great extend on the physical packaging of the fiber laser, where some applications even use the fiber laser as an acoustic sensor, in which the laser frequency is made highly sensitive to vibrations through the use of clever designed packaging. Acoustic noise is not treated any further.

Other noise terms such as nonlinear effects, ion-clustering, upconversion and heating effects in the cavity are also important, but they are likewise not treated here, despite the importance they have for the laser noise characteristics.

The first noise parameter to consider is the intensity noise of the laser. The electrical current power fluctuation spectrum is defined as

$$
S_p(\nu) = 4P_{\text{out}}^2 \langle |\varepsilon(\nu)|^2 \rangle,
$$

(3.89)
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where $<>$ is the ensemble average and a unit response photodiode is assumed. The two contributions to intensity fluctuations are pump power fluctuations, $P_p + \Delta_p(t)$ and spontaneous absorption or emission events which disturb the susceptibility as $\chi(t) = \Delta_c(t) + i \Delta_s(t)$. Without further justification, the power fluctuation spectrums for spontaneous emission noise and pump power fluctuations can be shown to be given as

$$S_p^{\text{Sp}}(\nu) = \frac{16\epsilon_0 c \omega_{\nu} \eta_i \mu^2}{\pi \nu^2 - \nu_0^2 \Gamma(\nu)^2} P_{\text{out}}^2$$  \text{Spontaneous} \quad (3.90)$$

$$S_p^{\text{P}}(\nu) = \frac{|\Delta_p(\nu)|^2}{P_p^2} \frac{\nu_0^2 \Gamma^2(\nu)}{|\nu^2 - \nu_0^2 \Gamma(\nu)|^2} P_{\text{out}}^2$$  \text{Pump,} \quad (3.91)$$

where $\eta_i = \frac{\sigma_{nc}}{\sigma_{nc} + \sigma_n} (1 + c \tau \sigma_{ns} \rho \Gamma_s)$, $c$ is the speed of light, $\Delta_p(\nu)$ is the pump power fluctuation spectrum and $\nu$ is the radial frequency. One should note here, that a correction has been appointed to equation 3.91 [A3] (see appendix A.5), which is different from the equation used in [18].

A common figure of merit for intensity noise is the Relative Intensity Noise (RIN) which is defined as the square of the output power normalized to the average square of the output power as

$$RIN_i = \frac{<|\Delta P|^2>}{P_{\text{avg}}^2} \approx 4 \frac{<\epsilon(\nu)^2>}{P_{\text{out}}^2} = \frac{S_p(\nu)}{P_{\text{out}}^2},$$  \quad (3.92)$$

typically specified in a 1 Hz bandwidth. A more practical measure of laser noise is the relative noise, which is the ratio of the RIN of the laser itself to the RIN of the pump laser, i.e.

$$RN = \frac{RIN_i}{RIN_p},$$  \quad (3.93)$$

where $RIN_p = \frac{|\Delta_p|^2}{P_p^2}$. The relative noise RN is a measure of the amplification of the RIN from the pump to the laser, which is easily identified in equation 3.91. The DC asymptotic of RN is then easily seen to be 1, which is a reasonable result when considering a laser above threshold has a linear ($P_p, P_{\text{out}}$)-curve. Thus, low frequency oscillations of the relative pump power are directly transferred to the relative laser output power.

Frequency noise will only be given here as a reference, since there has not been any detailed studies of the linewidth or frequency noise properties of the thulium doped lasers in this project. Furthermore, as mentioned, the very important contribution from acoustic noise is very difficult to analyze, however for
3.12. NOISE SOURCES

Reference, the most important frequency noise terms will be given. Frequency
noise, which stems from time variations in the phase of the laser field, originating
from spontaneous-, pump- and thermal noise events can be shown to be
[18, A3]

\[
S_v^s (\nu) = \frac{16 \epsilon_0 \nu \nu_c \eta}{\pi A^2} (1 + \alpha_g^2) \quad \text{Spontaneous} \tag{3.94}
\]

\[
S_v^p (\nu) = \frac{\nu^2 \nu_c^2 |\Delta_p(\nu)|^2}{P^2_s |\nu^2 - \nu_c^2 \Gamma(\nu)|} \quad \text{Pump} \tag{3.95}
\]

\[
S_v^t (\nu) = \frac{\omega^2 g k_b T^2 f_T(\nu)}{c A_m L_c} \quad \text{Thermal}, \tag{3.96}
\]

where \(\epsilon_0\) is the vacuum permeability, \(k_b\) is the Boltzmann constant, \(T\) is the
temperature, \(A_m\) is the mode-power area in the fiber, \(\alpha_g\) is the linewidth
enhancement factor and \(q\) is the sum of the temperature gradients of the refractive
index and strain of silica, which has a value of approximately \(2.7 \times 10^{-11} \frac{1}{K}\)
[56]. The function \(f_T(\nu)\) is a normalized form function, which has been derived
by Wanser to [57]

\[
f_T(\nu) = \frac{A_m}{2 \pi D} \ln \left[ \frac{\left( \frac{\nu}{\pi d_m} \right)^2 + \left( \frac{\pi d_m}{\nu} \right)^2 + \left( \frac{\nu}{2 \pi d_f} \right)^2}{\left( \frac{\nu}{2 \pi d_f} \right)^2 + \left( \frac{\pi d_f}{\nu} \right)^2 + \left( \frac{\nu}{2 \pi d_f} \right)^2} \right], \tag{3.97}
\]

where \(D \approx 8 \times 10^{-7} \frac{m^2}{s}\) is the diffusivity of silica, \(d_f\) and \(d_m\) are the fiber
diameter and mode-power diameter, respectively. In the frequency spectrum below
the resonance frequency of the laser, thermal noise will dominate the spontaneous
noise term by more than 30 dB, but as the frequencies become bigger than the
relaxation frequency the situation reverses, since the spontaneous noise term is
white and the thermal noise rolls of with approximately 20 dB/decade.

The frequency noise coming from the linewidth enhancement factor is unknown.
The reason is because of a small link in silica fibers between the refractive index
and the carrier density, which is well known in semiconductor materials. This link
is known as the linewidth enhancement factor \(\alpha_g = \frac{\delta n_r}{\delta n_i}\),
where \(n_r\) and \(n_i\) are the real and complex parts of the refractive index. The
actual value has never been measured in rare-earth doped silica fibers and only
very few approximative guesses have been made on its actual value. In a DFB
noise study, it was estimated that it had a value up to 2-4 at 1550 nm [36] which
is comparable to the value in semiconductors, but the value was only based on
very weak indications.

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3.13 Summary and outlook

The analytical model for a DFB fiber laser has been outlined in a short form with the intention of keeping focus on the main subject of an intuitive understanding of the laser characteristics. The model is very instructive and provides expressions for the output power, threshold pump power and noise properties. The analytical model has been used to predict trends of a general DFB fiber laser and optimized cavity parameters and an extension to include asymmetric cavities has been put forth. The analytical model has pros and cons and the useability of the model depends to some extend on the fact that the pump power is treated as a constant parameter, which is a weak assumption for DFB fiber lasers. One could for example solve the equations with constant pump power and then calculate the pump power absorbed from the obtained knowledge of the average gain. This could recursively lead to the correct operating point, $X$, of the laser. Future developments of the analytical model should concentrate on the pump power absorption for the model to be successful for short cavity DFB fiber lasers. Finally, noise properties of DFB fiber lasers have been reviewed and it was found that the dynamics of the laser can be modeled by a simple damped harmonic oscillator in which the grating strength is a very important factor in the damping coefficient.
Chapter 4

Theoretical Tm:DFB lasers

The work presented until now has aimed at establishing a theoretical model for thulium doped DFB fiber lasers. Simulations of Er and Er/Yb-DFB fiber lasers are numerous [58, 59, 60], while simulations of thulium doped lasers have only been performed on standard Fabry-Perot (FP) lasers relying on spectroscopic data of questionable accuracy [31]. There are two simple reason for this, one being that no one has previously demonstrated a thulium-doped DFB fiber laser and another that accurate and reliable spectroscopic data for thulium doped silica fibers have not been available. With the present work, an effort is made to change this.

Simulations will be performed with both the analytical and the numerical model, since the pump absorption in the relevant thulium doped fibers is relatively high. To avoid confusion, all results displayed in figures obtained by the analytical model are represented by solid lines, whereas results obtained by the numerical model are displayed by lines marked with crosses.

4.1 Laser performance

The simulations strive to reflect real laser performance. This is achieved using the fiber parameters just studied in chapter 2. This means that the study is limited to Al/La- and Al/Ge-codoped silica fibers with the effective doping areas and confinement factors as before, which are immediately usable in both the analytical and the numerical model. However, it is believed that thulium doped silicate fibers with different codopants are also well modelled by these
Figure 4.1: (a) Pump threshold as a function of laser wavelength for a pump wavelength of 1596 nm for a symmetric $\kappa L = 10.78$ cavity (b) Cavity value of $\kappa L = 4.9$.

parameters. It must be remembered that every model is an idealization of the real world and assumptions are necessary. The results presented here assume a perfect $\pi$-phase shifted cavity, no UV-induced losses, a perfect two-level laser model and no perturbations from the thermal effects etc.

### 4.1.1 Threshold condition

The first thing to simulate is the pump power necessary to reach threshold. As recalled, threshold is reached at a pump power, where the small signal gain equals the total cavity losses, explicitly given by equation 3.47. To keep things simple, the grating strength is kept constant at a value of $220 \, \text{1/m}$ which is near the optimum value according to equation 3.55 and the displacement of the $\pi$-phase shift is chosen to be $+3.5$ mm for a $5 \, \text{cm}$ cavity, according to equation 3.59. The background attenuation is found from figure 2.15 and the emission and absorption cross sections are from figures 2.10 and 2.11. It is assumed that the background attenuation is identical for the two fibers, despite the fact that it has only been measured for the Tm1 fiber. The threshold pump power, at a convenient pump wavelength of 1596 nm, as a function of laser wavelength then looks as in figure 4.1. As expected, the pump threshold drops quickly with
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Increasing laser wavelength as a consequence of the increased ratio of emission to absorption cross section, which is so since the background attenuation is negligible up to wavelengths around 2000 nm. For this low-loss fiber it is thus not the background attenuation that limits performance even up to wavelengths of 2000 nm. It is also noticed that the Tm2 fiber generally has a lower threshold, which is due to the higher pump absorption at the specific pump wavelength as compared to the Tm1 fiber. Another very interesting point is that it should be practically possible to achieve lasing at a wavelength as low as 1650 nm, where the ratio of emission to absorption cross section is as low as 0.4 – 0.5.

As expected, it is also seen by comparing figure 4.1a and 4.1b that the cavity strength has a large influence on the threshold condition. The reason why a grating strength of 100 $1/m$ has been chosen is that it reflects the value typically found in real fabricated laser cavities. Even though a higher grating strength is seen to be much better, there is in reality a limit to how high values of index changes that are possible to reach with the UV-sensitivity of the fibers. With the less confined cavity configuration of figure 4.1b, it is seen that lasing should only be expected above wavelengths of 1.7 $\mu$m and below 2.0 $\mu$m. Yet, reaching high grating strengths is not a physically fundamental problem, which is why one may expect improved performance as fibers with better UV-characteristics are developed.

It is also important to be able to predict the optimum pump wavelength. The immediate conclusion, that the higher the pump absorption, the lower the pump threshold and the higher the slope-efficiency is confirmed by figure 4.2 and 4.3. It is seen from the figure that choosing the optimal pump wavelength becomes less significant with respect to the threshold pump power as the signal emission to absorption cross section ratio increases. On the other hand, the effect of optimal pump wavelength has big influence on the output power in this respect, such that the optimum pump wavelengths in all cases fall between 1620 – 1630 nm. In reality, there is of course a trade-off, since not all wavelengths are available from present diode pumps. The pump module in this project is a L-band Erbium doped fiber amplifier (EDFA) pumped by 980 nm and 1480 nm diodes and seeded with a high power DFB diode laser, which will be treated in detail in chapter 5.5. The output power of such a system is not constant with wavelength because of the erbium gain curve and maximum output power is obtained at a wavelength of 1585 nm. Hence, a trade-off between available pump power and thulium pump absorption has to be made, such that the output power is maximized. This has also been illustrated in figure 4.2 and 4.3 by the light green curve, which assumes an EDFA pump-module output power scaled to 100 mW at 1595 nm. It is observed that the optimal
Figure 4.2: (a) Pump threshold and forward output power as a function of pump wavelength at a signal wavelength of 1740 nm for Tm2 fiber with optimal cavity. Dark green curve is for a constant pump power of 100 mW, while light green curve is for the output power of the EDFA pump module normalized to 100 mW at 1595 nm. (b) Signal wavelength of 1850 nm.
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Figure 4.3: (a) Pump threshold and forward output power as a function of pump wavelength at a signal wavelength of 1740 nm for Tm1 fiber with symmetric $\kappa = 100 \, \frac{1}{m}$, $L = 49$ mm. Dark green curve is for a constant pump power of 100 mW, while light green curve is for the output power of the EDFA pump module normalized to 100 mW at 1595 nm. (b) Signal wavelength of 1850 nm.
Figure 4.4: (a) Forward output power as a function of wavelength at a pump wavelength of 1596 nm for $P_p = 100$ mW and optimal cavity configuration. (b) Pump transmission as a function of signal wavelength for the Tm1 fiber for a pump wavelength of 1596 nm.

The output power as a function of laser wavelength for a pump wavelength of 1596 nm is seen in figure 4.4. The laser operates most efficiently at a wavelength around the peak of the emission cross section as seen from the figure. Beyond 2065 nm lasing is theoretically impossible, since it was found in the spectroscopy measurements that ESA was present beyond this wavelength, which resulted in negative small signal gain. At the low wavelength side, it is seen that the output power becomes very modest below 1650 nm.

The magnitude of the output power found from the analytical model is not correct since the pump absorption becomes non-negligible for practically all laser wavelengths at the specific pump wavelength. The relatively high ion-concentration causes significant pump absorption along the cavity as seen in figure 4.4b from a numerical simulation. The correct output power, when tak-
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Figure 4.5: (a) Forward output power at 1850 nm as a function of cavity length for \( x = 0 \) at a pump wavelength of 1596 nm for \( P_p = 100 \) mW. (b) Asymmetry introduced by \( x = 0.07 \times L \).

Introducing pump propagation into account, is also marked in figure 4.4a for the Tm1 fiber. Unfortunately, this prevents the use of the analytical model in simulating thulium doped DFB fiber lasers.

The optimum grating strength of 220 \( 1/m \) used until now relates to the cavity lengths and length of a symmetric laser. Changing the length would increase the output power, provided that the grating strength was adjusted due to the presence of background attenuation. This dependency is seen in figure 4.5 for the Tm1 fiber. For a 5 cm cavity, the predicted optimum cavity strength is close to the optimum, as the crest of the 220 \( 1/m \)-curve is close to the 5 cm. If the length is increased to just 7 cm, the output power becomes zero as the photons within the cavity performs too many roundtrips compared to the losses experienced. Nevertheless, one may improve the output power by using a longer length cavity, but with a lower grating strength.

Introducing an asymmetry of \( x = 0.07L \), which corresponds to a relative asymmetry of 0.57, drastically improves the output power from one end. This is seen by comparing figure 4.5a and 4.5b. The output power for a 5 cm cavity is nearly doubled and the grating strength can be increased since the crest of the black curve is now at 5 cm. The relative improvement in the output power when increasing the length for a symmetric and an assymetric is however nearly
Figure 4.6: (a) Output power as a function of launched pump power for a symmetrical cavity of strength 100 \( \frac{1}{\mu m} \) for 49 mm of Tm2 fiber. The line with diamond points is measured. (b) Same, but for the Tm1 fiber.

### 4.1.3 Theory vs. Experiment

As mentioned previously, it is the objective of every theoretical simulation to reflect real life experiments and with the results obtained above, it would undoubtedly be fortunate if the model predicted the actual laser behavior, as for example a thulium DFB laser with an efficiency of almost 50 % from figure 4.5b. Unfortunately, some results from the theory do not coincide with experiments, which will now be discussed.

There is a significant difference when comparing the theoretical results for the output powers to experimental results. Figure 4.6 shows the calculated \((P_p, P_{out})\)-curves for lasing wavelengths of 1740 nm and 1984 nm with a cavity strength of 100 \( \frac{1}{\mu m} \) and a total cavity length of 49 mm. The cavity strength is estimated from transmission measurements of the 20 dB-bandwidth of the cavity gratings as presented in chapter 5.2 and the phase mask length is estimated to 49 mm. The calculated slope-efficiencies for the launched pump power above threshold is seen to be around 4 – 6 % depending on fiber and wavelength. However, slope-efficiencies for real lasers are measured to be only around 1.6 –
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1.8 %, i.e. less than half as much as expected from theory. For example, the slope-efficiency is only 1.8 % for the 1740 nm laser of the Tm2 fiber, but simulated to be 6.1 %.

A nearby explanation for the discrepancy between experiment and simulations would be that the fiber background losses of figure 2.15 are too low, especially if one takes UV-induced background losses into account. But a grating strength of 100 $\text{m}^{-1}$ gives a cavity loss coefficient of $\gamma_c = 2\kappa e^{-\gamma_c \ell} \approx 1.5 \text{ m}^{-1}$ and for the background attenuation to be comparable to this, say $\alpha = 0.1 \text{ m}^{-1}$, would require UV-induced losses of more than 30 dB, which seems unrealistic. This is confirmed by the fact that the threshold pump power of 21 mW predicted by theory agrees well with that of the real laser of approximately 24 mW, remembering that uncertainties are involved in estimating splice and insertion losses which leads to the launched pump power. Moreover, if the UV-induced losses are wavelength independent, a $+30 \text{ dB}$ increase in the absorption leads to a threshold pump power at 1984 nm of 500 mW, which was not observed in practice. In conclusion, background attenuation is negligible at this low grating strength and the explanation of the discrepancies should be found in effects that have dependencies in the slope-efficiency, but not in the threshold condition. However, from equation 3.47 and 3.52 it is seen that it is not easy to adjust parameters that have influence on the slope-efficiency without influencing the threshold condition. So an explanation has to be found in details not included in the model.

A possible explanation could be the presence of ion clusters. Cluster induced quenching of a fraction, $k_{cl}$, of the ions effectively locks these ions to the ground-state because of fast exchange of energy within a cluster, which reduces the available gain. It has previously been found that ion quenching only affects the slope-efficiency of Er:DFB fiber lasers, because of an increased upconversion rate in the presence of a strong inter-cavity laser field [35]. A cluster fraction of 10 % was shown to reduce the slope-efficiency with nearly a factor of 2, leaving the threshold pump power unaltered. The threshold condition is unaffected by the ion clustering as long as the threshold inversion needed to obtain lasing is well below $1 - k_{cl}$.

However, it was seen in chapter 2 that fluorescent decay curves showed insignificant signs of clustering. Also, if it is assumed for a moment that clustering was actually present in the fibers, then the procedure leading to the scaling of the emission cross section would have false assumptions. The ratio of emission to absorption cross section determined from a max/min. gain measurement would thus be under-estimated, thereby counteracting the negative dependence of the slope-efficiency from ion quenching. For this reason, the difference in
4.2 Dynamic properties

It is also interesting to examine the dynamic properties of the Tm:DFB laser. The cavity design was seen in figure 3.13 to have significant influence on the damping of the 2nd order system and a root curve is shown in figure 4.7 for varying grating strength. It is again seen that a high grating strength is favourable in terms of the damping coefficient and that the resonance frequency is now peaking around 1.2 MHz for grating strengths around 150 \( \mu \text{m} \), remembering that the pump power is constant 100 mW. It is also seen that the damping coefficient is not significantly wavelength dependent and that it is very low for grating strengths below 100 \( \mu \text{m} \). A higher imaginary part of the roots is favoured by a small pump saturation, \( P_{\text{sp}} \), a long excited-state lifetime and a high threshold or small-signal gain, \( \gamma_\text{t} \). The influence from the grating strength comes through a secondary effect of decreasing the total cavity losses, which equates to a higher normalized intensity \( \hat{X} \), which is proportional to the real part of the roots. Thus, decreasing the total cavity losses or increasing the threshold gain causes the roots to move further into the complex plane and thus become better damped. If the background attenuation was dominating the cavity losses, then the damping coefficient would likewise increase with decreasing attenuation.

The spectrum of the intensity noise of the laser relative to the intensity noise of the pump of equation 3.93 can be seen in figure 4.8a. The typical behaviour of increased peak frequency and wider resonance widths are observed as the pump power is raised. The peak frequency of the RN-curves are plotted together with estimated peak frequencies from a measurement of the noise spectrum from a
4.2. DYNAMIC PROPERTIES

Figure 4.7: a) Root curves of the transfer function for a symmetric thulium doped DFB laser at two wavelengths for varying grating strengths 75 – 200 l/m. The pump power is constant at 100 mW and points are indicated for 100, 150, 200 l/m. Symmetry about the real axis left out.

b) Resonance frequency and damping coefficient for the same thulium doped laser at the wavelengths of 1.74 µm (−) and 1.85 µm (−)
Figure 4.8: a) Relative pump induced noise spectrums (RN) for three pump powers. Laser cavity grating strength of 100 \( \mu \text{m} \) lasing at 1.74 \( \mu \text{m} \).

b) Measured (red) and theoretically predicted (blue line) frequencies of the intensity noise peak as a function of launched pump power for a 1740 nm laser. The laser is made in the Tm2 fiber with an estimated grating strength of 100 \( \mu \text{m} \).
4.3 SUMMARY
eal laser in figure 4.8b. The model seems to overestimate the trend of the RIN peak frequency slightly, but the peak frequencies found from experiments are also more and more uncertain as the pump power is increased because of peak broadening and the noise limit of the detector.

Simulations of the RIN peak level have not been performed, mainly because of the discrepancies found in the values between the theoretical and real laser output powers. It was thus not possible to get a resonable match of the model RIN peak level with those observed in experiments and it would not be fair to attempt to model it without a decent explanation and an improved model that could lead to better agreement between measured and simulated results.

4.3 Summary

Results from thulium spectroscopy and numerical and analytical models of DFB fiber lasers have been combined to predict the behavior of thulium-doped DFB fiber laser in several operating conditions. Threshold pump power as a function of pump wavelength in the L-band has been treated. Introducing the gain curve of an EDFA module as the L-band source has been shown to lead to an optimal pump wavelength around 1595 nm and optimum operating wavelengths have been identified. The simulations have a tendency to overestimate the output power of the laser when compared to experimental results, which could be due to imperfections of the gratings and effects not accounted for, such as ion-clustering and upconversion. Thulium DFB noise properties have also been studied and relaxation oscillation frequencies are found to be close to the frequencies observed in reality.
Chapter 5

Experimental Tm:DFB fiber lasers

The primary goal throughout this study has been to demonstrate single-frequency lasing in thulium doped silica fibers. The line-of-thought in achieving this goal was to use existing knowledge of cavity design of DFB fiber lasers combined with an active fiber doped with thulium and with the use of a suitable pump source. Of these three requirements, only the DFB cavity designs and fabrication was readily at hand.

The active fiber used for laser fabrication should meet some basic specifications in order to be interesting for this particular purpose. First of all, the singlemode fiber had to be doped with a significant amount of thulium because of the relatively short cavity of a DFB fiber laser of only 3 – 10 cm. Secondly, the fiber had to be photosensitive, which essentially means that Ge-codoping is a requirement such that UV-induced Bragg gratings can be formed with existing equipment. Thirdly, the fiber should also have a considerable amount of Al-codoping to facilitate high rare-earth concentrations and reduce the ion clustering. Considering all these specifications and at the time of the project, such demands to a thulium doped fiber nearly excluded every commercially available fiber. Therefore a project of fabricating a new fiber with OFS Fitel Denmark was immediately decided.

In the early stages of the project, the only available pump source was a tunable bulk Ti:S laser which could pump the thulium ions at a wavelength of 790 nm. High power diodes with pigtailning were available at this wavelength,
but only from one single supplier and the lasers were multi-longitudinal moded. Pumping at 790 nm was replaced by \textit{L-band} pumping after discovering instabilities and other problems associated with the 790 nm pump wavelength as will be treated in chapter 5.4.1. \textit{L-band} refers to a term borrowed from fiber optical communications, which designates a wavelength band in the Long wavelength end of the erbium amplifier amplification spectrum, usually from 1570 nm to 1610 nm. In fact, \textit{L-band} pumping paved the way for the realization of a practical and simple thulium doped DFB fiber laser module.

The following sections will describe the results obtained from fabrication and maturing of the design of thulium doped DFB fiber lasers, which has involved the production of more than 60 lasers.

5.1 Grating fabrication

All gratings are fabricated with a pulsed 240 nm excimer laser and a UV-writing setup using the phase mask technique, which is explained in detail elsewhere [62]. The UV beam at a wavelength of 248 nm is directed orthogonal to a phase mask with a period of \( \Lambda \), which diffracts the incoming UV intensity into the \( \pm 1 \) order modes to form a periodic UV-intensity directly beneath the phase mask with a period of \( \Lambda/2 \). The photosensitive fiber is placed under the phase mask such that the periodic UV-intensity falls onto the bare fiber, forming a periodic index variation in the fiber core. A schematic of the setup can be seen in figure 5.1. The details of mechanisms in UV-sensitivity and the formation of permanent index changes by exposition of UV intensity are very complicated. The understanding of the mechanisms of UV-induced index changes are still improving but not yet fully solved and in this study, UV-sensitivity has only been treated as a necessity for the fabrication of the laser cavity. It has thus not been under detailed investigation and for a detailed description of the most recent theories of UV-induced index changes, the reader is referred to for example [63].

UV characterization of the thulium doped fibers was initially performed in a wavelength region where characterization equipment was readily at hand. So to begin with, the transmission spectra and Bragg wavelengths were monitored by inscription of test gratings using a mask period of 1067 nm. Two different methods were used in the characterization. One, in which a short grating of 13 mm was written with a stationary UV-beam, meanwhile monitoring the Bragg wavelength and the transmission minimum as a function of UV-fluence. And another, in which a longer grating of 24 mm (nearly equal to a half-grating in a DFB cavity) was written by scanning the relative position of the UV-beam.
5.1. GRATING FABRICATION

![Diagram of phase mask UV writing setup]

\[ \Lambda_0 = \frac{\Lambda}{2} \]

Figure 5.1: Phase mask UV writing setup. The phase mask has a period of \( \Lambda \) and the \( \pm 1 \)-order diffracted beams interfere to transfer the phase mask onto the fiber with a period of \( \frac{\Lambda}{2} \).

and the fiber, while afterwards monitoring the transmission spectrum for each UV-fluence used. The results can be seen in figure 5.2. It is first noted, that the Bragg wavelength indicates an effective index of approximately \( \frac{1557.7}{1067} = 1.459 \), which has the expected monotone slow increase as a function of UV fluence. However, the transmission minimum dependency as a function of UV fluence for the Tm2 fiber is seen to behave as a that of a fiber with a very high concentration of Ge[64, 65, 66]. The UV-sensitivity of fibers with high Ge-co-doping have previously been shown to behave as that seen in figure 5.2a [67], where a dip in the transmission occurs at a fluence of \( 300 \, \text{J/cm}^2 \). The behavior is seen in both the case of stationary and scanned UV-gratings and suggests that an initial grating is formed in the usual way proportional to the average index change. UV absorption saturation at the high UV intensity points and the limited visibility of the UV intensity pattern causes the grating transmission to increase for a certain dose of UV fluence, which results in the dip in the reflection. The rather fine transmission spectra of figure 5.2b were not always realized and often for fluences beyond \( 300 \, \text{J/cm}^2 \), the scanned gratings could have nearly unrecognizable spectra.

The fiber from OFS (Tm1) had a distinctively different behavior to UV-fluence. The fiber is doped with Ge in a ring around the fiber core. The fiber
was deuterium loaded prior to grating inscription and was only observed to form simple grating growth in the experiments performed. The index change induced as a function of UV-dose was nearly linear in the range of interest and the transmission spectra appeared close to that observed in theory.

The inscribed gratings were annealed at 120 °C for 30 minutes for non-loaded fibers and 10 hours for loaded fibers to accelerate aging and out-diffuse remaining $D_2$.

From the data collected on UV induced average index changes it is possible to estimate the effective refractive index of the fiber. This is very important, as a typical task will be to predict the phase mask period for future laser designs. Since the thulium gain bandwidth covers more than 300 nm it is necessary to include the material dispersion into the considerations. This is seen in figure 5.3, where a curve of the effective refractive index of each fiber has been extracted from two datapoints and a simple one-pole Sellmeier equation [68]. The extraction does not include waveguide dispersion, because of the relatively small significance in relation to material dispersion. It is noted, that two laser wavelengths have already been predicted using this curve at as high a wavelength as 1984 nm and 2090 nm, indicated by crosses in figure 5.3.
5.2 CAVITY DESIGN

Figure 5.3: Effective refractive index calculated by a simple Sellmeier method with one absorption band. The curve is based on the triangular points, whereas the points marked by a crosses are realized lasers at a later time.

5.2 Cavity Design

The cavity design of the laser is important with respect to slope efficiency and threshold pump power as seen in the results of chapter 4. However, it is also necessary to include practical considerations into the design of the cavity for the success of a laser design.

Forming a cavity with a high grating strength, near the optimum value of equation 3.55 is desirable with respect to improved damping of the harmonic oscillator and optimum output power. Yet, the grating strength predicted by this equation becomes rather high for the low background attenuation measured in figure 2.15, i.e. around 200 $1/m$. As the grating strength scales inversely with wavelength, it becomes increasingly difficult to write strong gratings in practice, because of saturation in the UV sensitivity and due to other effects such as the grating formation seen in figure 5.2b. In practice, the 20 dB-bandwidth of a strong grating provides a fairly good estimate of a grating strength since the usual condition of relating the grating strength to the minimum transmission of the grating is limited by the dynamic resolution of the instruments used. The 20 dB-bandwidth as a function of grating strength can be solved numerically.
Figure 5.4: 20 dB width and minimum transmission of Bragg gratings of varying grating strength for 3 important wavelengths. \( L = 5 \text{ cm} \). The dotted line shows the minimum transmission at the right ordinate axis.

from equation 3.1 and 3.2 and is shown in figure 5.4 for a grating length of 5 cm for three common wavelengths. Measured 20 dB-bandwidths of the test lasers at a Bragg wavelength of 1740 nm were nearly all between 70 – 100 pm for 5 cm cavities, which converts into grating strengths between 110 and 170 \( \frac{1}{\text{m}} \). Particularly in the Tm2 fiber, it was very difficult to write strong gratings of high quality and increasing the UV fluence often lead to the formation of strange artifacts in the grating spectra resulting in poor performance. A very strong grating can also have the negative effect of decreasing the threshold of higher order modes and thus jeopardize single-frequency operation and stability, such that apodization of the cavity grating becomes necessary [69]. However, this was not experienced to be the case with the particular fibers. Besides this, increasing the UV fluence also has the effect of increasing UV-induced losses to the cavity.

Introducing the phase shift is another practical problem, which has to be considered carefully and within practical limits. All lasers in the study are formed with a negative distributed optical phase shift and often a \( \pi/2 \) phase
shift was not found to be optimal because of thermal effects introduced under pumping. The phase shift is affected by the elongation of the fiber due to heating and the thermo-optic coefficient. It was thus often observed that the laser could change lasing mode during a full pump power span, which is of course not acceptable. Often, the mode changes occurred between the two natural polarization modes of the fiber. This leads to another practical design issue, that each longitudinal mode is in fact double-modeled due to polarization splitting. This is probably the most challenging obstacle of every design of single-frequency fiber lasers.

Achieving single polarization output of a DFB fiber laser can only be achieved if the threshold (or output power) of one of the polarization modes is suppressed with respect to the other. This means that the phase shift has to differ in each polarization. One way to realize this is through UV-induced birefringence of the optical phase shift. Achieving this in the Tm2 fiber was not a problem and the polarization mode beat frequency was usually found to be nearly 1 GHz. On the other hand, the Tm1 fiber was found to only have mode beat-frequencies in the range of 10 – 100 MHz and lasing in dual-polarization modes was found in almost all of the fabricated gratings. Nearly all UV writing parameters was changed in hope of a better polarization discrimination in the Tm1 fiber, but generally without luck. Only in the case of using UV light polarized perpendicular to the fiber axis it was occasionally possible to observe nearly single-polarization output from the lasers. Writing with polarized UV-light has previously been shown to enhance UV induced birefringence [70, 71].

The exact mechanism which leads to a polarization discriminated phase shift is a little speculative at the present moment. Besides UV-induced birefringence, there is also stress and index anisotropy because of the UV beam primarily illuminating only one side of the fiber. Also the orientation of the intrinsic birefringent axes of the fiber with respect to the polarization of the UV beam may be of consideration and perhaps fiber core eccentricity plays an important role. A recent and likely explanation could be radial stresses formed during fiber drawing which cause preferential UV absorption for light polarized in the radial direction [72].

5.3 Laser characterization

The decision to first demonstrate lasing at a wavelength of 1740 nm could at first seem surprising in relation to the gain curves of the thulium doped fibers shown in figure 2.10 and 2.11. However, the decision was not intended for optimum
performance but instead to facilitate methods of characterization using available high performance equipment, i.e. within the typical response of 1750 nm of most InGaAs based detector equipment.

The first lasers were fabricated by more or less guessing the cavity design. A few cavity gratings were written at 1565 nm and scanned with existing tunable SF-lasers to get a first impression of the induced phase shift necessary to get a transmission peak near the center of the grating stopband. As the first single-frequency laser was produced, the possibility arose to characterize the cavities with respect to phase shift magnitude and position by scanning the single-frequency laser using piezo-tuning across the wavelength span of the other laser cavities. This proved to be a valuable tool for analyzing and optimizing the cavity design and the method was refined to produce reliable and accurate results. A typical spectrum of such a scan is shown in figure 5.5a for both thulium doped fibers used. The small kinks on the spectra reveal the detuning positions of the phase shifts. If the polarization of the scanning laser is a mix of the two principal states in the fiber under test, then both phase shifts appear in a scan. However, if the polarization of the scanning laser is adjusted to interrogate only one of the principal polarization states of the cavity under test, then a scanned spectrum appears as in figure 5.5b. The phase shifts are separated in each scan, which also confirms that the scanning laser is single-frequency. The phase shift of polarization 1 is clearly shifted toward the high wavelength side of the stopband center, whereas it is nearly centered for polarization 2, thereby providing the before mentioned difference in lasing threshold.

5.4 Laser performance

Laser performance has mainly been focusing on threshold pump power and single-sided output power of lasers which fulfilled the primary condition of single-frequency lasing. Other specifications as noise, linewidth and tuning have only been addressed with very few lasers which were selected for further analysis and processing. This has to be viewed in the light that the results of this project are only the first in a hopefully long line of future improvements for thulium doped DFB fiber lasers.

5.4.1 790 nm pumping

Pumping into the $^3H_4$-band at a wavelength around 800 nm has for a long time been referred to as one of the primary advantages of thulium doped fiber
5.4. LASER PERFORMANCE

![Graphs showing laser grating transmission spectra](image)

Figure 5.5: (a): Typical laser grating transmission spectra scanned by a single-freq. Tm:DFB fiber laser for the two Tm-doped fibers.
(b): Laser grating transmission (Tm2) scanned at the two polarization states that match each phase shift.

The availability of mature high power AlGaAs diode pumps, used to pump Nd:YAG laser at a wavelength of 808 nm, could potentially scale output powers of thulium doped lasers to very high values. Motivated by this, the primary investigations into pumping of the thulium doped fibers in this project were carried out using a laboratory Ti:Sapphire laser tuned to 790 nm. This was of course with the intention of switching to diode pumping once lasing was demonstrated. A slope-efficiency curve for the first single-frequency laser demonstrated is shown in figure 5.6a pumped by the Ti:S laser. The slope efficiency with respect to launched pump power was only 0.2 % with a threshold pump power of approximately 50 mW. An output power of 1 mW was obtained at a pump power of 600 mW. The tunability of the Ti:S laser provided a curve of the threshold pump power as a function of wavelength, which is shown in figure 5.6b. The optimal pump wavelength is seen to be located in a narrow band around 785 – 790 nm.

The laser was characterized with respect to true single-frequency lasing with plane-mirror scanning Fabry-Perot interferometer having a resolution of 55 MHz. One such Free-Spectral-Range FPI scan is seen in figure 5.7b together with a trace from an optical spectrum analyzer. Only one peak is visible in
5.4. LASER PERFORMANCE

Figure 5.6: (a): Slope efficiency of the first-ever Tm:DFB laser to achieve stable single-frequency output. The slope efficiency with respect to launched pump power above threshold is a modest 0.2% at a pump wavelength of 786 nm.
(b): Laser threshold pump power when pumping into the $^3H_7$-band.

The FPI scan which indicates true single-frequency lasing and the signal-ASE ratio is seen from the analyzer scan to be more than 50 dB. In all, the laser performed fairly well with a relatively stable output and only few mode hops were observed as the pump power was increased and these results encouraged further efforts to improve efficiency and scalability.

As mentioned it was the intention to go for diode pumping once single-frequency lasing was demonstrated. Yet, although high-power multimoded diodes at 808 nm were easy to get, it proved difficult to get transversally single-moded diode laser pigtailed to SM-fibers at this wavelength with an output power needed according to figure 5.6b, despite the many references in the literature [73, 74, 75]. An affordable diode laser was found which could provide a multi longitudinal mode output at a wavelength of 784 nm pigtailed to SMF fiber. With an output power of more than 100 mW, it seemed the right solution as an effective pump module. The diode pump was also able to achieve lasing of the fiber lasers, but a serious stability issue of the fiber laser output was discovered. The signal from a 125 MHz detector coupled to the fiber laser output when pumped by the 784 nm diode can be seen in figure 5.8. The signal is seen to be very unstable switching between states where weakly damped oscillations
5.4. LASER PERFORMANCE

Figure 5.7: (a): Spectrum of the first-ever realized thulium doped DFB fiber laser (FPGA fiber). (b): Scanning Fabry-Perot spectrum of the first single-frequency laser. FPI finesse = 37 and FSR = 2 GHz

Figure 5.8: (a): Temporal evolution of the output from a DFB fiber laser pumped by a 786 nm diode. (b): Zoom on the relaxation oscillations. The turquoise curve is proportional to the pump current.
are excited at random times. The switching behavior was random in time even though the output power of the diode pump was checked to be stable. Stability problems are well documented and modeled for high-concentration Er-doped fiber lasers [76, 77, 78], in which ion-pairs are treated as intra-cavity saturable absorbers. Suggestions for work-arounds have been to introduce intensity dependent feedback into the cavity, increase the ratio of emission-to-absorption cross section to make the laser quasi 4-level and/or change to in-band pumping because of a lower achievable inversion. Stability problems have also recently been reported in conventional thulium doped silica fiber lasers [79, 80]. Self-sustained pulsing behavior laser emission around 2 $\mu$m was observed when pumping at 804, 1064 and 1319 nm. It was proposed to originate from ion clustering or un-pumped regions of fiber acting as a saturable absorber.

However, the damped oscillations seen in figure 5.8 indicate a stable system and self-sustained pulsing has not been observed except at pumping rates very close to the threshold. Furthermore, the instabilities were not seen when pumping with the Ti:S laser, which lead to the conclusion that exchange of pump power between the many longitudinal modes of the diode pump laser induced a fluctuating pump power absorption because of the very narrow bandwidth of the ($^2H_6$, $^4I_1$) transition, see figure 2.1b and 5.6a. A possible solution would be to grating stabilize the pump diode, but phase masks at the correct wavelength were not available.

The consequence of the stability problems was to search for other pump wavelengths. In that respect it was obvious to try in-band pumping with a high power DFB diode laser at a wavelength of 1590 nm. The result was to completely remove the instabilities and even provide improved efficiency.

5.4.2 1600 nm pumping

High power diodes at wavelengths above 1600 nm are not available, but combining a moderate output power diode with a L-band EDFA provides a simple and efficient way of realizing high power at wavelengths from 1580 – 1600 nm. Output powers and spectra of L-band pumped Tm:DFB fiber lasers are shown in figure 5.9. The slope-efficiency of the 1740 nm laser is now up to 1.8 %, which is nearly a factor of 10 better than in the case of 790 nm-pumping. The improvement is caused by several factors. First, the lasers are not directly comparable, as the laser with the bigger efficiency is an improved design in a much better thulium doped fiber. Second, more than a factor of 2 improvement in efficiency comes directly from the minimized quantum defect $\lambda_s/\lambda_p$ and thirdly, the overlap of the L-band pump modal field with the thulium ions is greatly improved.
5.4. LASER PERFORMANCE

Figure 5.9: (a): Output power as a function of pump power for a pump wavelength of 1597 nm for three realized lasers. Slope efficiencies with respect to launched pump power above thresholds are indicated. (b): Laser spectra recorded by a monochromator. The ripples on the low wavelength side of each spectrum originate from the response of the grating in the monochromator which are traced by the narrow linewidth lasers. The ripples are thus not higher order modes of the lasers.
because of a singlemode pump field. The thulium fiber guides up to 4 modes at a wavelength of 790 nm. The only drawback is that the pump absorption is reduced because of a lower ground-state absorption of the \(3F_4,3H_6\)-transition.

A scanned spectrum of each of the Tm:DFB lasers produced during the project is shown in figure 5.9b. In chronological order, a 1740 nm, 1984 nm and a 2087 nm laser have all been demonstrated to achieve single-frequency lasing pumped by a L-band DFB diode laser. By far, the 1740 nm laser is the most extensively studied, whereas the two other laser wavelengths until now have only been a proof-of-possibility. Actually, the lasing at 2087 nm is a bit of a puzzle, since as recalled, every small signal gain measurement made as a part of the studies in chapter 2 indicated ESA and negative small signal gain beyond a wavelength of 2065 nm! Yet, the laser is actually lasing in a single-frequency checked by a FPI-interferometer coupled to the reverse output port of a WDM and the wavelength is found from a monochromator scan as seen in the figure. Even though a positive small signal gain would exist, it was still a concern if the cavity losses were too high at this wavelength. So it was to some surprise that the laser actually worked and on top with a reasonable efficiency. Two DBR cavities were also written at 2087 nm, but neither of these showed lasing up to a pump power of 50 mW. This may be due to a too low cavity Q-value of the of the DBR lasers, which consisted of two identical 13 mm gratings with reflectivities of nearly 20 dB separated by 34 mm active Tm: fiber, but this was not checked further. The puzzle is not yet solved and a worrying thought is that the fiber is inhomogenous along the fiber length.

The problems with stability experienced with 790 nm pumping were not seen under 1600 nm pumping. The absorption band of the \(3F_4,3H_6\)-transition is very broad and the pump diodes used are single-frequency DFB semiconductor diodes. Yet, at pump rates very close to threshold, the laser becomes self-pulsing. The laser comes to rest with CW-output when the pump rate is increased above threshold. The situation is believed to be that of a saturable absorber mentioned earlier where the saturable absorber is an under-pumped region at the end of the cavity. The output is a pulsetrain with an approximate Pulse-Repetition-Frequency (PRF) of \( (8 \mu s)^{-1} = 125 \text{ kHz} \) and FWHM pulsewidths around 700 ns. The pulsetrain looks relatively stable in both PRF and peak pulse power and actually resembles what has also been observed in [79].

The laser is still very weakly damped because of the relatively low cavity Q-value. The relaxation oscillations are clearly visible under abrupt pumping rate changes as seen in figure 5.10. An inversion builds up until threshold is reached, which results in an output pulse that drains some of the inversion and turns
5.4. LASER PERFORMANCE

Figure 5.10: (a): Step response of the output from a DFB fiber laser pumped by a 1600 nm external cavity laser, showing damped oscillations. (b): Modulation of the pump at a frequency of 90 kHz captures one single relaxation oscillation to produce a pulsed output from the fiber laser with a FWHM pulsewidths around 480 ns. The turquoise curve is proportional to the pump current.
the laser off. The situation repeats in a damped way until stable cw-output is obtained. As with any other laser this opens the possibility of the most simple way of producing pulsed output from a laser by on/off switching of the pump rate at a repetition rate which just captures the first relaxation oscillation, also known as gain-switching. This can be seen in figure 5.10b. A regular step pump modulation of 90 kHz produced stable pulsed output from the Tm:DFB laser with pulsewidths of about 480 ns. Unfortunately, equipment was not at hand to measure pulse energy or peak power and this feature was not investigated further. The simplicity of the method is also a limitation because of the fixed maximum pulse repetition frequency (PRF) and the relatively long pulse widths. Pulsing of multi-longitudinal mode thulium doped silica fiber lasers have been success fully demonstrated, both in the form a singlemode mode-locking with pulsewidth of 190 ns and a PRF of 50 MHz [6] and multimode Q-switching with 150 ns, 30 kHz and peak power of 4.1 kW [5]. Yet, this is the first pulsed single-frequency operation demonstrated in thulium doped silica fiber, even though it has not been the intention to produce pulsed operation.

5.4.3 ASE source

One may also consider making an ASE-source in a thulium doped silica fiber, which could provide a high spectral density output of incoherent light in the wavelength range from 1750 nm to 2050 nm. An ASE spectrum of a 3 m long Tm1 fiber pumped by a seeded L-band amplifier is seen in figure 5.11. The ASE spectrum shows the typical behavior of shifting of the peak emission corresponding to the maximum in the gain inversion. The ASE is seen to be concentrated into a band from 1780 – 2000 nm with a maximum output of only 5 mW for 200 mW launched pump power. Above this pump rate, onset of lasing was observed even though the fiber were terminated in an APC connector, thus justifying that a longer length of fiber could provide more output power. The poor ASE power efficiency is a consequence of the low radiative quantum efficiency of the \( ^{3}F_{4} \rightarrow ^{3}H_{6} \) transition in thulium doped silica, which was found to be approximately 10 \% in chapter 2. The low quantum efficiency makes thulium doped silica fibers poor radiators of spontaneous emission. Recently, an ASE sources with a power conversion efficiency of 2 \% has been reported and a total output power of 40 mW in a Tm:Ho codoped DCLAD fiber was obtained [81]. The output power was also found to concentrate around 1850 nm, despite the use of Ho codoping, which should have shifted the gain peak to higher wavelengths.
5.4. LASER PERFORMANCE

Figure 5.11: (a): ASE power spectrum in a 1 nm bandwidth of 3 m of Tm1 fiber pumped by a 1597 nm seeded L-band amplifier. (b): Forward ASE power from 3 m of Tm1 fiber pumped at 1597 nm. A pump power of 200 mW is at the onset of lasing.

5.4.4 RIN and linewidth

With the laser emitting at a wavelength of 1740 nm, it was just within reach of fast, low-noise InGaAs photodiodes. Because of this, it was possible to measure a RIN spectrum accurately and calibration of the detection diode and spectrum analyzer could be performed by relating the values obtained to those found from a HP lightwave RIN measurement system. A typical RIN-spectrum is seen in figure 5.12. A relative RIN spectrum is found by recording the output from 125 MHz detector coupled to an electrical spectrum analyzer after division with the DC-voltage at the analyzer input, that is

\[ RIN = \frac{S}{V_{DC}} \quad (5.1) \]

where \( S \) is the power spectral density in units of [V/Hz]. The relative spectrum was then calibrated by comparing it to a known RIN level of a 1575 nm laser. Following this procedure, the RIN peak magnitude could be traced as a function of pump power, as seen in figure 5.12b. The RIN peak magnitude decreases as expected with increasing pump power having a value of \(-110 \, \text{dB/Hz} \) at a pump
Figure 5.12: (a): RIN spectrum for a 1740 nm laser pumped at 60 mW giving an output power of approximately 0.6 mW. (b): Estimated RIN peak for different pump powers calibrated to a RIN measurement on a 1575 nm Er-DFB-FL.

power of 50 mW, providing an output power of nearly 600 μW. The RIN of the pump is given from the manufacturer to be approximately $-130 \text{ dB/Hz}$.

Also the linewidth is an important figure-of-merit of a DFB fiber laser. The linewidth was measured using a 20 km fiber MZI with an AOM modulator in one of the arms as to shift the difference frequency away from DC. A self-heterodyne spectrum of a 1740 nm laser can be seen in figure 5.13. The linewidth is estimated to around be 20 kHz, which is only slightly wider than the linewidths of typical Er/Yb:DFB fiber lasers.

In conclusion, it has now been verified that not only single-frequency lasing is possible with Tm:DFB fiber lasers, but also low-noise, narrow-linewidth operation with specifications that are comparable to those obtained from Er/Yb:DFB fiber lasers and this being only the first preliminary results.

5.5 System configuration

The results of the previous sections are so convincing that they open up the possibility of taking the Tm:DFB fiber laser to the next level. Single-frequency operation has been demonstrated spanning a very wide range of wavelengths
5.5. SYSTEM CONFIGURATION

Figure 5.13: Measured linewidth using an AOM and a 20 km balanced MZI interferometer of a packaged 1740 nm pumped by an EDFA module. The 3 dB linewidth is approximately 20 kHz.
with reasonably good performance in terms of threshold pump power, output power and stability. This paves the way to the next goal of forming a system that together with the Tm:DFB laser provides simple means of achieving a single-frequency laser in the 1.7 – 2.0 μm range. However, the first results of demonstrating an idea using laboratory equipment is an academic proof. The way towards practical realization involves a series of other challenges, if at all practically possible.

During the study of the properties of thulium doped fibers, it was always a challenge to find suitable devices operating at these high wavelengths. For example, detectors at these wavelengths are relatively slow and noisy, WDMs are non-existing and isolators are bulky and expensive and finally the possibility of getting a simple pump source. It is always a huge advantage in fiber optics, if devices that are parts in the optical communications can be used because of superior performance and low price. A transmission spectrum (insertion loss) of a standard L-band 980/1590 nm WDM and a standard L-band isolator is shown in figure 5.14. The WDM functions as a laser-pump combiner/separator and it is seen from the figure that it performs best for pump wavelengths in the range 1575 – 1625 nm and signal wavelengths in the range 1900 – 2100 nm. One
5.5. SYSTEM CONFIGURATION

has to accept a rather high loss if the WDM is used at signal wavelengths below 1900 nm, which necessitates other means of pumping. Most 1480/1590 nm WDMs are based on a reflective thin film technology i.e. effectively a stopband filter and are thus not useable. Custom WDMs could be produced from fiber fusion to provide different periodicity of the response and would probably be the best comprise at the moment.

The isolation of the fiber laser is another issue. The isolation itself of standard isolators is not that problematic, but the transmission of the isolator is. The transmission spectrum of a standard L-band isolator is seen to be seriously degraded above 1700 nm and only very few are capable of producing bulk isolators around 2.0 µm with good specifications. Lasing without isolators is of course possible, but the impact on stability and noise is in most cases unacceptable.

A suitable pump source is also important. Expecting a Ti:S laser as the pump source is utopic and until 790 nm diodes mature, the only practical pump wavelength is an L-band source. One possible realization is to use a high power DFB diode laser with an output power above 10 mW as a high power seed source to a L-band EDFA module. The output characteristics of such a system can be seen in figure 5.15. The module produces nearly 300 mW of singlemode output at a wavelength of 1595 nm. It is relatively simple, all-fiber and has a small form factor. Pumping with an L-band DFB diode laser has another important practical application. Since the pump laser is narrow-band, it offers the opportunity of retro-pumping using a Bragg grating inscribed at the end of the cavity. It has been verified that the output power of Tm:DFB fiber lasers at 1740 nm shows a nearly 3 dB increase in output power when retro-pumped while at the same time providing a more uniform pump distribution in the cavity. However, the preliminary tests have also shown that particularly the linewidth is broadened by retro-pumping. Other effects have not been studied in detail at the moment, but retro-pumping could open up a new method of achieving improved efficiency. Power scaling is also possible at the L-band pumping wavelength. High power EDFA DCLAD modules are widespread today and subsequent amplification of the single-frequency laser output in a thulium amplifier provides means of reaching very high powers. As an illustration, the output power of the 1740 nm laser was easily boosted to 60 mW using only 20 cm of extra thulium doped fiber following the DFB cavity. The amplifier section was pumped by unabsorbed pump from the laser cavity.

Finally, all components have to be configured together to form a complete system. A few interesting variants are proposed in figure 5.16. Besides the seeded L-band EDFA there also exist a possibility of pumping with an Er:ASE
source or the very exciting possibility of intra-cavity pumping. A Tm:DFB laser is spliced into the cavity of a Er:DBR/FP laser, hopefully being able to pump the Er:DBR laser above threshold to provide a high power pump for the Tm:DFB laser. Assuming a Tm:Er fiber splice loss of 0.1 dB/splice and a GSA of 0.9 dB/cm from the thulium fiber, the erbium fiber has to provide more than 4.7 dB gain to overcome the inter-cavity losses associated with the thulium fiber, which does not seem completely unrealistic. The dynamics of such a system are probably not advantageous, but the idea is very exciting and is a complementary task in future research.

5.6 Summary

The actual demonstration of single-frequency oscillation at three different wavelengths have been presented in this chapter. From the first 1.74 μm laser to the recently demonstrated 2.00 μm laser, it has been proved that single-frequency emission can in fact be produced over the full range of emission wavelengths of thulium. The 1.74 nm laser was initially showing poor efficiency when pumped
5.6. **SUMMARY**

![Diagram of different proposals for L-band pump configurations of a 1.7–2.0 μm thulium doped DFB fiber laser.](image)

Figure 5.16: Different proposals for L-band pump configurations of a 1.7–2.0 μm thulium doped DFB fiber laser.
at 790 nm, but changing to 1590 nm pumping and through maturing of the cavity design, it was possible to obtain a reasonable slope-efficiency above 1% with respect to launched pump power. The laser was later configured in a MOPA configuration to provide 60 mW of output power using residual pump power from the 250 mW pump module. The RIN peak level of the laser was measured to be below 110 dB/Hz which was the limit for the detection system. Also the linewidth of the 1.74 nm laser was measured to only 20 kHz which proves that the lasers are actually able to emit highly coherent laser radiation. The good results of the improved configuration actually allows for a future potential for the thulium doped DFB fiber laser beyond laboratory environments.
Chapter 6

Coherent CW fiber-based LIDAR

Light Detection And Ranging (LIDAR) is the laser cousin of the more common RADAR. A coherent LIDAR works in its most basic operation by launching a laser-beam or pulse towards a target through a transmitting telescope. The target can be naturally occurring aerosols or particles (Mie-scattering) suspended in the air or molecular scattering (Rayleigh scattering). A very small fraction of the incident light is backscattered and collected by a receiving telescope. The backscattered light is then coherently mixed to down-convert the Doppler frequency shift of the scattered light. The Doppler shifted frequency, which is directly proportional to the targets relative velocity, is then detected. A measurement technique for determining the mean wind speed is then established. Range estimation can also be obtained by modulation.

A wavelength of 1.55 μm is the first choice for developing fiber based LIDARs, since the fiber optical communications technology provides good and reliable off-the-shelf components. A 2 μm system could have the advantage of high power operation using mature 800 nm pumping modules. With this technique, Coherent Technologies have reported a 2 μm LIDAR system providing 0.5 J, however not fiber based[82]. The advantage of a fiber based system is its relative simplicity, coherent signal processing and very robust design. The fiber-optic technology ensures stable operation in harsh and shaken environments. This makes the system ideally suited for airplane installation, also due to its light weight, air-cooling and relatively low power consumption, which are

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important factors in aeronautics. The infrared CW lidar also has the advantage of being *eye-safe*, see further discussion in section 6.5.

The work presented here was carried out at the Research Facility, EADS GmbH in Munich, Germany. The particular setup investigated was intended for on-board aircraft, real-time wind velocity measurements at close ranges (< 50 m) with a fiber based system operating at 1.55 μm. The idea behind this is manifold, but specifically to measure wind profiles around the aircraft, true-airspeed indication and as a reference system for other instruments.

### 6.1 LIDAR theory

The fiber based LIDAR system investigated in this work has the schematic setup shown in figure 6.1.

![LIDAR diagram](image)

**Figure 6.1:** Setup of the two investigated principles of the coherent LIDAR. Uppermost is the monostatic setup where the T/R switch is a circulator and below the bistatic setup where T/R arms are physically separated

The setup basically consists of a master oscillator, an amplifier, a Transmit/Receive (T/R) switch, a detector and a telescope. All components are
6.1. LIDAR THEORY

connected with standard singlemode optical fiber. The setup can be either monostatic, in which a \( T/R\)-switch (usually a circulator or a beam-splitter) separates the sending and receiving signals or it can be bistatic, in which the sending and receiving signals are physically separated. The master oscillator is a 1.55 \( \mu \)m single-frequency DFB fiber laser with a narrow linewidth, which is amplified by an Er/Yb double-clad fiber amplifier to levels around 1 W. The amplified signal is transmitted through the atmosphere through a telescope and focused at a target volume, typically 5-100 m away. The target acts as a source and scatters a very small fraction of the incident light directly back towards the receiver. Also a part of the master oscillator is coupled along with the backscattered signal and optically mixed to form a beat-frequency. This provides a signal with the difference frequency, exactly equal to the back-reflected signals Doppler frequency shift, given by

\[
\Delta f_d = \frac{2\Delta v \cos \theta}{\lambda},
\]  

(6.1)

where \( \lambda \) is the incident wavelength, \( \Delta v \) is the relative particle velocity and \( \theta \) is the angle between the incident wave-vector and the particle velocity vector. At \( \theta = 0 \) and \( \lambda = 1.55 \mu \)m the Doppler shift amounts to \( \Delta f_d = 1.29 \frac{MHz}{\mu m} \). The signal is then detected by a square-law photodetector and signal-processed.

A schematic of the free-space optics involved in the telescope of the LIDAR is seen in figure 6.2. The lenses used were achromats of diameter \( D \), which were AR coated for 1.5 \( \mu \)m. The fiber output is placed in a micrometer adjustable mount at a distance a small fraction larger than the focal length of the lens. This provides a focused sounding volume, some distance \( S'' \) away.

The Signal-to-Noise Ratio (SNR) of a shot-noise limited, tightly focused coherent LIDAR operating without atmospheric disturbances and with an ideally matched local oscillator has the very simple form of\[83\]

\[
SNR_{mono} = \frac{\pi \eta P_t \beta \lambda}{2Bh\nu},
\]  

(6.2)

where \( \eta \) is the quantum efficiency, \( \beta \) is the 180° backscatter coefficient and \( \nu \) is the optical frequency, \( B \) is the system detection bandwidth and \( P_t \) is the transmitted optical power. The subscript \( mono \) signifies a system which shares the same arm for transmitting and receiving. The backscatter coefficient \( \beta = N\sigma \), where \( N \) is the aerosol density and \( \sigma \) the backscatter cross section, is a very important figure in LIDAR discussions. To have a SNR of 1 with a 1 W transmitted power, one must have a backscatter coefficient of the order
\[ \beta = 10^{-5} \frac{1}{\text{m} \text{sr}} \] and the backscattered power is of the order of pW. Hence coherent techniques must be employed in order to detect such weak signals and the DFB fiber laser is ideally suited for this purpose.

Notice, that the SNR is independent of the telescope optics and target range. This originates from the fact that the signal power per particle increases with a smaller focal spot, but the sounding volume decreases proportionally. However, this is only valid if the sounding volume encompasses many scattering centers and this is not the case for this particular setup, that operates on single-scattering events. If there is only one particle in the sounding volume at a time, the SNR becomes dramatically dependent on the optics\[84\]. The focal volume is proportional to the product of focused beam area and the Rayleigh length of the beam. For a diffraction limited beam, the smallest spotsize diameter achievable is given by\[85\]

\[ d_{\text{min}} = 2.44\lambda \frac{1}{2NA''}, \quad (6.3) \]

where \( NA'' \) is the image plane’s numerical aperture given by \( \frac{D}{2S''} \). The Rayleigh length is related to the focusing of the beam and is given by

\[ z_r = \frac{\pi d_{\text{min}}^2}{4\lambda}, \quad (6.4) \]
6.2. SYSTEM DESIGN

which gives a sounding volume, $V$, proportional as

$$V \propto \lambda^3 \left( \frac{1}{NA^2} \right)^4$$  \hspace{1cm} (6.5)

This strong dependency of the sounding volume on the optical arrangement is very important for the operation of the single-particle coherent LIDAR.

Single-particle scattering events gives another relation for the SNR than the one in equation 6.2. It is proposed in [84] that the LIDAR operating in a single-particle scattering regime obeys the SNR equation as

$$SNR_c = \frac{\pi \eta P \sigma}{256 \lambda Bhe \left( \frac{L}{D} \right)^4}$$ \hspace{1cm} (6.6)

which for a typical sea level particle concentration $N = 5000 \frac{1}{m^3}$ and a lens radius of $75 \text{mm}$ gives a factor of 40 (or 16 dB) improvement, i.e. more than one order of magnitude. This means that for $1 \text{W}$ launched power, this LIDAR system has a minimum detectable backscatter coefficient of the order $10^{-6} \frac{1}{m^2}$, which is close to the value near sea level.

6.2 System design

The system can be either monostatic or bistatic, see figure 6.1. A switch or splitter separates the transmitted signal from the received signal in the monostatic setup, whereas the transmit and receive arms are physically separated in the bistatic setup. The bistatic setup has the advantages of being insensitive to the transmit power and is suitable for high power operation, i.e. $> 10W$, because there is no leak power from the transmitting arm to the receiving arm. It may also operate with one transmitting channel and several receiving channels, which is advantageous for retrieving all 3 components of the wind vector. The monostatic setup can only retrieve one velocity component at any instant and a leak power from the transmitter is always present in the receiving arm which disturbs the total received signal. However, the monostatic setup is the far best solution, which will now be clarified.

The bistatic setup suffers from an inherent sensitivity degradation because of overlapping beam volumes. The volume overlap integral of two Gaussian beams focused at the same spot, but with beam axis separated by an angle $\theta$ is
given by [86]

\[ S(z) = \frac{w_0^2}{w(z)^2} \exp \left( -\frac{2z\theta}{w(z)} \right)^2, \]

(6.7)

where \( w_0 \) is the beam waist radius at focus and \( w(z) \) the beam waist at a distance \( z \) from the focus. The monostatic setup has \( \theta = 0 \). The relative signal is plotted in figure 6.3 for the monostatic case and the bistatic case with a normalized separation of \( \frac{\theta}{\theta_d} = 1.84 \), where \( \theta_d = \frac{\lambda}{\pi w_0} \) is the half-angle beam divergence at the focus point. The overall sensitivity is proportional to the area under the

![Graph showing theoretical sensitivity](image_url)

Figure 6.3: Theoretical sensitivity of the monostatic and bistatic setup along with measured values (red and black crosses) as a function of normalized length. \( R_c \) is the Rayleigh length.

curves, which for a separation value of \( \frac{\theta}{\theta_d} = 1.84 \) amounts to a sensitivity loss of 80% for the bistatic setup as compared to the monostatic setup.

Furthermore, a profound advantage of the monostatic setup is that it requires no alignment and is insensitive to small disturbances! This issue cannot be underrated, especially for airborne applications where disturbances are unavoidable. The critical component of the monostatic setup is the T/R switch, usually a polarization insensitive fiber circulator. The disadvantage is that the circulator has a finite crosstalk of about -60 dB, which means that a +30 dBm
transmitted signal leaves an unwanted leak-power of $-30$ dBm which is much stronger than the returned Doppler signal. The leak-power provides a homodyned DC signal and may eventually saturate the detector and thereby a loss of available signal. State-of-the art fiber circulators have a crosstalk no better than $-60$ dB and furthermore they tend to be power limited to around 5 W of input power. The leak noise is also inversely dependent on the laser coherence time and the relative delay time squared[87]. This favours the use of very narrow linewidth lasers and short connecting fibers.

To overcome these limitations, a new setup is proposed that may solve the T/R switch problem of the monostatic setup. The idea is to separate the high power side from the low power side while still retaining the obvious advantages of the monostatic setup. If the amplifier, and thus the high signal power, is moved away to the transmit side, problems with high power coherent signals mixing at the detector may be avoided. A schematic of the setup is shown in figure 6.4. Still using a circulator to receive the scattered signal, but instead coupling it to

![Diagram](https://via.placeholder.com/150)

**Figure 6.4:** A new setup proposed to overcome the limitations of leaking signal power. The high power signal is isolated to the transmit arm, while the advantages of the monostatic setup are maintained.

a narrow passband filter before mixing at the detector will drastically decrease the leak power. However, for this to work, the backward propagating ASE from the amplifier has to be kept at a minimum and the filter has to be as narrow just to encompass the doppler shifted bandwidth. To achieve this in practise, a circulator and a Bragg grating is coupled together to form narrow passband filter. One may estimate the ASE power transmitted to the detector by using
the simple relation for a fiber amplifier noise

$$P_{\text{ASE}} = 2h \frac{C}{\lambda} F(G - 1)B,$$

(6.8)

where $h$ is the plank constant, $F$ is the amplifier noise figure, $G$ is the amplifier gain and $B$ is the filter bandwidth. Inserting $\lambda = 1.55 \, \mu \text{m}, F = 2, G = 1000$ and a filter bandwidth of 10 GHz gives an estimated ASE power of $-23 \, \text{dBm}$. This is seen to be a 10 dB improvement with respect to the other setup. Additionally, the performance degradation of the ASE signal is believed to be much smaller, because of the incoherent nature of ASE light. Mixing of the leak signal with the local oscillator is thus not present with this setup, however saturation of the detector is still of concern.

6.3 Atmospheric optics

The atmosphere always backscatters some fraction of the incident light and the contribution can be divided into two main groups. Molecular and Aerosol backscattering. Molecular scattering, which is always present, has a $\frac{1}{\lambda^4}$ dependency on the backscatter cross section. The scattering is elastic and more or less angular symmetric. On the contrary, aerosol scattering is only present when an aerosol or particle is within the sensitive volume and the particle has a size comparable to or greater than the incident wavelength. Aerosol scattering is a very complicated function of particle size, wavelength and profound in the forward direction for large particles.

The aerosol distribution as a function of altitude in the atmosphere is very inhomogeneous and depends on many factors. Also the particle density and size distribution is difficult to predict. A model distribution of particle diameters is shown in figure 6.5.

The figure clearly demonstrates that the mean diameter of the aerosols in the atmosphere is around 0.01 $\mu \text{m}$, which is much smaller than the wavelength used in an NIR LIDAR system. This is a fundamental limitation of a NIR system, since the backscattering cross-section is proportional to the physical size of the aerosols and for particles much smaller than the wavelength, the backscatter cross-section has a $\frac{1}{\lambda^4}$ dependency as Rayleigh (molecular) scattering.
6.4 Backscattered signal

The characteristics of the LIDAR system is most easily tested against a solid target, which backscatters a significant amount of the incoming energy. This is achieved using a rotating metal-plate inclined at some angle relative to the beam direction. A typical return signal is shown in figure 6.6. The return signal from a hard target can off course only simulate a signal originating from the atmosphere, yet basic features are more easily compared and investigated using this approach. The hard-target simulates the situation of an atmosphere with many scattering particles and the SNR obeys equation 6.2. This has been experimentally verified by measuring the SNR of two different optics, ($f200$; $D50$) and ($f840$; $D150$), were measured to give 38.8 dB and 40.0 dB respectively with an uncertainty of 0.5 dB. This is in agreement with equation 6.2 which states that the optics has negligible influence on the SNR.

The operation of the system was decided to be monostatic because of the before mentioned advantages. Optimizing the setup for this configuration actually improved performance considerably. A backscattered signal received from an
aerosol target is seen in figure 6.7. The figure shows a single particle scattering event as a function of time (upper left subfigure) and the corresponding spectrum (lower subfigure). It is possible to capture scattering events from single particles since the sounding volume is only of the order of cm$^3$ at short distances. In figure 6.7, the finite resting time of the particle in the sounding volume is approximately 160 $\mu$s, giving a signal burst of high SNR within this timeframe. The aerosol signals come in bursts at random times and a challenging task is hence to capture this signal in the most efficient way.

A time-series trace of single-scattering events is shown in figure 6.8. The captured events are frozen and displayed in a stacked event diagram in the left part of the figure. In this way, one can track fast aerosol returns and estimate longer timescale variations in the wind velocity. The challenge is to have every signal event captured and this was not possible with the equipment at hand and will probably require custom made signal processing equipment.

For an on-board real-time system, the update rate of aerosol signal events is of critical importance. Update rates around 20 Hz is necessary for the signal to appear live for the human eye and for flight instruments it has to be even higher. This is a challenging requirement, especially for a system that has to
fly at high altitudes, because aerosol density and size usually decreases with increasing altitude. The backscatter coefficient $\beta$ drops from around $10^{-6}$ $1/\text{m·sr}$ at ground level to $10^{-9}$ $1/\text{m·sr}$ at 20 km height \cite{89}. Thus the SNR is 30 dB lower at 20 km than at ground level and moreover the particle density has dropped, giving reduced update rates. The update rate obtained with this setup was only 4-6 Hz, even after the many improvements on optics and going to a monostatic setup and this was even indoor at ground level. However, a better update rate should be possible with better data capturing.

The aerosols signal events can be drastically increased by increasing the transmitted output power. This is because smaller particle signal events rise above the noise floor and the density of particles is increasing with decreasing size, see figure 6.5. This gives rise to a negative exponential signal probability distribution as can be seen in figure 6.9.

The smaller optics of $(f20, D50)$ has a sounding volume which is 80 times bigger than the volume of the $(f840, D150)$ optics. If the intensity of the beam is high enough, that the smaller particles have signals above the noise floor, the bigger sounding volume effectively boosts the update rate, which is proportional to the number of events above the noise events in figure 6.5. However, in
Figure 6.8: Time series data of aerosol events. The left figure shows the spectrum of a single event and the right figure shows contour diagram of events as a function of time (vertical axis) and frequency (horizontal axis). The colorscale shows each event’s SNR.

An atmosphere with very few particles, the bigger optics is superior, since the peak SNR is seen to be an order of magnitude higher than the smaller optics. This trade-off in optical arrangement of a tightly focused CW LIDAR system operating at close ranges is a very important design tool, especially when the update rate is considered.

An estimate of the update rates function of transmitted power is seen in figure 6.10. The smaller optics performs best at higher output power, since the atmosphere sounded in this experiment is relatively dense (ground level and indoor environment). The update rates shown in the figure are not real, since they are obtained by successive data captures at low sweep rates. The actual maximum update rates achieved with the system was approximately 4-6 Hz, also at ground level.
Figure 6.9: Aerosol signal statistics for two different optics. The distribution is a negative exponential. The smaller optics has more low signal events, but no high signal events.

6.5 Laser radiation safety

One very important feature and motivation of the NIR CW LIDAR described above is that it can be considered eye-safe. Laser systems are being more widespread in applications and the output power of laser radiation is ever increasing. When the laser beam, or reflections hereof, is directed into the surroundings where people are located, care should be taken to ensure safety from radiation. With a common term, this is usually referred to as eye-safety or eye-safe radiation.

Lasers are light sources that are highly concentrated in space and spectrum. Even though the output power of some lasers may seem relatively low with respect to incandescent lamps, they can be very harmful to the eye, since the radiation from lasers is much more concentrated. Even more, laser light can be tightly focused and hence the focusing ability of the eye can create a spot on the retina with very high intensity. The pupil diameter is around 7 mm and the smallest spot achievable on the retina is approximately 25 μm from the focusing of the eye, giving an area demagnification factor of nearly 80,000."
Figure 6.10: Pseudo update rate as a function of transmitted power. The smaller optics is superior as the smaller particles signal values rise above the noise floor. The update rate is not real-time, since it is obtained by successively stored sweeps from a scanning spectrum analyzer.

The peak sensitivity of the eye is in the wavelength range from around 400 nm to 700 nm, which is the visible region of the electromagnetic spectrum. Incident radiation on the eye in the wavelength range from 400 nm to 1400 nm is focused by the cornea and the eye-lens at the retina, but radiation from 700 nm to 1400 nm is invisible to the eye and hence the natural blinking or closing of the eyelid is not in function. Weak damages to the retina may heal, but damages from severe exposure levels are mostly permanent. Radiation in the IR region from 1400 – 3000 nm is not focused on the retina, but mainly absorbed by the cornea. Since exposure limit for the cornea (and lens) is somewhat higher than for the retina, the maximum radiation limit is higher in this wavelength range. Damage to the cornea can to some degree be healed or repaired. This is why the wavelength range from 1400 – 3000 nm is said to be eye-safe.

Exposure limits are defined by MPE values (Maximum Permissible Exposure) which is a complicated function of wavelength and exposure time (standard tables, see [91]). The lowest MPE values are found in the visible region of the spectrum where the eye has highest sensitivity. The most eye-safe wavelength...
6.6. SUMMARY

range, or more precise, the wavelength range with the highest MPE values is found between 1500 nm and 1800 nm. Nevertheless, it is never safe to look directly into a laser beam and one should always use common sense as the first safety measure.

A 1 W-class fiber based LIDAR in the 1.5 μm to 2.6 μm wavelength range is well suited for eye-safe operation, especially in CW operation, where the hazardous zone is only in the vicinity of the focal point. Therefore, there is a growing interest in developing LIDARs in this wavelength range.

6.6 Summary

A CW fiber laser system for measuring wind speeds has been studied with respect to dependency on optics and design. The performance has been greatly improved in terms of SNR and update rate by choosing a monostatic setup and optimizing the optics involved. A new design for the monostatic case has been suggested, in which the high power signals are isolated from the low power return signals to overcome the problems with coherent mixing of leak power from the transmit/receive switch. The system has been demonstrated to provide an update rate of approximately 4-6 Hz on capturing live aerosol data events.
Chapter 7

Conclusion

The study of thulium doped silica fibers in this project has covered a wide area. From basic research of spectroscopic properties to theoretical simulations of Tm:DFB fiber lasers and realizations of single-frequency thulium lasing ending up with a system application of a narrow-linewidth fiber laser. The project thus includes both basic and applied research results, always with the primary aim of performing research for applications.

Spectroscopic parameters have for the first time been measured accurately in silica fibers using a combination of methods selected from the criterion of having as few assumptions and uncertainties as possible. The excited-state lifetime of the thulium doped silica fibers used have been found by curve-fitting to fluorescent decay experiments to be 568 ms and 650 ms for Al/Ge codoping and Al/La codoping, respectively. The emission and absorption cross sections have been scaled using the ratios of maximum gain to maximum loss and the emission cross section spectrum has been found by applying a gain tilt method. Absorption cross section and rare-earth dopant concentration has then been calculated from GSA measurements and numerical calculations of the effective overlap between the optical mode and a relative dopant distribution. The peak absorption cross section was found to be 4.5 and $4.2 \times 10^{-25}$ m$^2$ for the two fibers. The peak emission cross section was only found to be 3.5 and $3.9 \times 10^{-25}$ m$^2$, which is somewhat lower than what previous results indicate. A 10 % quantum efficiency of the (3F$_4$,3H$_6$) transition of thulium in silica was found because of significant phonon quenching of this level.

With the purpose of using the spectroscopic data in theoretical work, an analytical model of DFB fiber lasers was deeply studied. The model was further
developed to account for asymmetric cavity designs and simple general equations were derived from the theory for approximate determination of optimal grating strength and asymmetry. The model was compared to numerical calculations to show excellent agreement in the situation of negligible pump power absorption. In cases of advanced cavity designs or high pump power absorption, it is necessary to simulate DFB fiber lasers with a numerical routine.

The numerical routine was implemented with the spectroscopic data of thulium doped silica fibers and simulations of threshold condition, output power and noise properties of thulium DFB fiber laser were performed. The results indicated an efficient operation of the DFB fiber laser under L-band pumping within practically possible pumping rates. However, comparison to experimental results showed a factor of 2 over-estimation of the slope-efficiency while fair agreement was obtained with the threshold pump power. The discrepancies are unclear at the moment, the best explanation being ion-clustering effects in the fibers.

The first experimental proof of single-frequency lasing in thulium DFB fiber lasers were obtained at a wavelength of 1.74 μm with a pump threshold of 50 mW and a slope-efficiency with respect to launched pump power of 0.2 % using a Ti:S laser tuned to 790 nm as pump laser. Maturing the cavity design and configuring for L-band pumping at a wavelength of 1595 nm provided a remarkable improvement in the slope-efficiency to nearly 2 % with a threshold pump power of 20 mW at the same laser wavelength of 1.74 μm. Pushing the limits for long-wavelength single-frequency emission was achieved with the demonstration of first a 1.98 μm and later a 2.09 μm single-frequency laser with slope-efficiencies around 1 %. Also power scaling was also demonstrated as a 60 mW module was formed using a very simple MOPA configuration in which unabsorbed pump power was used to pump an extra 20 cm of thulium fiber following the laser cavity. Power scaling using a MOPA configuration is thus an efficient way of achieving higher output power.

Finally, an application of a DFB fiber laser as a coherent source for a CW fiber lidar system was studied during a 4 month stay in the Corporate Research Center at EADS, Munich, Germany. The update rate and SNR dependency on receiving and transmitting optics was explored and it was found that a mono-static setup was far superior in terms of overall performance achieving an update rate around 5 – 8 Hz with a launched power of 500 mW. Coherent fiber-based lidars have a potential in aviation sensor systems provided that the output powers are scaled above 10 W and power leaking problems are solved.

Summing up, the successful demonstration of single-frequency emission from thulium doped DFB fiber lasers at wavelengths of 1.74 μm and 2.09 μm achieved
in this study broadens the fan of single-frequency emission wavelengths from DFB fiber lasers. The potential of the Tm:DFB laser is promising and goes beyond research laboratory environments. It is the hope of the author that results of this study will pave the way for further research and improvements of Tm:DFB fiber lasers.
Bibliography


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Appendix A

Derivations for the DFB model

In this appendix a couple of longer derivations are presented from the analytical model for a DFB fiber laser originally proposed by Scott Foster [17, 18, 48]. The derivations are given here and not in the main text in order not to lose focus on that main issues in the models. But the derivations are nevertheless considered important as they do not appear in [17, 18, 48] and they are essential in order to verify and deeply understand the model. Not to mention the situations where smaller misprints or even more serious mistakes have been made.

A.1 Gain rate equation

Consider a two-level system as shown is figure 3.4. The rate equation for the inversion is given by

\[
\frac{dx}{dt} = (1 - x)(P_a + S_a) - x\left(\frac{1}{\tau_{fi}} + S_e + P_e\right)
\]  

(A.1)

which is solved in the steady state for \( \dot{x} \) to give

\[
\frac{n_2}{n_2 + n_1} = \dot{x} = \frac{P_a + S_a}{P_a + P_e + S_a + S_e + \frac{1}{\tau_{fi}}}
\]  

(A.2)
\section{A.1. Gain Rate Equation}

Now a zero is subtracted from the inversion equation

\begin{equation}
\frac{d\hat{x}}{dt} = -(1 - \hat{x})(P_a + S_a) + \hat{x}\left(\frac{1}{\tau_{fl}} + S_c + P_c\right) \tag{A.3}
\end{equation}

\begin{equation}
\frac{d(x - \hat{x})}{dt} = (1 - x)(P_a + S_a) + x\left(\frac{1}{\tau_{fl}} + S_c + P_c\right) - (1 - \hat{x})(P_a + S_a) + \hat{x}\left(\frac{1}{\tau_{fl}} + S_c + P_c\right) \tag{A.4}
\end{equation}

\begin{equation}
\frac{d(x - \hat{x})}{dt} = -(P_a + S_a + \frac{1}{\tau_{fl}} + S_c + P_c)(x - \hat{x}) \tag{A.5}
\end{equation}

, which is inserted into the gain equation 3.29 after using the same approach for this

\begin{equation}
\frac{d\gamma}{dt} = \frac{d(\gamma - \hat{\gamma})}{dt} = \frac{d(\rho(\sigma_{es} + \sigma_{as})(x - \hat{x}))}{dt} = -\rho(\sigma_{es} + \sigma_{as})(x - \hat{x})(P_a + S_a + \frac{1}{\tau_{fl}} + S_c + P_c) \nonumber
\end{equation}

\begin{equation}
\frac{d\gamma}{dt} = -(\gamma - \hat{\gamma})(P_a + S_a + \frac{1}{\tau_{fl}})(1 + P\frac{P_a}{P_s}) \nonumber
\end{equation}

which is then rewritten to

\begin{equation}
\frac{d\gamma}{dt} = -\left(\gamma - \frac{\gamma_l}{1 + \frac{|E|^2}{P_s}}\right)\left(\frac{P_a}{P_a} + 1\right)\left(1 + \frac{|E|^2}{P_s}\right) = \frac{1}{\tau_g} \left(-\gamma\left(1 + \frac{|E|^2}{P_s}\right) + \gamma_l\right) \tag{A.6}
\end{equation}

where a gain medium lifetime has been defined as

\begin{equation}
\tau_g = \frac{\tau_{fl}}{1 + P\frac{P_a}{P_s}} \tag{A.7}
\end{equation}

and the equations 3.35, 3.36 has been recovered.

It is here noted, that the derived equation is the intensity gain, with \(\tau_{fl}\) being an intensity lifetime. This can cause confusion in the value for \(\tau_g\), so looking closer at the expression for this gives

\begin{equation}
P_a + P_c + \frac{1}{\tau_{fl}} = \sigma_{ap}\frac{P_a}{\hbar\omega_p}\Gamma_p + \sigma_{ep}\frac{P_p}{\hbar\omega_p}\Gamma_p + \frac{1}{\tau_{fl}} \tag{A.8}
\end{equation}

which is converted into an amplitude equation by making the substitutions \(\tau_a = 2\tau_{fl}\), \(a_p = \frac{1}{2}\sigma_{ap}\rho\Gamma_p\), \(g_p = \frac{1}{2}\sigma_{ep}\rho\Gamma_p\) which gives

\begin{equation}
\tau_g^{-1} = 2\sigma_{ap}\frac{P_p}{\hbar\omega_p\rho} + 2\sigma_{ep}\frac{P_p}{\hbar\omega_p\rho} + \frac{2}{\tau_a} = \frac{2}{\tau_a}\left(P_p\left(\frac{\tau_a(a_p + g_p)}{\hbar\omega_p}\right) + 1\right) = \frac{2\left(\frac{P_p}{\tau_a} + 1\right)}{\tau_a} \tag{A.9}
\end{equation}
It is seen that $P_{y}$ is not changed by using either amplitude or intensity coefficients since the product $\tau\left(a_{y} + g_{y}\right)$ cancels this dependency. It is therefore seen that equation (A.7) is again recovered.

### A.2 Average gain integral

The following illustrates the calculation of the important gain integral for a DFB fiber laser [92]. Consider the integral for the average gain $\gamma_{avg}$

$$\gamma_{avg} = \gamma_{f} \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{e^{-2k|x|} \frac{1}{T_{r}} (1 + \sin(2k_{b}z + \phi))}{1 + \frac{4\pi}{\lambda_{r}} e^{-2k|x|} \frac{1}{T_{r}} (1 + \sin(2k_{b}z + \phi))} dz,$$

(A.10)

which is fundamental for the DFB laser theory.

To integrate a fast varying function and a slowly varying function, one integrates by period and sum $I = \sum_{n=0}^{x_{n}} I_{n}(x_{n})$, $x_{n} = \frac{2\pi}{k_{b}}$, where

$$I_{n}(x_{n}) \approx \int_{x_{n}}^{x_{n+1}} \frac{\frac{1}{2} f(x_{n}) (1 + \sin(2k_{b}x))}{1 + \frac{4\pi}{\lambda_{r}} K f(x_{n}) + \frac{1}{2} K f(x_{n}) \sin(2k_{b}x)} dx,$$

(A.11)

which is equivalent to the integral to obtain the average gain. Use that $\sin(2k_{b}x) = \frac{1}{2i}(z - 1/z)$, where $z = e^{2k_{b}x}$ and $dz = \frac{1}{i2k_{b}x} dx$ to obtain

$$I_{n}(x_{n}) = \int_{iz}^{iz} \frac{f(x_{n})(z + i)^{2}}{iz [(4 + 2K f(x_{n})z + K f(x_{n})(z^{2} - 1))] dz$$

(A.12)

The denominator is has a 2nd order polynomial, i.e $K f(x_{n})z(z - z^{+})(z - z^{-})$ such that $z^{+}z^{-} = 1$ and $|z^{\mp}| < 1$. The integral is then reduced to

$$I_{n}(x_{n}) = \frac{1}{K} \int_{iz}^{iz} \frac{(z + i)^{2}}{z(z - z^{+})(z - z^{-})} dz$$

(A.13)

which has poles inside unit circle for $z = 0$ and $z = z^{+}$. Using residual calculus of complex integrals, i.e.

$$\int_{c} f(x) dx = i2\pi \sum_{i} Res_{z=p_{i}} (f(x))$$

(A.14)

$$Res \left( \frac{g(x)}{h(x)} \right) = \frac{g(x_{0})}{h'(x_{0})}$$

(A.15)
the integral gives

$$I_n(x_n) = \frac{i2\pi}{K} \left( 1 + \frac{(z^+ + i)^2}{z^+(z^+ - z^-)} \right)$$

(A.16)

In which $z^+$ and $z^-$ are substituted to give

$$I_n(x_n) = \frac{2\pi}{K} \frac{Kf(x_n)}{1 + Kf(x_n) + \sqrt{(1 + Kf(x_n)}}$$

(A.17)

and we can go back to integration from summation

$$I \approx \frac{2\pi}{k_b K} \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{K e^{-2\kappa |x|}}{1 + K e^{-2\kappa |x|} + \sqrt{1 + K e^{-2\kappa |x|}} \Delta x}$$

(A.18)

$$= \frac{2}{K \kappa} \ln \left( \frac{1 + \sqrt{1 + K}}{1 + \sqrt{1 + K e^{-2\kappa L}}} \right)$$

(A.19)

$$\approx \frac{2}{K \kappa} \ln \left( \frac{1 + \sqrt{1 + K}}{2} \right),$$

(A.20)

where the integration has been done using the substitution $y = \sqrt{1 + K e^{-2\kappa |x|}}$

and using the integrated function is symmetric about the origin. Finally

$$\gamma_{avg} = \frac{2\gamma_t}{L_c} I = 4\gamma_t \frac{\ln \left( \frac{1 + \sqrt{1 + X}}{2} \right)}{X}$$

(A.21)

In the case of a phase shift placed at a distance $x$ to the right of the cavity center, the integral gives

$$\gamma_{avg} \approx 2\gamma_t \left[ \ln \left( \frac{1 + \sqrt{1 + X}}{2} \right) + \ln \left( \frac{1 + \sqrt{1 + X e^{-2\kappa (\frac{L}{2} - x)}}}{X} \right) \right],$$

(A.22)

from which it is seen that if $x = \frac{L}{2}$, the integral becomes half the value of that with a symmetric phase shift. Solving for $X$ in this equation yields almost the same result as using equation A.20 unless $x$ becomes a considerable fraction of $\frac{L}{2}$. 

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A.3. DIFFERENTIATION WITH RESPECT $\kappa$ AND $x$

A.3 Differentiation with respect $\kappa$ and $x$

For the optimization of the output power with respect to the parameters $\kappa$ and $x$, the output power equation is differentiated. The single-sided output power can be written in the high pump power limit as

$$P_{\text{out}} = E(L/2)^2 - \frac{2}{L_c} e^{-2\kappa(\frac{L}{2} - x)} \frac{\bar{P}_s L_c}{2} = 2\gamma_t P_s e^{-2\kappa(\frac{L}{2} - x)} \frac{\bar{P}_s}{\gamma_c + \alpha} \ln \left( \frac{2\gamma_t}{\gamma_c + \alpha} \right)$$

$$= A e^{-2\kappa(\frac{L}{2} - x)} \frac{2\gamma_t}{\gamma_c + \alpha} \ln \left( \frac{2\gamma_t}{\gamma_c + \alpha} \right), \quad (A.23)$$

where terms irrelevant to the differentiation are left in the constant $A$. First, it is noted that the gradient $\nabla_{\kappa,x} P_{\text{out}}$ is a rather messy function where it is impossible to separate either of the variables. Therefore, the approach of first assuming $x = 0$ for the differentiation with respect to $\kappa$ and then inserting this into the differentiated with respect to $x$ is adopted.

First, differentiating with respect to $\kappa$ and equating the differential to zero, while noting that $(2\kappa e^{-\kappa L})' = 2e^{-\kappa L}(1 - \kappa L)$ and assuming for simplicity that $x = 0$ gives exactly

$$2e^{-2\kappa L} (\kappa L - 1) = (\alpha L e^{-\kappa L} + 2e^{-2\kappa L}) \ln \left( \frac{2\gamma_t}{\gamma_c + \alpha} \right) \quad (A.24)$$

Secondly, differentiating with respect to $x$ and equating to zero gives

$$\left[ \cosh(2\kappa x) - \sinh(2\kappa x) + \frac{2\kappa e^{\kappa L}}{\sigma} \right] \ln \left( \frac{2\gamma_t}{\gamma_c + \alpha} \right) = \sinh(2\kappa x), \quad (A.26)$$

and the equations in the main text can be derived from these.

A.4 G(x) and relaxation oscillations

The important function $G(X) = \frac{1}{\sigma}$ appears frequently in the DFB fiber laser context. It is defined as

$$G(x) = \int_{-\frac{Z}{2}}^{\frac{Z}{2}} \frac{|\varepsilon(z)|^2}{1 + \frac{1}{Z} e^{\kappa L} |\varepsilon(z)|^2} dz, \quad (A.27)$$

For a DFB fiber laser with a constant grating strength $\kappa$ and a central $\pi$ phase shift, the function is shown above to be

$$G(x) = \frac{1}{x} \ln \left( \frac{1 + \sqrt{1 + x}}{2} \right) \quad (A.28)$$
A.4. $G(x)$ AND RELAXATION OSCILLATIONS

Figure A.1: Blue curve shows $G(x) = \sigma^{-1}$ and red curves are small and large argument expansions respectively. Inset figure is zoom on $G(x)$ for small arguments.

where a small and large argument expansion are

$$G(x) = 1 - \frac{3}{8}x, \quad |x| < 1 \quad (A.29)$$

$$G(x) = \frac{3 \ln x}{2x}, \quad x \text{ large} \quad (A.30)$$

The functions are plotted in figure A.1

The solution, or the characteristic equation, of the perturbed gain and amplitude equations in the Fourier domain was found to be

$$\nu^2 - \nu_0^2 G\left(\frac{\hat{X}}{i\nu \tau_g + 1}\right) + \nu_0^2 G(\hat{X}) = 0. \quad (A.31)$$

Now use that $i\nu \tau_g + 1 \approx i\nu \tau_g$, since $\nu$ will be close to $\nu_0$ and that $\nu_0 \tau_g >> 1$ as apparent from the solution. Moreover, use that $\frac{\hat{X}}{i\nu \tau_g}$ is small, such that the first
A.5. PUMP INTENSITY NOISE

G-function can be small argument expanded to give

\[ \nu^2 - \nu_0^2(1 - G(\hat{X})) + \frac{3 \nu_0^2 \hat{X}}{8 \tau_0 \nu} = 0 \]  \hspace{1cm} (A.32)

Once again use that \( G(\hat{X}) << 1 \) since \( \hat{X} \) is large and solve the equation recursively to

\[ \nu_n = \pm \nu_0 \sqrt{1 - \frac{3 \hat{X}}{8 \tau_0 \nu_{n-1}}} \]  \hspace{1cm} (A.33)

The square root is expanded to 1st order \( (\sqrt{1 - x} \approx 1 - \frac{1}{2} x) \) and once again by recursive substitution to give

\[ \nu = \pm \nu_0 + \frac{3 \hat{X}}{16 \tau_0} \]  \hspace{1cm} (A.34)

where the solution with the negative imaginary part has been trashed since the optical field will go like \( e^{i \nu t} \). Also the apparent solution \( \nu = 0 \) is neglected, since it is non-physical. The assumptions behind this result can be summed up to \( 1 << \hat{X} << \nu_0 \tau_0, \nu_0 \tau_0 \sim 10^3 \). \( X \) will therefore generally be around 100. For practical DFB fiber lasers this is only the case near the threshold of the laser. In the high pump regime \( P_p >> P_t, X \) will typically have a value around \( 10^3 \) and the actual relaxation frequency will be smaller than \( \nu_0 \).

A.5 Pump intensity noise

Assume that the pump power is of the form \( P_p + \Delta_p(t) \) then the gain equation is rewritten to

\[ \frac{d\gamma}{dt} = \frac{1}{\tau_g} \left[ -\gamma \left( 1 + \frac{|E|^2}{P_s} \right) + \gamma_t \right] + \frac{q \Delta_p}{\tau_g} \]  \hspace{1cm} (A.35)

where

\[ q = \frac{\gamma_{avg} |E|^2}{P_s} \left( \frac{g_l - \gamma_t}{P_t + P_p} \right) + \frac{g_l - \gamma_t}{P_t + P_p} \]  \hspace{1cm} (A.36)

\[ = \frac{\frac{2}{\tau} - \hat{\gamma}}{P_t + P_p}, \]  \hspace{1cm} (A.37)
where it is remembered that $\dot{\gamma} = \frac{\gamma}{1 + \frac{\nu}{\tau_g}}$ and that $|E|^2 = |\tilde{A}|^2|c(z)|^2$. This is inserted in the gain equation to become

$$
\frac{d\gamma}{dt} = \frac{1}{\tau_g} \left[ -(\dot{\gamma} + \eta) \left( 1 + \frac{\dot{A}^2(1 + 2\varepsilon)|c|^2}{P_s} \right) + \gamma \right] + \frac{q\Delta_p}{\tau_g} \quad (A.38)
$$

Multiplying by $\tau_g$ and isolating with respect to $\eta$ after having substituted $\frac{d\eta}{dt} = \frac{d\eta}{dt}$ and Fourier transformed. This gives in the Fourier domain

$$
\eta = \frac{2\dot{A}^2\gamma|c(z)|^2\varepsilon}{(i\nu\tau_g + 1 + \frac{\dot{A}^2|c|^2}{P_s}) P_s} + \frac{q\Delta_p}{i\nu\tau_g + 1 + \frac{\dot{A}^2|c|^2}{P_s}} \quad (A.40)
$$

$$
= \frac{2\dot{A}^2\gamma|c(z)|^2\varepsilon}{(i\nu\tau_g + 1 + \frac{\dot{A}^2|c|^2}{P_s}) P_s \left( 1 + \frac{\dot{A}^2|c|^2}{P_s} \right)} + \frac{q\Delta_p}{i\nu\tau_g + 1 + \frac{\dot{A}^2|c|^2}{P_s}} \quad (A.41)
$$

From where the first terms are identical to those in [18]. Concentrating on the second term gives

$$
\eta_{2nd} = \frac{g_t\Delta_p}{P_t + P_p} \frac{1}{1 + i\tau_g\nu + \frac{\dot{A}^2|c|^2}{P_s}} - \frac{\gamma_t\Delta_p}{P_t + P_p \left( 1 + i\tau_g\nu + \frac{\dot{A}^2|c|^2}{P_s} \right) \left( 1 + \frac{\dot{A}^2|c|^2}{P_s} \right)}
$$

$$
= \frac{g_t\Delta_p}{P_t + P_p} \frac{1}{1 + i\tau_g\nu + Z} - \frac{\gamma_t\Delta_p}{P_t + P_p \left( 1 + i\tau_g\nu + Z \right) (1 + Z)} \quad (A.42)
$$

Previously, a $Z$ was also in the numerator, but not now, which gives a decomposition of the last fraction to

$$
-\frac{1}{i\tau_g\nu + Z} + \frac{1}{i\tau_g\nu + Z} \quad (A.43)
$$

where the sign is remembered

$$
\eta_{2nd} = \frac{g_t\Delta_p}{P_t + P_p} \frac{1}{1 + i\tau_g\nu + Z} - \frac{\gamma_t\Delta_p}{P_t + P_p \left( 1 + i\tau_g\nu + Z \right) (1 + Z)}
$$
A.5. PUMP INTENSITY NOISE

If it is assumed that $P_p >> P_r$, such that $\gamma_t = 1/2g_l$ the whole thing reduces to

$$\eta_{2\text{nd}} = \frac{g_l \Delta P}{2P_p} \left( \frac{1 + i \tau_g \nu}{1 + i \tau_g \nu + Z} - \frac{1}{1 + Z} \right)$$

$$= \frac{g_l \Delta P}{2P_p i \nu \gamma_{avg}} \left( \frac{1}{1 + \frac{Z}{i \nu \tau_g}} - \frac{1}{1 + Z} \right),$$

(A.44)

which can be transformed into $h(\nu)$ by multiplication with $|e(z)|^2$ and integrating, which gives

$$h(\nu) = \frac{g_l \Delta P}{2P_p i \nu \gamma_{avg}} \left( G \left( \frac{\hat{X}}{i \nu \tau_g + 1} \right) - G \left( \frac{\hat{X}}{1 + 1} \right) \right) = \frac{\Delta P \nu_0^2 \tau_c}{P_p 2i \nu} \Gamma(\hat{X})$$

where $\nu_0^2 = \frac{2\gamma c}{\tau_g}$ is the small-signal relaxations-frequency and when the laser is running the cavity lifetime is the inverse of the average gain $\tau_c = \frac{1}{\gamma_{avg}}$. The Matrix now becomes

$$\begin{bmatrix} i \nu & -\frac{1}{\nu_0^2} \\ \nu_0^2 \tau_c \Gamma & i \nu \end{bmatrix} \begin{bmatrix} \frac{\delta}{h} \\ 0 \end{bmatrix} = \frac{\Delta P \nu_0^2 \tau_c \Gamma}{P_p} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

such that the power spectral density in the photodiode output is given by

$$S_p(\nu) = \frac{|\Delta P(\nu)|^2}{P_p^2} \frac{\nu_0^4 \Gamma^2(\nu)}{[\nu^2 - \nu_0^4 \Gamma(\nu)]^2} P_p^2,$$

(A.45)
Appendix B

Cylindrical 1D FEM modesolver

This is a short description of a one dimensional finite element modesolver used throughout the thesis for calculating modal profiles and eigenvalues for a circularly symmetric fiber with arbitrary index profile.

The eigenvalue equation to be solved for a circularly symmetric optical fiber with refractive index profile \( n(r) \) is

\[
\nabla^2 E_z + (n^2 k^2 - \beta^2) E_z = 0, \tag{B.1}
\]

where \( E_z \) is the longitudinal component of the electric field vector. It is then assumed that the field can be expanded in a sum of \( N \) trial functions \( \varphi_i \) like

\[
E_z = \sum_{i=1}^{N} \varphi_i a_i, \tag{B.2}
\]

where \( a_i \) is the \( i^{th} \) amplitude of the trial functions \( \varphi_i = \tilde{\varphi}_i e^{j m \phi} \) being triangular as

\[
\tilde{\varphi}_i(r) = \begin{cases} 
1 + \frac{r-r_i}{r_{i+1}-r_i}, & r < r_i \\
1 + \frac{r-r_{i+1}}{r_{i+1}-r_i}, & r > r_i \\
0_i, & \text{elsewhere}
\end{cases} \tag{B.3}
\]

Inserting equation B.2 into B.1, while multiplying with the \( j^{th} \) trialfunction
and integrating over the transverse cross-section gives

\[ \sum_{i=1}^{N} \left( L_{i,j} + k^2 N_{i,j} \right) a_i = \beta^2 \sum_{i=1}^{N} E_{i,j} a_i, \quad (B.4) \]

where the matrix elements \( L_{i,j}, N_{i,j} \) and \( E_{i,j} \) are given by

\[
L_{i,j} = \int_A \phi_j^* \nabla_i^2 \phi_i dS \quad (B.5)
\]

\[
N_{i,j} = \int_A \phi_j^* n^2 \phi_i dS \quad (B.6)
\]

\[
E_{i,j} = \int_A \phi_j^* \phi_i dS. \quad (B.7)
\]

In matrix form, the equation is reduced to a generalized eigenvalue equation

\[ \mathbf{K} \mathbf{a} = \beta^2 \mathbf{E} \mathbf{a}, \quad (B.8) \]

where \( \mathbf{K} = \mathbf{L} + k^2 \mathbf{N} \) and the problem can be solved by standard methods. The problem is then reduced to calculating simple overlap integrals of powers in \( r \) and keeping track of indices, while finally solving a sparse generalized eigenvalue matrix equation.

Figure B.1: Triangular trial functions for different indices. Left- and rightmost indices have separate treatment, otherwise the functions have both a left side, \( \varphi_l \) and a rightside \( \varphi_r \).
In general, the integral of the triangular functions can be split into integrals of leftside functions $\varphi_l$ and rightside functions $\varphi_r$, see figure B.1. The special case where the second quantum number $m$ is zero is treated separately, while for $m \neq 0$ it will only influence the matrix elements of $\mathbf{N}$ and $\mathbf{L}$. The matrix elements are calculated in the following, starting with the $\mathbf{E}$ matrix, which is the self-overlap matrix. For complete overlap, $j=1$ and

$$
E_{i,i} = 2\pi \int_{r_i}^{r_{i+1}} \varphi_i^* \varphi_i r dr = 2\pi \left[ \int_{r_i}^{r_i} \varphi_i^2 r dr + \int_{r_i}^{r_{i+1}} \varphi_i^2 r dr \right]
= 2\pi \left[ \int_{r_i}^{r_i} \left( 1 + \frac{r - r_i}{r_i - r_{i-1}} \right) ^2 r dr + \int_{r_i}^{r_{i+1}} \left( 1 + \frac{r - r_i}{r_i - r_{i+1}} \right) ^2 r dr \right],
$$

$$(B.9)$$

A new set of variables are defined as $r' = r - r_{i-1}$ and $r' = r - r_{i+1}$ and using substitution in the integrals gives

$$
2\pi \left[ \int_0^{\Delta_{i-1}} \left( 1 + \frac{r' - \Delta_{i-1}}{\Delta_{i-1}} \right) ^2 (r' + r_{i-1}) dr' + \int_0^{\Delta_{i+1}} \left( 1 + \frac{r' - \Delta_{i+1}}{\Delta_{i+1}} \right) ^2 (r' + r_{i+1}) dr' \right]
= 2\pi \left[ \int_0^{\Delta_{i-1}} \frac{r'}{\Delta_{i-1}} (r' + r_{i-1}) dr' + \int_0^{\Delta_{i+1}} \frac{r'}{\Delta_{i+1}} (r' + r_{i+1}) dr' \right]
= 2\pi \left( \frac{\Delta_i^2}{4} + \frac{r_{i-1} \Delta_{i-1}}{3} - \frac{\Delta_i^2}{4} + \frac{r_{i+1} \Delta_{i+1}}{3} \right),
$$

$$(B.10)$$

where $\Delta_{i-1} = r_i - r_{i-1}$ and $\Delta_{i+1} = r_i - r_{i+1}$. In the case that $j = i \pm 1$ there is partly overlap and the matrix element becomes

$$
E_{i,i+1} = 2\pi \int_{r_i}^{r_{i+1}} \varphi_i \varphi_{i+1} r dr
= 2\pi \int_{r_i}^{r_{i+1}} \left( 1 + \frac{r - r_i}{r_i - r_{i+1}} \right) \left( 1 + \frac{r - r_{i+1}}{r_{i+1} - r_i} \right) r dr
= 2\pi \int_{\Delta_{i+1}}^{0} \left( 1 + \frac{r' - \Delta_{i+1}}{\Delta_{i+1}} \right) \left( 1 - \frac{r'}{\Delta_{i+1}} \right) (r' + r_{i+1}) dr'
= -2\pi \left( \frac{1}{12} \Delta_{i+1}^2 + \frac{1}{6} r_{i+1} \Delta_{i+1} \right).
$$

$$(B.11)$$

The integral is of course invariant for the substitution of $i + 1 \rightarrow i - 1$ with a
change of sign, i.e.

\[
E_{i-1,j} = 2\pi \left( \frac{1}{12} \Delta_{i-1}^2 + \frac{1}{6} r_{i-1} \Delta_{i-1} \right).
\]  

For all other pairs of \(i, j\) the integral vanishes and the self-overlap matrix becomes a tridiagonal sparse matrix.

The matrix elements for the index matrix \(\mathbf{N}\) depends on the refractive index profile of the fiber. If the sampling of a measured index profile \(n_m(m = 1..M)\) does not coincide with the grid of the finite element trialfunction expansion, then it is converted into this using the linear expansion

\[
n^2(r) = \sum_{k=1}^{N} \left( \sum_{m=1}^{M} n_m^2(m) \varphi_m(r_k) \right) \varphi_k(r) = \sum_{k=1}^{N} n_k^2 \varphi_k(r),
\]

which is then inserted in equation B.6

\[
N_{i,j} = \int_{A} \varphi_j^* \sum_{k=1}^{N} n_k^2 \varphi_k(r) \varphi_i dS = \sum_{k=1}^{N} n_k^2 \int_{A} \varphi_j^* \varphi_k \varphi_i dA_i
\]

which gives values different from zero for \(j = i \land k = i \pm 1, i\). This means that there are 2 basic situations of overlap between the trialfunctions and all others are combinations hereof. The first basic integral is where two trialfunctions are overlapping and the third partly overlapping, for example \(j = i \land k = i + 1\)

\[
N_{i,i} = 2\pi \int_{r_i}^{r_{i+1}} \left( 1 + \frac{r - r_i}{r_i r_{i+1}} \right) \left( 1 + \frac{r - r_{i+1}}{r_i r_{i+1}} \right) r dr \\
= 2\pi \int_{\Delta_{i+1}}^{r_{i+1}} \left( \frac{r'}{\Delta_{i+1}} \right) \left( 1 - \frac{r'}{\Delta_{i+1}} \right) (r' + r_{i+1}) dr' \\
= -2\pi \left( \frac{1}{20} \Delta_{i+1}^2 + \frac{1}{12} r_{i+1} \Delta_{i+1} \right),
\]

and the situation for \(j = i \land k = i - 1\) gives the same with a change of sign and the substitution \(i + 1 \rightarrow i - 1\). The last case of complete overlap, \(j = k = i\), of
all three trialfunctions gives

\[
N_{i,i} = 2\pi \left[ \int_{r_{i-1}}^{r_i} \left( 1 + \frac{r - r_i}{r_i - r_{i-1}} \right)^3 r \, dr + \int_{r_i}^{r_{i+1}} \left( 1 + \frac{r - r_i}{r_i - r_{i+1}} \right)^3 r \, dr \right]
= 2\pi \left[ \int_0^{\Delta_{i-1}} \left( \frac{r'}{\Delta_{i-1}} \right)^3 (r' + r_{i-1}) \, dr' - \int_0^{\Delta_{i+1}} \left( \frac{r'}{\Delta_{i+1}} \right)^3 (r' - r_{i+1}) \, dr' \right]
= 2\pi \left[ \frac{1}{5} \Delta_{i-1}^2 + \frac{1}{4} r_{i-1} \Delta_{i-1} - \frac{1}{5} \Delta_{i+1}^2 - \frac{1}{4} r_{i+1} \Delta_{i+1} \right].
\] (B.16)

There is an extra case, where one trialfunction is at the evaluation point and the two other trialfunctions are displaced by one index, for instance \( j = i - 1 \) and \( k = i - 1 \)

\[
N_{i-1,i} = 2\pi \left[ \int_{r_{i-1}}^{r_i} \left( 1 - \frac{r - r_{i-1}}{r_i - r_{i-1}} \right)^2 \left( 1 + \frac{r - r_i}{r_i - r_{i-1}} \right) r \, dr \right]
= 2\pi \left[ \int_0^{\Delta_{i-1}} \left( 1 - \frac{r'}{\Delta_{i-1}} \right)^2 \left( \frac{r'}{\Delta_{i-1}} \right) (r' + r_{i-1}) \, dr' \right]
= 2\pi \left[ \frac{1}{30} \Delta_{i-1}^2 + \frac{1}{12} r_{i-1} \Delta_{i-1} \right].
\] (B.17)

The Laplacian matrix elements can be calculated after simplification using integration by parts. The Laplacian term in cylindrical coordinates are from eq. B.5

\[
L_{i,j} = \int_A \left( \frac{d^2}{dr^2} \frac{1}{r} \frac{d}{dr} \right) \varphi_i \, dS
= 2\pi \left( \int_{r_i}^{r_{i+1}} \left( \varphi_j^* \frac{d\varphi_i}{dr} + \varphi_j \frac{d\varphi_i}{dr} \right) \, dr + \int_{r_{i-1}}^{r_i} \left( \varphi_j^* \frac{d\varphi_i}{dr} + \varphi_j \frac{d\varphi_i}{dr} \right) \, dr \right)
= -2\pi \int_0^{r_{i+1}} \varphi_j^* \frac{d\varphi_i}{dr} \, dr,
\] (B.18)

where it has been used that a guided mode must vanish at infinity. The ma-
triple element for \( j = i \) is then

\[
L_{i,i} = -2\pi \int_{r_{i-1}}^{r_i} \frac{d\varphi_i^+}{dr} d\varphi_i r dr
\]

\[
= -2\pi \left[ \int_{r_{i-1}}^{r_i} \frac{1}{(r_i - r_{i-1})^2} r dr + \int_{r_i}^{r_{i+1}} \frac{1}{(r_i - r_{i+1})^2} r dr \right]
\]

\[
= -2\pi \left[ \int_0^{\Delta_{i-1}} \frac{1}{\Delta_{i-1}^2} (r' + r_{i-1}) dr' - \int_0^{\Delta_{i+1}} \frac{1}{\Delta_{i+1}^2} (r' + r_{i+1}) dr' \right]
\]

\[
= 2\pi \left( \frac{r_{i+1}}{\Delta_{i+1}} - \frac{r_{i-1}}{\Delta_{i-1}} \right), \quad \text{(B.19)}
\]

and for the case of \( j = i - 1 \) gives

\[
L_{i,i-1} = -2\pi \int_0^{\Delta_{i-1}} \frac{1}{\Delta_{i-1}^2} (r' + r_{i-1}) dr'
\]

\[
= 2\pi \left( \frac{1}{2} + \frac{r_{i-1}}{\Delta_{i-1}} \right), \quad \text{(B.20)}
\]

with the usual sign change and substitution of \( i - 1 \rightarrow i + 1 \) for the case of \( j = i + 1 \).

If the second quantum number \( m \) is different from zero, the Laplacian changes such that

\[ m \neq 0 \quad L_{i,j} = \int_A \varphi_j^+ \left( \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} \right) \varphi_i dS \]

\[
= -2\pi \int \frac{\partial \varphi_j^+}{\partial r} \varphi_i r dr + \int \int \varphi_j^+ \frac{\partial^2 \varphi_i}{\partial \phi^2} \frac{1}{r} dr d\phi
\]

\[
= -2\pi \int \frac{d\varphi_j^+}{dr} \frac{d\varphi_i}{dr} r dr - 2\pi m^2 \int \varphi_j^+ \varphi_i \frac{1}{r} dr, \quad \text{(B.21)}
\]

The first integral is identical to B.18. The last integral is the new term and it gives for the case of complete overlap \( j = i \)

\[
-2\pi m^2 \left( \int_{r_{i-1}}^{r_i} \left( 1 + \frac{r - r_i}{r_{i-1} - r_i} \right)^2 \frac{1}{r} dr + \int_{r_i}^{r_{i+1}} \left( 1 + \frac{r - r_i}{r_i - r_{i+1}} \right)^2 \frac{1}{r} dr \right)
\]

\[
= -2\pi m^2 \left( \frac{1}{\Delta_{i-1}^2} \int_0^{\Delta_{i-1}} \frac{r'^2}{r'} dr' - \frac{1}{\Delta_{i+1}^2} \int_0^{\Delta_{i+1}} \frac{r'^2}{r'} dr' \right)
\]

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where it is again seen, that it is only necessary to calculate one of the integrals and use substitution for the other, i.e. for the leftmost integral

\[
-2\pi m^2 \frac{1}{\Delta_{i-1}} \left( \frac{(\Delta_{i-1} + r_{i-1})^2}{2} - 2r_{i-1}(\Delta_{i-1} + r_{i-1}) + r_{i-1}^2 \ln(\Delta_{i-1} + r_{i-1}) \right) - \frac{r_{i-1}^2}{2} + 2r_{i-1}^2 - r_{i-1}^2 \ln(r_{i-1}) \right) \\
= -2\pi m^2 \left( \frac{r_i - 3r_{i-1}}{2\Delta_{i-1}} + \frac{r_{i-1}^2}{\Delta_{i-1}} \ln\frac{r_i}{r_{i-1}} \right),
\]

(B.22)

and in the case of partial overlap, \( j = i - 1, m \neq 0 \)

\[
\mathbf{L}_{i,i-1} = -2\pi m^2 \int_{r_{i-1}}^{r_i} \left( 1 - \frac{r - r_{i-1}}{r_i - r_{i-1}} \right) \left( 1 + \frac{r - r_i}{r_i - r_{i-1}} \right) \frac{1}{r} dr \\
= -2\pi m^2 \int_{r_{i-1}}^{\Delta_{i-1}} \left( 1 - \frac{r'}{\Delta_{i-1}} \right) \left( 1 + \frac{r'}{\Delta_{i-1}} \right) \frac{1}{r + r_{i-1}} dr' \\
= -2\pi m^2 \left[ \left( 1 + \frac{r_{i-1}}{\Delta_{i-1}} \ln\frac{r_{i-1}}{r_{i}} \right) - \left( \frac{r_i - 3r_{i-1}}{2\Delta_{i-1}} + \frac{r_{i-1}^2}{\Delta_{i-1}} \ln\frac{r_i}{r_{i-1}} \right) \right] \\
\]

and again substitution and change of sign for \( j = i + 1 \).

To evaluate the method and the actual program, the performance has to be compared to a well known situation, preferably in which a analytical expression of the modal properties are available. This is the case for the parabolic index distribution, which is given by

\[
n_p^2(r) = n_2^2 \left( 1 - 2\Delta n \frac{r^2}{a^2} \right),
\]

(B.24)

where \( n_2 \) is the peak index and \( \Delta n \) is the index difference and \( a \) is the fiber core radius. The eigenmodes can be grouped in \( LP_{\ell p} \) modes as in the step index fiber, where \( \ell \) is the azimuthal quantum number and \( p \) is the radial quantum number. A total quantum number \( m \) is defined as \( m = 2p + l - 1 \), which is the modal number, i.e. \( m = 1 \) is the fundamental mode. The effective index of a circular symmetric fiber with the index distribution of B.24 is given by[93]

\[
n_{e,p} = n_2 \sqrt{1 - \frac{2m\sqrt{2\Delta n}}{n_2 ka}}
\]

(B.25)

For the test distribution seen in figure B.2, the \( LP_{0,1} \) mode has an analytical effective index of 1.456828477226172 and the numerically calculated value is in
good agreement with 1.456829614049853. The accuracy is determined by the truncation of the profile.

![Figure B.2: Parabolic index function and the guided modes for $\lambda = 1.28\mu m$. The eigenfunctions are Gauss-Laguerre functions!](image)

### B.1 Confinement factor

A very important figure in modeling active waveguide structures is the confinement factor, which is the overlap integral of the mode-profile with the dopant distribution within the waveguide. The confinement factor appears as a figure in the amplifier coefficients, for example the attenuation per length

$$
\alpha = (1 - x)\sigma_o 2\pi \int_0^{\infty} \varphi^*(r) N(r) \varphi(r) r dr,
$$

where $N$ is the circular symmetric dopant distribution density and $\sigma_o$ is the absorption cross section and $x$ is the local inversion. By defining a constant
average dopant concentration within some radius \( R_l \) the relation reduces to

\[
\alpha = (1 - x)\sigma_a 2\pi N_{\text{avg}} \int_0^\infty I_n(r) \frac{N(r)}{N_{\text{avg}}} r dr,
\]

(B.27)

where \( I_n(r) \) is the normalized intensity distribution where \( R_e \) is defined as a weighted integral as [26]

\[
A_e = \pi R_e^2 = 2\pi \int_0^\infty \frac{N(r)}{\max (N(r))} r dr
\]

(B.28)

and the average dopant distribution is equal to the integrated distribution and assuming that the dopant distribution is proportional to the index difference gives

\[
N_{\text{avg}} = \frac{2 \int_0^\infty N(r) r dr}{R_e^2} = \frac{2\alpha \int_0^\infty \Delta n(r) r dr}{R_e^2},
\]

such that the equation for the attenuation is simplified to

\[
\alpha = (1 - x)\sigma_a N_{\text{avg}} \frac{\int_0^\infty I_n(r) \Delta n(r) r dr}{\max (\Delta n)} = (1 - x)\sigma_a N_{\text{avg}} \Gamma,
\]

(B.30)

where \( \Gamma \) is the confinement factor defined from the above equation. It is seen that a confinement factor of 1 corresponds to a transversal uniformly doped fiber. The definition is equivalent to a top-hat distributed function with a constant average dopant concentration occupying an area of \( A_e \). Furthermore, optical intensities used in the models are found from \( I = \frac{4}{A_e} \).
Appendix C

Gaseous absorption lines

A selection of gases with absorption lines covered by the emission wavelengths of thulium doped fiber lasers are presented in the table below.
<table>
<thead>
<tr>
<th>Trace gas</th>
<th>Absorption line ($\mu m$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HCl</td>
<td>1.74</td>
</tr>
<tr>
<td>NH$_3$</td>
<td>1.79</td>
</tr>
<tr>
<td>N$_2$O</td>
<td>1.95</td>
</tr>
<tr>
<td>CO$_2$</td>
<td>1.96</td>
</tr>
<tr>
<td>HBF$_6$</td>
<td>1.96</td>
</tr>
<tr>
<td>NO</td>
<td>1.81</td>
</tr>
<tr>
<td>H$_2$CO</td>
<td>1.93</td>
</tr>
<tr>
<td>CH$_2$O</td>
<td>1.76</td>
</tr>
<tr>
<td>H$_2$O vapor</td>
<td>1.82-1.92</td>
</tr>
</tbody>
</table>

Table C.1: Trace gases and water vapor absorption lines covered by the emission band of thulium
Appendix D

Rare-earth concentrations

A number of designations for the concentrations of rare-earths dopants are used throughout the literature. The following gives an explanation of the different terms used and their definitions, primarily taken from [21].

The molecule molar masses of common constituents are in $\frac{g}{mol}$:

\[
Z_{SiO_2} = 60.1 \quad Z_{Al_2O_3} = 102.0 \quad Z_{GeO_2} = 104.6 \quad Z_{P_2O_5} = 142.0
\]
\[
Z_{Er_2O_3} = 382.6 \quad Z_{Yb_2O_3} = 394.1 \quad Z_{Tm_2O_3} = 385.9 \quad Z_{Nd_2O_3} = 336.3
\]

(D.1)

The molar fraction of the i’th constituent is $x_i$ which gives the total weight of the glass as

\[
W = \sum_i x_i Z_i \quad \text{(D.2)}
\]

The number of rare-earth ions per cubic-centimeter is then given by

\[
\rho_{RE} = x_{RE,molecule} \frac{2DN}{W}, \quad \text{(D.3)}
\]

where $D$ is the glass density and $N = 6.02 \cdot 10^{23} \frac{1}{mol}$ is Avogadro’s number. The glass density is approximately equal to $2.9 \frac{g}{cm^3}$ for the silica glasses considered.

Molar concentrations are given for the molecules and a typical number is given in ppm (parts per million) as

\[
\rho_{molecule,mol} = 10^6 \frac{n_{RE-molecule}}{n_{SiO_2}} = \frac{10^6 W}{x_{SiO_2} 2DN^2 \rho_{RE}} \quad \text{(D.4)}
\]
and weight concentrations are given for both molecules and ions as

\[
\rho_{\text{molecule, wt\%}} = 100 \frac{\rho_{\text{RE}}}{2DN} Z_{\text{molecule}}
\]  \hspace{1cm} (D.5)

and

\[
\rho_{\text{RE, wt\%}} = 100 \frac{\rho_{\text{RE}}}{DN} Z_{\text{RE}}
\]  \hspace{1cm} (D.6)

and another common form is ppm wt given by

\[
\rho_{\text{RE, wt}} = 10^6 \frac{\rho_{\text{RE}}}{DN} Z_{\text{RE}}
\]  \hspace{1cm} (D.7)
Bibliography


