Optimization of the raw material use at Danish slaughterhouses

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Optimization of the Raw Material Use at Danish Slaughterhouses

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Summary

The pig industry is an important part of Danish economy with an export value of more than DKK 28 billions in 2006 [Danish Meat Association (2007)]. The competition increases, and Danish slaughterhouses are pressed by the large farmers, who are also co-operative owners of the slaughterhouses, to increase payments for the pigs delivered. Therefore, it is more important than ever to optimize all aspects of Danish pig production and slaughtering processes.

The basis for the current project is an expectation: That operations research methods can be used to improve the basis of the slaughterhouses for making decisions regarding raw material use. This hypothesis has been tested and proved valid regarding a number of important strategic decisions made by the slaughterhouses on a regular basis.

This thesis develops different models for solving optimization problems. These models can support the slaughterhouses in their strategic decision making and can support the following decisions:

- How much would investments to obtain improved quality measurements at the slaughterhouses be worth?
- What is the value of a general increase in the slaughter weight for the pigs, and how would the increase influence the average price per kg?
- What is the cost of having logistic limitations in the production, and how much would it be worth investing in order to eliminate the effects of these limitations?
- How good are different sorting strategies, and how much can the profit be increased by adding additional sorting groups?

The main conclusion is that the Mixed Integer Programming models developed and described in the four papers constituting the main body of the thesis can be used to improve the slaughterhouses basis for making decisions. Before the slaughterhouses can use the models for actual decision making, these must be developed into full-fledged models with more products as input to the models, furthermore a price and cost study should be performed. Recommendations for such a study have been made.
**Resumé**

Svineproduktion og slakteriøkonomi er en vigtig del af den danske økonomi og havde en eksportværdi på mere end DKK 28 milliarder i 2006 [Danish Meat Association (2007)]. Konkurrencen er stigende, og de danske slakterier er presset af de største svineproducenter, som også er andelshavere (ejere) i (af) slakterierne til at øge afregningspriserne for de leverede grise. Det er derfor mere og mere vigtigt, at alle aspekter af den danske svineproduktion og slagtning optimeres.

Udgangspunktet for nærværende projekt er en forventning om: At operationsanalyse kan anvendes til at forbedre slakteriernes beslutningsgrundlag i forbindelse med råvareanvendelsen. Denne hypotese er blevet testet og fundet gyldig vedrørende en række vigtige strategiske beslutninger, der slakterierne udfører jævnligt.

Afhandlingen indeholder fire tekniske rapporter, der er inkluderet i appendiks og omhandler modeller til løsning af forskellige optimeringsproblemer. Modellerne kan hjælpe slakterierne i forbindelse med en række vigtige strategiske beslutninger. Eksempler på forskellige beslutninger, der kan understøttes af de udviklede modeller er:

- Hvor meget vil det være værd at investere for at forbedre kvalitetsmålingerne på slakterierne?
- Hvad er værdien af en generel forøgelse af slagtevægten for grise, og hvilken indflydelse vil denne have på de gennemsnitlige hjembragte priser?
- Hvad er omkostningerne ved at have de nuværende logistiske begrænsninger i produktionen, og hvor meget vil det være værd at investere for at mindske eller fjerne disse begrænsninger?
- Hvor gode er forskellige sorteringsstrategier? Hvor meget kan der opnås ved at indføre yderligere sorteringsgrupper?

Det kan konkluderes, at de udviklede ”Mixed Integer Programming”-modeller beskrevet i de fire tekniske rapporter kan være med til at forbedre slakteriernes beslutningsstøtte. Før modellerne benyttes til egentlig beslutningsstøtte på slakterierne, bør de udvides til at anvende flere produkter som input, ligesom der bør gennemføres pris- og omkostningsstudier. Afhandlingen indeholder anbefalinger i forbindelse hermed.
Preface

This thesis was prepared at the Department of Management Engineering at the Technical University of Denmark (DTU) in cooperation with the Danish Meat Research Institute as a partial fulfilment of the requirements for acquiring the Ph.D. degree.

The Section of Operations Research where the study has taken place was until January 1, 2008 part of the Department of Informatics and Mathematical Modelling at the Technical University of Denmark. Due to re-organization at DTU the section is per this date part of the Department of Management Engineering.

The thesis supports a number of important strategic decisions made at the slaughterhouses, all based on optimization of raw material use. The Ph.D. project has been supervised by Professor Jens Clausen at Department of Management Engineering and by Research Manager Eli Vibeke Olsen and Development Manager Claus Hagdrup both from the Danish Meat Research Institute. Associate Professor Peter Jacobsen from the Department of Management Engineering at the Technical University of Denmark has been co-supervisor for the project.

The thesis includes four papers addressing different optimization problems of major importance for Danish slaughterhouses.

Lyngby, April 2008

Niels Kjærgaard
Papers included in the thesis


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Chapter 1

Introduction

The pig industry is an important part of Danish economy with an export value in 2006 of more than DKK 28 billions [Danish Meat Association (2007)]. Competition in the pig industry is substantial. Historical low USD and increased feeding costs without the possibility of using genetically modified crops as their American competitors do, have had a hard impact on the European farmers’ profitability. The Danish slaughterhouses are also in hard competition to attract pigs for slaughtering. During the last couple of years a substantial number of Danish farmers have delivered part of their pigs to German slaughterhouses. It is therefore important that all aspects of Danish pig production, slaughtering and delivery are optimized.

The slaughterhouse industry is different from the traditional industry in a number of ways. Most important is the natural (and large) variation in raw material regarding quality, weight, size, fat layer and meat content etc., as a consequence of pigs being a biological material. The industry is dealing with this large variation by sorting the pigs into groups consisting of pigs with almost the same characteristics. The variation within the individual sorting groups is thereby reduced substantially. The important factor for how much the variations can actually be reduced is the accuracy of the quality measurements. For the years to come, substantial investments are expected to be made in order to improve the measurement systems further. The model described in paper A [Kjærsgaard, N. (2008a)] can help the slaughterhouses in finding the value of improved quality measurements and thereby in deciding how much it will be worth investing in new measurement systems. The models described in the papers B and C [Kjærsgaard, N. (2008b, c)] are improved further and provides even more accurate results.

There have been varying opinions within the industry concerning an improved profit for the slaughterhouses and the farmers when increasing the slaughter weight. The argument for an increased slaughter weight is based on the fact that some of the costs at the slaughterhouses are unit costs, and it might therefore be interesting if the slaughter weight could be increased. If the slaughter weight was increased and the number of items produced almost unchanged the costs per kg produced meat would decrease [Kjærsgaard, N. (2008b)]. The savings in costs should be compared to the possible
decrease in the average sales price. These are considerations which are continuously taking place in the industry, but some more accurate tools to investigate the economic consequences are desired. The model described in paper B can be used by the slaughterhouses to improve their basis for making decisions in connection with finding economic consequences of a general increase in the slaughter weight.

The models described in papers A and B [Kjærsgaard, N. (2008a, b)] makes it possible to find economic effects of a general increase in slaughter weight or of improved quality measurements. However, logistic limitations in the production and warehouse facilities imply that the entire potential of e.g. improved measurements cannot be reached. By taking logistic limitations into consideration the accuracy of the computations can be improved further. This is illustrated in paper C taking the special conditions in the equalization room into consideration. Furthermore, the model can be used to estimate the value of logistical changes in connection with rebuilding of warehouses and production facilities.

Sorting is one of the most important ways to ensure good raw material use in the slaughterhouses. In principle, sorting can be based on all kinds of quality characteristics, such as fat layer, lean meat percentage, slaughter weight or whether the pigs are special production pigs or not. In paper D [Kjærsgaard, N. (2008d)], sorting is illustrated by using the fat layer and the slaughter weight. The model used in paper D is the same model as described in paper C [Kjærsgaard, N. (2008c)] but is in this context used to compute the profit of different sorting strategies. Based on these computations a graphical tool has been set up to help the slaughterhouses in designing their sorting strategies.

The prices and costs used as input in the models are essential for how good the computations reflect the slaughterhouses economy. In chapter 6.2 recommendations for performing a cost study is given, including a brief description of the most important cost allocation principles.
Chapter 2

The Slaughterhouses

2.1 Special conditions at the slaughterhouses

The slaughterhouses differ from traditional industrial companies in a number of ways:

- The raw materials (the pigs) are not uniform but have a very large variation in size, shape, weight, weight of different potential products, fat layer etc. as well as the distribution of fat and meat on the pigs.

- In the traditional industrial companies such as car manufacturers, the manufacturers assemble a large number of parts to one final product, the car. The slaughterhouses have the reverse production process, where they split one raw material (the pig) into a number of different products.

- Both the raw materials and the final products have a limited shelf life. The shelf life can be prolonged by freezing, but additional costs may apply. Furthermore, the prices obtainable can be reduced as some customers perceive frozen products as of less quality than fresh meat.

![Diagram of assembling process](image1)

![Diagram of splitting process](image2)
• The ownership structure of the largest Danish slaughterhouses is a co-operative structure. The strong advantage of such a co-operative structure is the potential very solid relations to the suppliers (the farmers). The structure can certainly also have some disadvantages, especially regarding financial consolidation and providing additional equity for large investments. In general, the owners do not appreciate financial consolidation within the slaughterhouses as they cannot sell their part of the equity as normal shareholders can. The slaughterhouses have entered into long term contracts with the farmers, in which the slaughterhouses guarantee to buy all pigs produced by the farmers. Due to the long term contracts, the Danish slaughterhouses are less flexible than traditional industrial companies as they, in the short term, cannot decide how much to produce (the production volume is more or less given) but can only decide what to produce and how to produce it.

• The value added in the slaughterhouses is relatively limited. The Danish slaughterhouses annual accounts [Danish Crown (2007) and Tican (2007)] have been analysed in order to investigate the cost structure. Due to Danish Crown’s accounting principles their annual accounts are less informative regarding their cost structure. Therefore the analysis is based on Tican’s annual accounts, but it is expected that the cost structure for Danish Crown is not that different. Figure 3 below describes how a turnover of DKK 100 is used to cover different types of costs.

![Distribution of Cost and Profit](image)

Figure 3. Distribution of costs and profit at Tican a.m.b.a.
Figure 3 shows that approx. 65% of the turnover is used for direct payments to the farmers for the raw material (pigs) received. Even though the above figures are not consolidated group accounts but only related to the parent company Tican a.m.b.a., the figures include the secondary industry where the products are processed into slices of meat, sausages etc. for the end user. The annual account does not provide information of how revenue and costs are split between primary (slaughtering etc.) and secondary industry. If only slaughtering processes are considered the raw materials share of the revenue will be even larger than 65%.

The cost structure above can be used to illustrate how effective different rationalization initiatives can be in improving slaughterhouse economy. If the raw material use can be improved to an extent which improves the total yield by 1%, this will be equivalent to:

- Decreasing the administrative cost by 64%, or
- Decreasing the administration and distribution costs by 15%, or
- Decreasing the production wages by 7%

Improving the use of raw materials is likely to be the most important way for the slaughterhouses to improve their earning power. If a similar improvement should be made by decreasing the costs of production wages, relatively large reductions (7%) of the manpower in the production should be made. In practice, the reductions in manpower should be much higher as there would be considerable investments in automations in connection with such plans.

Improved sorting enabling the slaughterhouses to deliver the exact quality required by the customers may be the best approach for the slaughterhouses to stand up to the hard and increasing competition.

The Danish slaughterhouses and DMRI are working on improving current measurement systems and are investigating the possibility of using on-line CT scanner technology. In the years to come large investments are expected. Potentially, the CT scanner technology can improve the measurements substantially and provide almost full knowledge of the quality of the pigs. This will make it possible to improve sorting very considerably.
2.2 The raw material use at the slaughterhouses

The overall production flow is almost the same at different pig slaughterhouses in Denmark, but the number of slaughtering lines and other capacities may differ. The production flow is described in the paper C [Kjærgaard, N. (2008c)] but is repeated here to improve the readability of the thesis. The description of the production flow and the modelling in this thesis is primarily based on the conditions in one of the largest Danish slaughterhouses, which is used as the model slaughterhouse.

The overall production flow at the model slaughterhouse can be seen in Figure 4 below and in a larger version in Appendix 1.

Figure 4. Graphical model of the slaughterhouse.

When the pigs arrive from the farmers they are placed at the pig lairage area. The pigs stay in the lairage area for a few hours in order to reduce their stress levels (ensures good meat quality). After a couple of hours, the pigs are driven from the lairage area to
one of the stunning centres with subsequent sticking and debleeding. Then they continue at one of the four slaughtering lines to scalding, de-hairing and singeing.

The carcasses are cut open and heart, liver, kidney and intestines etc. are taken out. The carcasses are split in halves, but are still held together as one by their jawbone. The carcass as well as heart, liver, kidney and intestines are inspected for deceases etc. If deceases are found, the carcass can be rejected or utilized for other purposes (e.g. as heat-treated products). The carcasses are cleaned and then weighted and graded in the classification centre. Here sundry measurements are performed, such as lean meat percentage and fat layer for the total carcass as well as for the individual main parts (fore end, middle piece and ham). The weight and other measurements as well as information about whether the pigs are special production pigs (e.g. Antonius or welfare pigs to the UK market) are of the utmost importance for the subsequent sorting of the carcasses. At the model slaughterhouse each of the four slaughtering lines has a capacity of 350 pigs per hour, i.e. in total 1,400 pigs per hour. The model slaughterhouse is only using 1 shift of approx. 8 hours at the slaughter lines.

After slaughtering, the carcasses are led through a freezing tunnel which quickly reduces the temperature of the carcasses surfaces. Depending on predefined sorting groups the carcasses are placed on bars in the equalization room (cold storage), where they are hang for a period of minimum 16 hours to ensure a uniform temperature throughout the entire carcass. In general, these 16 hours will pass if the pigs are used for further processing the day after they have been slaughtered and placed at the equalization room.

The equalization room consists of 180 bars with a capacity of 80 pigs each. Each bar on which the carcasses are hung can only be emptied from the opposite side of the filling side, and consequently they have to be emptied in the same order as they were filled. See Figure 5 below:
For practical reasons whole bars are emptied at a time, making it possible to dispose freely of the bars once more. The physical limitations in the equalization room (the number and capacity of the bars) are of the utmost importance for the number of active sorting groups that can be handled effectively. In practice, the slaughterhouses are sorting based on the middle piece and the ham, which are the most valuable parts of the pigs and hence the largest sorting potential. As the main parts (fore end, middle piece and ham) are not yet separated it is only possible to sort by one criterion, which is primarily the middle piece. To some extent the sorting can be based on both the middle piece and the ham at the same time, but the number of combinations (and the need for further sorting groups) increases rapidly thereby.

Due to logistic reasons the bars numbered 1 to 108 are primarily used for cutting line 1 and the bars from 109 to 180 for cutting line 2. To some degree, however, it is possible to relocate all carcasses from one bar to other bars, but this is a time consuming procedure, which is not much used.

The placement in the equalization room is based on forecasts for how the pigs being slaughtered that day will be placed in sorting groups. The planning task here is to decide which bars should be allocated for different sorting groups. If possible, the carcasses are placed so that one bar consists of carcasses from only one sorting group and preferably from the same slaughtering day so the entire bar can be used in production at the same time for the same production batch. Sorting groups consisting of only few carcasses, for instance carcasses taken out for tests, damaged carcasses etc. may be mixed at the same
bar due to space considerations. Due to quality and yield considerations the carcasses should be used for further processing the day after slaughtering and not stocked further at the equalization room.

During the stay in the equalization room, each carcass has not yet been parted into the three main parts: Fore end, middle piece and ham. This limits the possible number of sorting groups as a combinatorial “explosion” takes place. If there are e.g. 10 sorting groups for the middle piece, 10 for the ham and 3 for the fore end 300 sorting groups are required if each of these combinations should be possible. Therefore, in practice, a limited number of sorting groups can be handled and primarily based on the quality of the middle piece and only to some extent on the ham.

The number of active sorting groups can vary over time, but there are typically 15-20 main groups. In addition to these, there are approx. 30 smaller sorting groups for pigs with salmonella, diseases, pigs used in sundry experiments, special production pigs as well as damaged pigs or pigs with missing identification etc.

After 16 hours of placement in the equalization room, the carcasses are taken to one of the two cutting lines. Here the tenderloin and head are cut off and each half of the carcass is parted into three main parts. The middle piece can further be split into two pieces. The middle piece and the ham are the most valuable parts and can be sorted further by weight. The sorting by weight is automated, and the middle piece and the ham can be sorted separately into a maximum of 3 sorting groups each at a time.

At the end of the cutting line the parts are placed on stands each consisting of 20 items. The stands are placed in a buffer storage before they are taken to different cutting departments for further processing. After further processing the products are packed and delivered to the customers.

**2.3 Products, prices and costs**

In the papers A, B, C and D [Kjærgaard, N. (2008a, b, c, d)] only a relatively small number of products have been used as the purpose was to demonstrate the models for proof-of-concept and not an actual decision support.

In paper A [Kjersgaard, N. (2008a)] regarding the value of improved measurements the computations are only made for the backs. Five different back products are used, but the
model can easily be modified to include more products of the back, the ham or other parts of the carcasses. In the other three papers [Kjærsgaard, N. (2008b, c, d)] 17 main products and four different alternative uses of the carcasses are used. The alternative uses for each of the three main pieces of the carcasses can be seen in Figure 6 below. Each alternative use can be considered a “package” of products made from the three different main parts. The back and the ham have two alternative uses each and the fore end, which is the least valuable, has one. In total, this gives 4 different alternative uses of each pig.

![Diagram of a pig showing alternative uses](image)

**Figure 6. Alternative uses of the pig.**
The slaughterhouses offer far more products than used here. See for instance Tican’s homepage [www.tican.dk](http://www.tican.dk) for an overview of their products. Figure 7 below shows an example from Tican’s homepage regarding products from the ham.

![Figure 7. Homepage for Tican.](image)

There is a natural variation in prices from one week to another. As the market situation changes continuously it was not always possible to gain access to the latest updated prices, but information of typical prevailing prices have been received from Danish slaughterhouses.

Before the models described in the four papers A-D can be used as basis for actual decision support, a price and cost survey should be performed. As the information is quite sensitive for the slaughterhouses it is recommended that the slaughterhouses perform this survey themselves. Such a price and cost study is vitally important for making good decisions based on the models and recommendations for such a study has been made. See Chapter 6.2 for recommendations.
2.4 Framework for the project

In the following, the overall framework will be described for the interaction between different parts of the project and other areas of relevance for raw material use at the slaughterhouses. The framework can be seen in Figure 8 below:

![Figure 8. Overall framework for the project.](image-url)
The measuring systems were originally developed for classification purposes, i.e. for making it possible for the slaughterhouses to determine payments to the farmers reflecting the quality of the pigs delivered. This is still an important task for the measuring systems, but another application just as important is using the measuring data for sorting purposes. In an automated environment with a throughput of 1,400 pigs per hour, manual sorting is not an option and improved measurements are considered a prerequisite for improved sorting. The current measuring system at the slaughterhouses was developed almost 15 years ago and a major project regarding development of a new measuring system has been started by Danish Meat Research Institute and the Danish slaughterhouses.

New technology has been investigated and the CT scanner technology has been identified to be a promising candidate of the greatest importance for obtaining information when performing measurements.

Potentially, CT scanner technology can provide full knowledge of the entire carcass, including a 3D view, information of the lean meat percentage and fat layer of different parts of the carcass as well as length, size, shape and weight of different product cuts. The huge improvement in information gathered compared to the old measuring systems, will be illustrated in the following example using a back product which is cut into cutlets. See Figure 9 below:

Figure 9. 3D-view of the back.
The quality of the cutlets (e.g. the fat layer, the intramuscular fat and the position of meat and fat) can vary substantially even for cutlets from the same back. CT scanners make it possible to learn the exact quality of each cutlet before processing it into cutlets. Figure 10 below gives an example of different cutlets from a back.

Figure 10. CT scanned cutlets from the same back.

If different backs is compared the cutlets may differ much more in quality.

The current technology based on ultrasound is not even close to providing the same amount of information. In Figure 11 below an ultrasound picture of a carcass can be seen.
The ultrasound picture is much harder to interpret and use for determination of the quality of the carcass. In fact, today the ultrasound picture is only used for estimating the lean meat percentages for the entire pig and for the different main parts. Furthermore, the estimated lean meat percentages and fat layers consist of considerable measuring error.

Improved measurements based on the CT scanner technology can improve the yield data substantially. This may be of benefit to many different R&D projects within the Danish meat industry as manual dissections can be avoided. Most often in today’s R&D projects only a limited number of pigs are dissected as it is both costly and time consuming. Improved measurement systems based on CT-scanners potentially make it possible to obtain full information of an almost unlimited number of pigs. Furthermore, each pig used in various R&D projects can be used virtually for an unlimited number of times. Improved yield data can also be used for improved production/yield control by benchmarking different cutting department’s yields with the estimated yields based on the actual raw materials received.
R&D within meat quality and breeding influences the improved measurements by helping to decide which parameters characterize different qualities in the best way and which parameters are worth measuring.

The purpose of improved sorting is to be able to use raw materials in a more efficient way, meaning a way that adds more value. Improved sorting can be obtained by designing new improved sorting strategies. Decision support for designing and evaluating such sorting strategies, based upon operations research methods, are important tools to improve sorting.

Operations research methods and simulation of the raw material flow at the slaughterhouses can also improve raw material use in other ways than sorting. Decision support based on operations research methods can improve the slaughterhouses basis for making decisions in a number of ways. One way is to improve the product mix so that the total prices obtained less production and sales costs are increased. Other important examples will be given later in this chapter. Automation and other logistic changes are more traditional ways used to improve raw material use and to lower unit costs. During the last 20 years the Danish Meat Research Institute and the Danish slaughterhouses have automated most parts of the slaughter line. Improved automation continues but other initiatives have to be made as well in order to improve the competitiveness further.

The improved raw material use can be related to savings in the production at the slaughterhouses as a consequence of the raw materials being more uniform. This may apply both for the different cutting departments and as a consequence of improved utilization of the automated machines at the slaughter lines. One example:
The slaughterhouse has two different raw materials which differ substantially in quality. The back to the left in Figure 12 below has a substantially fat layer and the one to the right has a more moderate fat layer.

![Figure 12. Two different raw materials (backs)](image)

Even though the raw materials differ in quality they may both be used to produce the same type of product (see Figure 13 below).

![Figure 13. Example of a processed back product.](image)

The end user may receive the exact same quality when the raw material has been processed, but the slaughterhouses yields and economy may be very different depending on which raw material is used. It is therefore important that the right raw materials are used to produce the product which they fit best. If the left back in Figure 12 is used to produce the product in Figure 13 it will result in lower yield and probably also more work (cutting time). The case is just for illustrative purposes and it may not be possible to produce the same product for the two raw materials seen in Figure 12 as there are substantial intra muscular fat in the left back.

The raw material use can also be improved by optimizing the product mix. Furthermore, the slaughterhouses will be able to add value to their customers by delivering the exact quality in the desired volume just as their customers demand. The customers may obtain
savings in their production and will be able to deliver the exact desired quality to their customers. Many of the products from both Danish slaughterhouses and from their customers end up in cool display in supermarkets.

It is important for the supermarkets to receive the exact quality ordered. Receiving a worse quality than paid for is obviously unacceptable for the supermarkets, but even receiving a better quality may not be acceptable. The supermarkets may wish to be able to deliver different qualities to their customers. An example is back bacon delivered to some UK supermarkets, which requires three different qualities to offer to their customers: a prime (extra premium) product, an ordinary product and a discount (low cost) product. If the quality of the discount product and/or the ordinary product is too good it will influence the sale of the more expensive products. Therefore, there is a substantial value to be added when delivering the exact quality desired by the customers.

Operations research methods can be used to improve the basis of the slaughterhouses for making decisions regarding raw material use within a whole number of areas (see Figure 8 above):

1. **Computing the value of improved measurements.** As the slaughterhouse industry in Denmark has to consider large investments in new measuring systems it is important that the economic consequences of such investments can be evaluated. An important decision has to be made regarding how much it will be worth investing in order to improve the accuracy of the measurement systems. How much will it for instance be worth investing to improve the measuring accuracy by 50%, or how much more will it alternatively be worth investing to obtain an even higher accuracy? This topic has been dealt with in detail in paper A [Kjærsgaard, N. (2008a)], but the model described in paper C [Kjærsgaard, N. (2008c)] is a further improvement.

2. **Evaluating the payment structure to the farmers.** One of the important decisions is to establish the optimal slaughter weight for the pigs delivered to the slaughterhouses. The model described in paper B [Kjærsgaard, N. (2008b)] and further improved in paper C [Kjærsgaard, N. (2008c)] evaluates the consequences of different changes in the slaughter weight. The payment structure is aimed to reflect the value of the pigs delivered, so the farmers receive a larger amount if the pigs delivered are more valuable for the slaughterhouse. Today, the slaughterhouses use two parameters; slaughter weight and lean meat percentage to establish the payments.
3. **Pricing of new products.** The improved yield data for new products together with sale support based on the models developed can help the sales personnel significantly in computing the “right prices”. Today, there is a risk that the pricing for new products is too low. Even accepting a high premium product may not be profitable if this results in a larger part of the pig being sold as low price products. These relations can be very hard to identify as accepting new products may influence the overall product mix as well as the yields for other products. If the pricing of new products is too high, business may be lost. The model described in paper C [Kjærsgaard, N. (2008c)] can be modified to be used for sale support.

4. **Test of logistic changes at the slaughterhouses.** From time to time the slaughterhouses are rebuilding parts of their production facilities (or even building new ones) in order to remove or reduce some of the logistic limitations. This often implies very large investments and it is desirable to know the economic consequences of different layout alternatives. Paper C [Kjærsgaard, N. (2008c)] addresses the value of reducing/removing such logistic limitations.

5. **Design of new sorting strategies.** Sorting is considered one of the most important ways of improving raw material use. Paper D [Kjærsgaard, N. (2008d)] concerns the computations of the economic consequences in relation to different sorting strategies, and a tool has been developed to help the slaughterhouses design different sorting strategies.

6. **Improved product mix and product planning.** The aim of the models developed and described in the four papers A-D [Kjærsgaard, N. (2008a, b, c, d)] was to provide decision support to the slaughterhouses in connection with a number of important strategic decisions. When the models are slightly modified, especially regarding the input, they can be used for more operative decisions as well. The models can already be used to find optimal product mix, but can be changed to support the daily production planning as well. This requires that the model described in paper C is augmented to include time frames for production and delivery of the products.

7. **Education of production planners.** Production planners can investigate the consequences of different changes in their production plans and compare the profitability of different production plans with each other. The model described in paper C can be used as basis for such computations. Preferably the model should be augmented to include time frames for production and delivery of products.
Furthermore, the last four items can be supported by the use of simulation of the entire raw material flow at the slaughterhouses. The Danish Meat Research Institute has already simulated part of the raw material flow at different slaughterhouses by using the graphical simulation tool Enterprise Dynamics, but a simulation model covering the entire logistic flow in detail has not yet been set up.
Chapter 3

Literature survey

The literature regarding optimized raw material use at the slaughterhouses has been addressed in the paper “The Value of Improved Measurements in a Pig Slaughterhouse” [Kjærsgaard, N. (2008a)]. It is repeated here and in the three other papers B-D [Kjærsgaard, N. (2008b,c,d)] as it should be possible to read each paper independently.

The amount of literature addressing improved or optimized raw material use in the food industry is substantial. However, the main part of the contributions is related to different aspects regarding either optimization of meat quality or different production processes. Examples of this are optimization of the industrial thermal sterilization of canned foods [Garcia, M. et. al. (2006)] and pigs stunning optimization [Dupuis, P. et. al. (2004)]. These types of optimizations are not relevant for this project as they are either based on statistical analysis without optimization of a mathematical model or the mathematical models are very different from the models, which are used in this Ph.D. project regarding optimization of the raw material use at the slaughterhouses.

Within the pork industry relatively few contributions have been found regarding optimization based on operations research methods. In the paper “Location of slaughterhouses under economies of scale” [Broek et. al. (2006)] optimization is used to investigate the savings potential of reducing the number of slaughterhouses in Norway and investing in additional capacity in the remaining facilities in order to obtain economies of scale. Another facility location problem is described in the paper “The impact of changes in livestock supply on the optimum number, size and location of slaughterhouses in East Macedonia” [Kamenidis, C. & Sorensen, V. (1978)]. In the paper ”Economic optimization of pork production – marketing chains. II. Modelling outcome” [Ouden et. al. (1996)] the authors are using Dynamic Linear programming to evaluate the development of pork chain concepts that also takes animal welfare into consideration. Kure in his Ph.D. thesis “Marketing Management Support in Slaughter Pig Production” [Kure, H. (1997)] uses Dynamic Programming to solve parts of the “slaughter pig marketing management problem”, which regards how the farmers should select and market their pigs to the slaughterhouses.
The above mentioned four examples of optimization problems within the pork industry are all somewhat different from the problem of optimizing the raw material use at the slaughterhouses. More similar problems have been found in the following contributions:

In 1990-1992 a project regarding optimization of the raw material use at the slaughterhouses was performed as a cooperation between Danish Meat Research Institute and the Royal Veterinary and Agricultural University (now the Faculty of Life Sciences at University of Copenhagen). Several reports were made:

A Linear Programming (LP) model for production planning and control for the hog slaughterhouses was developed and reported in [Rasmussen, S. & Thomsen, M. (1991)] and [Rasmussen, S. (1992)]. The model is a 2-stage model. First stage concerns a planning horizon of 3 months and the second stage one weeks day to day planning. In [Fertin, C. (1992)] the long term planning model (stage 1) is validated. In his Ph.D. thesis [Fertin [1995]] Fertin describes and further develops and validates the model.

We have searched for literature in other food related industries, e.g. poultry and beef slaughterhouses and the fish industry, but no relevant literature has been found.

Other industries have similar problems as the slaughterhouses regarding raw material use. An example is the refineries, but unlike the slaughterhouses the refineries have the option of blending different qualities in order to change the quality characteristics of the products. Another example is the lumber and wood industry. A few papers of the product mix problem within the wood industry have been identified. In the paper “An Optimization-Based Decision Support System for a Product Mix Problem” [Roy et. al. (1982)] an LP-model has been used to solve a plywood product mix problem for Ponderosa Industrial in Mexico.

Even though literature within food optimization is substantial, the main part of the contributions are related to optimization based on e.g. statistical analysis without optimization of a mathematical model. Other models are very different from the models used in this Ph.D. project. Except for the contributions from the Royal Veterinary and Agricultural University and the Danish Meat Research Institute not much literature of relevance for the Ph.D. project has been identified.
Chapter 4

The Models

Three different models have been developed and used in the four papers. In paper A [Kjærgaard, N. (2008a)] regarding the value of improved measurements, a relatively simple model is developed. This model only uses one product type at a time and the product weights are not estimated separately but based on average weights.

In paper B [Kjærgaard, N. (2008b)], regarding the value of a general increase in slaughter weight, the model is using the entire pig, and product weights for all potential products are estimated based on slaughter data and a simulated measuring error.

In the third paper, paper C [Kjærgaard, N. (2008c)] regarding logistic limitations in production and stocks, the model has been further refined to take the physical conditions in the equalization room into consideration.

The model used in paper D [Kjærgaard, N. (2008d)] is the same model as described in paper C, but in this connection used to evaluate different sorting strategies. A graphical tool to help the slaughterhouses design their sorting strategies is introduced. This is based on the model too.

The different models developed are described in the following section successively. We then comment explicitly on each paper.

4.1 Different models

The model used in paper A regarding the value of improved measurements is used to find the profit at two different levels of measuring accuracy. We have a set of backs from carcasses which are placed into different sorting groups \( I = \{1, \ldots, I\} \) based on their measured quality. Each carcass can be used to produce a set of different back products \( J = \{1, \ldots, J\} \). The decision variable \( x_{ij} \) indicates the number of pigs from sorting group \( i \) used to produce product \( j \). The variable \( y_{ij} \) is a binary variable with the value 1 if sorting group \( i \) is used to produce product \( j \) and 0 otherwise and is used to control the number of
products made of a quality without of specifications. The problem is to find the optimal utilization of each sorting group and the total profit for the optimal solution given the carcasses’ distribution on sorting groups \( \text{Distribution}_i \) and restrictions in the maximum sale of each product \( \text{MaxSale}_j \). Furthermore, there are restrictions that the number of products not living up to the quality specifications \( \text{QualityLow}_j \) cannot exceed the acceptable level \( \text{AcceptableQuality} \).

### 4.1.1 Model 1

The objective function:

1) \[ \text{Max. } Z = \sum_{i,j} \text{Price}_{i,j} \cdot x_{i,j} \cdot \text{AverageWeight} \]

Subject to:

2) \[ \sum_j x_{i,j} \leq \text{Distribution}_i \quad \forall \ i \in J \]

3) \[ \sum_i x_{i,j} / (N \cdot 100) \leq \text{MaxSale}_j \quad \forall \ j \in J \]

4) \[ y_{i,j} \leq x_{i,j} \quad \forall \ i \in I, \ j \in J \]

5) \[ y_{i,j} \cdot M \geq x_{i,j} \quad \forall \ i \in I, \ j \in J \]

6) \[ \text{SumQualityLow}_j = \sum_i \text{QualityLow}_{i,j} \cdot y_{i,j} \quad \forall \ j \in J \]

7) \[ \text{SumQualityLow}_j \leq \text{AcceptableQuality} \cdot \sum_i x_{i,j} \quad \forall \ j \in J \]

8) \[ x_{i,j} \geq 0 \quad \forall \ i \in I, \ j \in J \]

9) \[ y_{i,j} = \begin{cases} 1 & \text{if production} \\ 0 & \text{else} \end{cases} \quad \forall \ i \in I, \ j \in J \]

Indices:

- \( i \): sorting group \( i \) based on the measured meat quality.
- \( j \): product \( j \).

Variables:

- \( x_{i,j} \): Number of pigs from sorting group \( i \) used to produce product \( j \).
- \( y_{i,j} \): 1 if sorting group \( i \) is used to produce product \( j \); else 0.
Parameters:
Distribution\(_i\): Number of pigs available in sorting group \(i\).
MaxSale\(_j\): Maximum sale of products \(j\) in percentage of the total number
Price\(_{i,j}\): Net price for product \(j\) if raw materials come from sorting
group \(i\).
QualityLow\(_{i,j}\): Number of items in sorting group \(i\) not living up to the desired
specifications for producing product \(j\).
SumQualityLow\(_j\): Sum of items not living up to specifications for producing product
\(j\).
AcceptableQuality: Acceptable level (percentage) for items not living up to
specifications for producing the products. The acceptable level is
valid for each product.
AverageWeight: Average weight of back.
N: Number of pigs in the sample.
M: Big M is a constraint with a sufficient large value. By setting the
value equal to \(N\) it forces the value of \(y_{i,j}\) to obtain the desired value
depending on the value of \(x_{i,j}\)

The model was developed in 2005 as the first model and is a relative simple model only
using one product type at a time. The model was used to obtain knowledge of short
comings and other desired requirements, which could be implemented in the succeeding
models.

Model 1 is only investigating the effects for one type of product at a time. In paper A
the type of product used is the back product, and five different back products are used.
The weights of the products are not estimated based on information from the
slaughtering, but are just set to an average weight of 11 kg product per pig. As input to
the model simulated quality measurements (the fat layer) have been used based on the
actual slaughter data as well as simulated measuring errors at different levels.

The objective function (1) maximizes the total price obtained for the products by
finding the best use for the different parts. Constraint 2 ensures that the production of
pigs does not exceed the number of pig in each sorting group. Constraint 3 controls that
the maximum sale of different products is not exceeded. MaxSale\(_j\) is the percentage of
the total items allowed to be sold as product \(j\). Constraints 4 and 5 controls that the
binary variable \(y_{i,j}\) receives the value 1 if sorting group \(i\) is used to produce product \(j\)
and 0 otherwise. For each product, constraint 6 finds the number of products produced
from pigs not living up to product specifications. Constraint 7 is a quality constraint
controlling that the number of items produced from raw materials not living up to
specification does not exceed certain levels. Constraint 8 states that the number of pigs
used to produce different products cannot be negative, and equation 9 that $y_{i,j}$ is a binary variable.

### 4.1.2 Model 2

In paper B regarding a general increase in slaughter weight, a new model is introduced. Even though the purpose of the model is to investigate the economic effects of a general increase in slaughter weight, the model can be used as basis for a number of different other applications. The model is a clear improvement of the previous model and differentiates itself by using the entire pig and by estimating the product weights based on real slaughter data.

We have a set of carcasses $I=\{1,...,I\}$. Each carcass can be used to produce different products in the set $J=\{1,...,J\}$. Finally a number of different weight scenarios $K=\{1,...,K\}$ for the slaughter weight are computed. The decision variable $x_{i,j,k}$ is a binary variable with the value 1 if pig $i$ is used to produce product $j$ in weight scenario $k$ and 0 otherwise. The problem is to find the optimal use of each carcass at each weight scenario and the total profit of the optimal solution for each weight scenario:

The objective function:

1) \[ \text{Maximize } Z = \sum_{i,j,k} \left( \text{Price}_j + \text{PriceCoeff}_j \cdot \text{FatLayer}_i,k \right) \cdot \text{ProdWeight}_i,j,k \cdot x_{i,j,k} \]

Subject to:

2) \[ x_{i,P\_Breast2,k} \cdot \text{ProdWeight}_{P\_Breast2,k} \leq 4 \]
3) \[ x_{i,P\_Ham,k} \cdot \text{FatLayer}_i,k \leq 14 \]
4) \[ x_{i,P\_Shoulder,k} = x_{i,P\_Neck,k} \quad \forall \ i \in \mathcal{J}, \ k \in \mathcal{K} \]
5) \[ x_{i,P\_Neck,k} = x_{i,P\_CutOff1,k} \quad \forall \ i \in \mathcal{J}, \ k \in \mathcal{K} \]
6) \[ x_{i,P\_CutOff1,k} = x_{i,P\_Sundry1,k} \quad \forall \ i \in \mathcal{J}, \ k \in \mathcal{K} \]
7) \[ x_{i,P\_Backs (with bones),k} = x_{i,P\_Breast1,k} \quad \forall \ i \in \mathcal{J}, \ k \in \mathcal{K} \]
8) \[ x_{i,P\_Breast1,k} = x_{i,P\_CutOff2,k} \quad \forall \ i \in \mathcal{J}, \ k \in \mathcal{K} \]
9) \[ x_{i,P\_CutOff2,k} = x_{i,P\_Sundry2,k} \quad \forall \ i \in \mathcal{J}, \ k \in \mathcal{K} \]
10) \[ x_{i,P\_Backs (boneless),k} = x_{i,P\_Breast2,k} \quad \forall \ i \in \mathcal{J}, \ k \in \mathcal{K} \]
Indices:
i: Pig i
j: Product j
k: Weight increase/scenarios

Decision variables:
x_{i,j,k}: Decision variable with value 1 if product j is produced of pig i at weight scenario k, otherwise 0.

Parameters:
PigWeight (i): Slaughter weight of pig i
ProdWeight (i, j): Weight of product j produced from pig i
FatLayer (i): Layer of fat (in mm) for pig i
Price(j): Price per kg (in DKK) for product j
PriceCoeff(j): Price coefficient for product j (price increase in DKK as a consequence of a decrease in the fat layer by 1 mm)

The objective function is a “dummy” function, which optimizes the sum of total net sales of the optimal solutions for all the different weight scenarios. This means that optimal solutions for each weight scenario are found at the same time, as there are no inter-connection between the individual weight scenarios. The price for each product consist of a fixed price per kg and a price coefficient depending on the fat layer and stipulating how much the price per kg changes as a consequence of a 1 mm change in the fat layer. The product weights are estimated for each pig at each weight scenario based on the slaughter weight and the fat layer.
Constraints 2 and 3 control that some quality demands have to be fulfilled before certain products can be produced from the pig. The slaughterhouses have some constraints regarding which products can be produced. When the slaughterhouse decides to use a carcass to produce one product other specific products often have to be produced as well. Constraints 4-15 ensure that when an alternative use of the pig is chosen, this implies the choice of a whole package of products. For instance, constraint 4 controls that if P_Schoulder is produced then P_Neck is produced as well. If this is the case, constraints 5 and 6 control that also P_CutOff1 and P_Sundry1 have to be produced as well.

Finally constraints 16 and 17 control that the back and the ham can only be used for one product alternative at a time and equation 18 that $x_{i,j,k}$ is a binary variable with the value 1 if pig i is used to produce product j in weight scenario k, otherwise the value is 0.

### 4.1.3 Model 3

In paper C regarding the logistic limitations and its effect on the slaughterhouses economy, model 2 has been changed. The objective function now optimizes the value of the pigs placed at each bar, when all pigs placed on the same bar are used for the same product alternative.

Furthermore, the model has been reformulated so it uses product packages, which can be specified in a matrix (in GAMS a table) in a very compressed manner instead of having a number of constraints for each product package:
Figure 14. Table from GAMS specifying possible product alternatives.

It can be seen in Figure 14 that each product alternative consists of just one column with the value of 1 if the product is included and 0 otherwise. This makes it much easier to handle several products and product alternatives than in model 2. Model 2 uses several constraints for handling each product alternative. Furthermore, the model has been reformulated by using parameters instead of having the value calculations directly in the objective function. The reformulation makes the model more apparent.

We have a set of carcasses \( J = \{1, \ldots, I\} \). Each carcass can be used to produce different product alternatives in the set \( N = \{1, \ldots, N\} \) and each product alternative consists of a number of different products in the set \( J = \{1, \ldots, J\} \). Finally, the carcasses are hung on a set of bars \( K = \{1, \ldots, K\} \) in the equalization room. The decision variable \( y_{k,n} \) is a binary variable with the value 1 if the pigs placed on bar \( k \) are used to produce product alternative \( n \) and otherwise 0. The problem is to find the optimal utilization (product alternatives) for the carcasses placed at each bar and the total profit for the optimal solution:

The objective function:

1) \[ \text{Maximize } Z = \sum_{k,n} \text{ValueBar}_{k,n} \cdot y_{k,n} \]
Subject to:

2) \[ \sum_n y_{k,n} = 1 \quad \forall \ k \in \mathcal{K} \]

3) \[ y_{k,n} = \begin{cases} 1 & \text{if product alternative } n \text{ is produced by pigs placed on bar } k, \text{otherwise } 0 \end{cases} \]

4) \[ \text{Value}_{\text{Pig},i,n} = \sum_{i,n} (\text{Price}_j + \text{PriceCoeff}_j \cdot \text{FatLayerDeviation}_i - \text{QualityDeduction}_{i,j}) \cdot \text{ProdWeight}_{i,j} \cdot \text{AltUse}_{j,n} \quad \forall \ i \in \mathcal{J}, \ n \in \mathcal{N} \]

5) \[ \text{Value}_{\text{Bar},k,n} = \sum_k \text{Value}_{\text{Pig},k,n} \quad \forall \ k \in \mathcal{K}, \ n \in \mathcal{N} \]

Indices:
\[ i : \text{pig} \quad k : \text{bar} \quad j : \text{product} \quad n : \text{alternative use} \]

Decision variables:
\[ y_{k,n} : \text{Decision variable with value } 1 \text{ if the pigs placed on bar } k \text{ are used for product alternative } n, \text{otherwise } 0. \]

Parameters:
\[ \text{Value}_{\text{Pig},i,n} : \text{Value of pig } i, \text{when used to produce alternative } n. \]
\[ \text{Value}_{\text{Bar},k,n} : \text{Value of the pigs placed on bar } k \text{ when used to produce alternative } n. \]
\[ \text{Price}_j : \text{Fixed net price per kg when used to produce product } j. \]
\[ \text{PriceCoeff}_j : \text{Change in net price per kg for product } j \text{ when the fat layer in creases by 1 mm.} \]
\[ \text{FatLayerDeviation}_i : \text{Deviation in the fat layer of pig } i \text{ to the average layer of fat.} \]
\[ \text{QualityDeduction}_{i,j} : \text{Price deduction per kg if quality demands are not being met when pig } i \text{ is used to produce product } j. \]
\[ \text{ProdWeight}_{i,j} : \text{Estimated weight of product } j, \text{when produced from pig } i. \]
\[ \text{AltUse}_{j,n} : \text{Alternative use (product package) with value } 1 \text{ if product } j \text{ is part of product package } n, \text{otherwise } 0. \]

The objective function is the sum of the value of carcasses placed at each bar by finding the best alternative use for each bar, when all pigs placed on the same bar are used for same product alternative. Constraint 2 controls that the pigs placed at each bar are only used for one product alternative.
4.2 Solution times

The computations in the four papers A-D [Kjærsgaard, N. (2008a,b,c,d)] are primarily used for illustrative purposes. With 17 products and 4 product alternatives as used in the papers B-D the solution times are fast. The most advanced model described in paper C is solved to optimality in just 18 seconds. The model is solved on DTU’s Sun Fire E6900 server (48 UltraSPARC IV CPUs, 1200 MHz/dual-core/8 MB L2-cache per core and 96 GB memory), but are only using one of the CPU’s. The solution time using a PC is expected to be even faster.

Before the models are used as actual decision support, more products and product alternatives should be used. The number of products and product alternatives are increased by adding new fictive products and product alternatives and perform new computations. When using 54 products and 108 product alternatives the number of variables increases from 3,980 and 178,696 constraints to 63,180 variables and 4,809,712 constraints and the model is solved to optimality in 398 seconds.

Whether a solver time like this can be considered acceptable is depending on the context in which the application will be used. A number of different applications have been proposed in this thesis:

1) Value of improved measurements

Deciding on the level of measuring accuracy worth investing in is a strategic decision which is not performed often, but nevertheless is very important as the investments are large. The solution time for the computations is not critical as the decisions do not have to be made fast.

2) The value of logistic changes at the slaughterhouses

From time to time the slaughterhouses make decisions regarding changes of the production facilities at the slaughterhouses. These are strategic decisions which are not made often, but they are of major importance as the investments can be very large. The solution time is not critical, but how the application is used may differ depending on the solution time. If it takes hours to solve the model, it will probably mostly be used to
evaluate a relatively few final layout candidates and not used as a tool to evaluate more incremental changes.

3) **Value of a general increase in slaughter weight**

Decisions regarding a general increase in the slaughter weight are either tactical decisions or strategic decisions. If good decision support exists, such evaluations would probably be performed on a regular basis. The solver time is not critical.

4) **Design of new sorting strategies**

The market conditions change rapidly and influence prices for different products. Evaluating existing and designing new sorting strategies is a tactical decision which is made on a regular basis, especially if good tools exist. Solution time is not considered critical. However, it may influence on the use of the application. If the solution time is in hours, computations may be performed as input to starting the design process and at the end when evaluating a few promising sorting strategies. If the solution time is 398 seconds, the computations can be performed several times during the design process with smaller changes in the criteria.

For all the applications above, a solution time of 398 seconds is considered very acceptable.

As proposed, in the future the model can be modified for other applications as well. Some of the future applications mentioned in this paper are:

5) **Production planning**

An interesting application is to modify the model with a.o. time frames and use it for production planning. Such operational decisions are performed on a daily basis to find optimal production plans where the raw materials are used to produce the products for which they are best suited. The solution time is not critical, but it may influence the use of the application. If the solution time is e.g. 20 minutes, new computations can be made during the day when changes in the production occur (e.g. having problems with machinery etc.). If the solution time is in hours, computations are still very interesting but will only be performed to be ready for the production planners the next morning.
6) Sales support

The model can be modified to support the sales personnel in connection with setting prices for new and existing products. Two different uses of the model are considered for sales support. As support for tactical decisions performed on a regular basis supporting how new products should be priced and evaluating the prices of existing products. For this application the solver times is not critical. However, if the solution time is too long the application may not be used much for evaluating effects of changes in the price assumptions etc.

The model can also be used for operational decisions performed on a daily basis supporting the sales personnel with price proposals computed while having telephone contact to the customer. For such application the solution time is critical and even one minute may not be acceptable.
Chapter 5

Papers

The four papers being part of this Ph.D. thesis are presented in short in this chapter. The problems, models and results are briefly described.

5.1 Paper A - The Value of Improved Measurements in a Pig Slaughterhouse

The paper A [Kjærsgaard, N. (2008a)] presents the problems of the Danish slaughterhouses regarding measuring accuracy in connection with the classification systems, and the subsequent sorting of raw materials into sorting groups. The measurements have a quite indirect nature with substantial measuring error. As the Danish slaughterhouses have planned to invest considerably in new measuring systems in the years to come, it is desired to be able to estimate the value of improved measurements. Such computations will make it possible to investigate how much it will be worth investing to improve the measurements to different levels of accuracy.

Model 1 developed is a Mixed Integer Programming Model which is able to compute the value of improved measurements for one product type at a time. The model can significantly improve the basis for Danish Meat Research Institute and the Danish slaughterhouses for making decisions.

The model was developed in 2005 as the first model and is a relatively simple model using only one product type at a time. The model was used to obtain knowledge of short comings and other desired requirements, which could be implemented in the succeeding models.

It was found that the pricing and the assumptions herewith are of the utmost importance for how well the computations provide a fair and true view of the economy of the slaughterhouses. The paper holds recommendations with regard to splitting the costs in different price and cost contributions.
5.2 Paper B - The Value of a General Increase in Slaughter Weight for Pigs

There have been varying opinions within the industry concerning an improved profit for the slaughterhouses and the farmers when increasing the slaughter weight. The argument for an increased slaughter weight is based on the fact that some of the costs at the slaughterhouses and at the farmers are fixed, others are variable to the number of pigs produced and other again are variable to the volume or kg produced meat. A general increase in slaughter weight can therefore decrease the unit costs per kg produced meat. The paper B [Kjærsgaard, N. (2008b)] presents the important problems of Danish slaughterhouses regarding economic consequences of such a general increase in slaughter weight.

Compared to model 1 described in paper A, model 2 uses the entire pig instead of just one product type at a time and is furthermore improved with a more accurate pricing, as the pricing is based on estimated product weights. Model 2 is a clear improvement of the current methods used by the slaughterhouses, as it also takes the changes in the fat layer as a consequence of the increased slaughter weight into consideration.

5.3 Paper C - Limitations in the Production and its Effect in the Profitability

The paper C [Kjærsgaard, N. (2008c)] concerns the logistic limitations in the slaughterhouses and how these limitations influence the raw material flow and the economy of the slaughterhouses. By taking the logistic limitations into consideration, model 3 can be more accurate than the previous models described in papers A and B (model 1 and 2). Furthermore, model 3 can be used to find the economic effects of eliminating or decreasing the influence of these different logistic limitations in connection with investments in changes of the production facilities.

Some of the most important limitations regarding the raw material flow are limitations in the equalization room, where the pigs are placed on bars for a minimum of 16 hours in order for the temperature to equalize over the entire carcass. The carcasses are placed on the bar from one side and the bar can only be emptied from the other side. This
results in the limitation that each bar in principle can only be used for one main production order, and that the pigs in principle should have the same alternative use (same mix of products).

Model 3 described in paper C is based on model 2 described in paper B. The objective function has been changed to maximize the value of the carcasses placed at each bar by finding the best alternative use of each bar, when all the pigs placed on one bar are used for the same alternative use. Apart from that, the model is in principal the same, but has been reformulated in order to handle a larger number of products and alternative uses.

The carcasses are sorted into different sorting groups based on the actual slaughter weight as well as the simulated measuring error and fat layer. The carcasses are placed on bars depending on sorting groups, i.e. each bar only contains carcasses from the same sorting group.

5.4 Paper D - Simulation of Different Sorting Criteria and Strategies

Designing the right sorting criteria and strategies is one of the most important aspects of optimizing raw material use at the slaughterhouses. In principle, sorting can be based on all kinds of quality characteristics; in this paper sorting is illustrated by the use of two quality characteristics: namely the fat layer and the slaughter weight.

The model used in paper D [Kjærsgaard, N. (2008d)] is the same model as used in paper C, i.e. model 3 and it has been used to compute the value of different sorting strategies. A graphical tool has been developed to support the design of different good sorting strategies. The tool can be an important decision support for the slaughterhouses to design their sorting strategies, which can then be further examined by computing and comparing profits of the different sorting strategies.
5.5 The Papers Relation to the Overall Framework

The overall framework for the project was described in chapter 2.4 and shown in Figure 8. 7 topics for improving the slaughterhouses’ basis for making decisions were made. The 7 topics are:

1. Computing the value of improved measurements
2. Evaluating the payment structure to the farmers
3. Pricing of new products
4. Test of logistic changes at the slaughterhouses
5. Design of new sorting strategies
6. Improved product mix and product planning
7. Education of production planners

The topics 1, 2, 4 and 5 are directly supported by the models described in the four papers A-D. Topic 3, 6 and 7 can be based on the models described but requires that the models are modified.

For convenience Figure 8 is repeated in Figure 15 below and the activities in the overall framework which are or can be supported by the models developed has grey colour as background. The activities marked with red are the activities which are directly related to one of the seven topics (number of the topic is added). The remaining activities with grey background are indirectly supported as a consequence of the other activities. For instance if the measurements are improved it can also result in improved yield data. It can be seen that most activities are covered, except for the R&D activities within meat quality, breeding and new technology and optimization/OR and simulation. These activities can be considered supporting activities for improving the measurements, sorting and raw material use.
Figure 15. The overall framework for the project.
Chapter 6

Future Work

6.1 Future Work

Before the slaughterhouses can use the models for actual decision making more products and alternative uses should be used as input in the models and a price and cost study should be performed. Furthermore, the accuracy of the models can be improved further by taking more limitations in the production into consideration. Especially areas such as the automatic sorting where the middle pieces and the hams are sorted by weight and the buffer storage before the cutting departments would be interesting to include.

The price and cost study should be performed by the slaughterhouses as information of prices and costs is considered most sensitive for the slaughterhouses and they may not wish to share this information with others. The prices and costs are vital for the computations and it is recommended that prices and costs are split into different contributions and estimated separately for each product:

- Price per kg product made from raw materials living up to specifications.
- Costs of delivering a better quality than the specifications require.
- Contribution from delivering products from raw materials not within specifications.
- Waste in connection with extra trimming as a consequence of raw materials not being within specifications.
- Costs for extra cutting and handling etc. as a consequence of raw materials not being within specifications.

In order for the models to cover more products it will be necessary to obtain information from the slaughterhouses regarding product yields from all products used in the model. The slaughterhouses have substantial yield statistics, but additional surveys may be necessary, e.g. by performing yield studies with the help of CT scanners.
Finally, the models can - instead of being used in connection with making strategic decisions - be modified and used in connection with operative decisions, such as support in connection with the daily production planning. The model can be modified to include scheduling within the day, e.g. take into consideration that 2,000 hams should be available for delivery to a customer in France at 14:00. Furthermore, the model can be modified to include restrictions in the minimum or maximum sales of individual products. Different prices can be used for different intervals of sales volume, which can improve the way the model reflects the slaughterhouses actual demand curves.

Other areas of interest:

- **Pricing of new products.** The improved yields data for new products together with sale support based on the models developed can help the sales personnel significantly to compute the right prices. The model described in paper C can be amended to be used for sale support.

- **Production planning.** The models can already be used to find optimal product mix, but can be changed to support the daily production planning as well. This requires that the model described in paper C [Kjærgaard, N. (2008c)] is modified to include time frames for production and delivery of the products.

- **Education of production planners.** Production planners can investigate the consequences of different changes in their production plans and compare the profitability of different production plans with each other. The model described in paper C can be used as basis, preferably modified to include time frames for production and delivery of the products.

- **Strategic value of delivering the exact quality as customer’s desire.** Improved sorting makes it possible for the slaughterhouses to deliver the exact quality to the customers as they desire and thereby being the preferred supplier for their customers. There is consensus within the industry that the value of being able to do so is very considerable, but the exact value is very hard to estimate. Analysis can be performed investigating this by estimating the value added in different parts of the value chain and then use different scenarios for how the value added can be split between the slaughterhouses and its customers.

- **Simulation of the raw material use at the slaughterhouses.** Already today, the Danish Meat Research Institute and the slaughterhouses have simulated part of the raw material flow of slaughterhouses. Until now simulation has primarily been used in connection with new or rebuilding of slaughterhouses, but
simulation of the entire raw material flow in detail can be a tool to further improve the raw material use.

- **The use of Information at the slaughterhouse.** Potentially CT scanners can revolutionize the measurements and provide full knowledge of the quality of the carcasses. Today, the measurement systems are placed at the end of the slaughter line before the pigs are placed in the equalization room. It should be examined if this is still the best placement or if full knowledge of the quality makes it desirable to perform the measurements later in the processes. By having the measurements performed later, the information can more easily be used in the subsequent automated machines. On the other hand, it is also desirable to receive the information early to be used for placement of the pigs in the equalization room.

- **Improved yield data.** CT scanners make it possible to improve the yield data substantially. The Danish Meat Research Institute and the Danish slaughterhouses are working on a project “The virtual butcher” which potentially makes it possible to establish a data base consisting of a considerable number of scanned carcasses. The data base can then be used to estimate product weight and yields for different cuts based on all carcasses in the data base. A special challenge will be to establish a system ensuring that the yields are also realised at the slaughterhouses.

### 6.2 Recommendations for Cost Study

In this chapter recommendations for the cost survey will be given, including a short description of the most important cost allocation principles. However, first a short introduction to how costs can be categorized. Costs are traditionally categorized in the following types of costs:

- **Variable costs** are costs which are variable according to the activity level (e.g. the production or sales volume measured in pieces or in kg).

- **Fixed costs** are costs which are fixed even if the production or sales volume changes. In principle, the fixed costs will still exist if the production/sales vanish totally, however only for a period of time.
All costs can be categorized as either variable or fixed. In some cases the costs are also categorized as either direct or indirect costs. Direct costs are costs which can directly be related to a given activity (here production of a given product), whereas indirect costs cannot. So based on these two different categorizations, we have four kinds of costs:

- **Direct variable costs** are costs directly related to the production of a single product. An example is raw materials.

- **Indirect variable costs** are costs that vary with the production volume, but it cannot be established to which product it is actually referred. An example can be maintenance of machinery etc., where the machinery is used for several products

- **Direct fixed costs** are fixed costs related to product specific activities. An example is costs connected to product specific machinery.

- **Indirect fixed costs** are fixed costs which cannot directly be related to a given product and do not vary with the production volume.

These four kinds of costs will be used when explaining the principles of Activity Based Costing.

As mentioned, a number of different principles can be used to allocate costs for different products. The most important and most used are:

- The contribution cost method (in Danish “dækningsbidragsmetoden”)
- The full cost method (in Danish “fuldkostmetoden”) and
- Activity based costing.

Which principle is the best suited depends on the purpose of the allocation and to some extend also on the time horizon chosen. Classification of costs as being either direct or indirect depends on the purpose as well as on the choice of cost drivers. Whether a cost can be considered as variable or fixed depends to a certain degree on the time horizon chosen, as a number of costs, which in the short term can be considered as fixed costs, can be considered as variable costs in a long term perspective. This is due to the fact that over a long period of time most costs can be reduced with the activity level (e.g. production or sales volume). It must be noticed, that the definition of variable and fixed costs may not entirely be the same in the three different costs allocation methods, but this is of no importance for the recommendations. Each method has its advantages and disadvantages, which will be explained further in the following.
6.2.1 The Contribution Cost Method

The contribution cost method allocates the direct and indirect variable costs, while the fixed costs are neglected. Variable costs (when using the contribution cost method) can be defined as “costs, which in any situation is given by the activity level and the activity’s specific character”\(^1\). In other words, this means that the costs are variable with the product (the number of items or volume sold or produced of the product).

The advantage of the contribution cost method is that it supports perfectly the business economic marginal considerations used when examining the economic consequences of e.g. producing one unit more or less of a given product or whether or not to accept an isolated order from a customer.

The criticism regarding the contribution cost method is that the production processes today are characterized by a continuously larger degree of automation. This has influenced the cost structure for production companies. Today, typically fewer costs are directly variable to the number of produced items. When the fixed cost’s share of the company’s total costs for many industries (including the slaughterhouses) has been increasing in this way, and the method does not take the considerable fixed costs into account, the method has become less attractive for many purposes.

6.2.2 Full Cost Method

Another method is the full cost method. The principle of the full cost method is that all costs of the company, i.e. both the direct and indirect variable and fixed costs, are allocated to products etc.

The advantage of this method is that all costs of the company (including fixed costs which are often considerable) are actually allocated to different products etc. The disadvantage of this method is that the allocation of costs does not always reflect how much different products etc. are actually employing the company's resources. The calculated profitability of the products will therefore not always provide a true and fair view of the resources actually used.

\(^1\) As defined by late Professor Zakken Worre, Copenhagen Business School.
6.2.3 Activity Based Costing

When using activity based costing the direct variable and fixed costs are allocated, whereas the indirect costs are not taken into consideration. The method hereby allocates a considerable part of the fixed costs to orders, products and customers etc.

Activity based costing operates in a hierarchy of four levels, which can be seen in Figure 16 below, whereas the contribution cost method only uses one level (the product unit, similar to the unit level in activity based costing). By using these four levels more costs can be allocated and less are considered as non allocated fixed costs.

![Cost hierarchy](#)

**Figure 16. Cost hierarchy.**

**Unit level activities**

The unit level activities cover activities which are directly related to the production of the individual product unit. An example of such an activity is the use of materials, which varies with the number of items produced or sold.

**Batch level activities**

Batch level activities cover activities, related to the individual production batch and in principle not to the size of the batch. As an example of such activities can be mentioned adjustments of machinery to produce a certain product.
Product sustaining level activities

These activities cover activities, related to the individual product type and are independent of the number of production batches or the volume produced. As an example of such activities can be mentioned product development.

Facility sustaining level activities

Some of the company’s activities are necessary whether or not production takes place. As an example can be mentioned the top management, as there has to be a managing director no matter if the production volume is small or large or whether or not it covers one or more products. These activities, which are often referred to as “the rule of one” are not allocated to products.

Cost allocation principle

The rationale in activity based costing is that resources are spent on activities, and that products and services are a result of activities. Activity based costing traces the use of resources to activities (or cost centres) and links the activity costs to cost objects, such as products, services or customers by the use of cost drivers. See Figure 17 below:

![Cost allocation principle diagram](image)

Figure 17. Cost allocation principle.
Cost drivers

Cost drivers are used to allocate costs from resources to activities (or cost centres) and the costs of activities to cost objects, such as products, services or customers etc. Cost drivers are characterized by that the costs in general should be variable to the utilization of the cost drivers and can be divided into the following three main groups:

- Transaction dependent cost drivers
- Duration/time dependent cost drivers
- Intensity dependent cost drivers

Transaction dependent cost drivers allocate costs every time the cost driver concerned is used. This type of cost drivers can e.g. be set up of machinery, handling of a production order etc. Transaction dependent cost drivers are the most inexpensive cost drivers to establish and just require a simple registration. Costs are allocated with the same amount each time the cost driver is utilized.

Duration dependent cost drivers take basis in the duration of the concerned activity. This type of cost drivers can similarly be set up of machinery etc., handling of a production order etc., but here the costs are allocated based on the actual time spend on the activity. This type of cost driver requires more intense registration and is with additional costs as a consequence. Especially in situations where there are large fluctuations in the actual time of the activity, the duration dependent cost drivers are highly suited.

The intensity dependent cost drivers are the most accurate, but also the most expensive ones to establish. The intensity dependent cost drivers allocate the costs directly to the product for the time the resources have been used. Using the example with e.g. the set up of machinery, different types of employees might have been involved with the set up. Different types of employees might spend different time at the set up, and their wages might vary too.

The size of cost drivers can be determined based on the budgeted (or historical) activity level or alternatively based on the capacity of the activity concerned. These two principles will be illustrated in the following example.

The annual costs of e.g. sales administration amount to DKK 1,000,000 and as cost driver is chosen the number of sales orders. When using the activity level this can be the budgeted or historical number of sales orders (based on forecast or e.g. last year’s...
numbers of sales orders). In the example the budgeted activity level is 2,000 sales orders and capacity in the sales administration has been set to be 25% higher than this level, i.e. 2,500 sales orders annually. With these two principles the cost driver can be calculated as follows:

<table>
<thead>
<tr>
<th>Principles</th>
<th>costs</th>
<th>sales orders</th>
<th>cost driver</th>
</tr>
</thead>
<tbody>
<tr>
<td>Activity level</td>
<td>1,000,000</td>
<td>2,000</td>
<td>500</td>
</tr>
<tr>
<td>Capacity</td>
<td>1,000,000</td>
<td>2,500</td>
<td>400</td>
</tr>
</tbody>
</table>

Table 1. Cost drivers determined based on activity level and capacity

If the principle regarding the expected activity level is used and the number of sales orders will be as expected (i.e. 2,000) all costs of 1,000,000 will be allocated to the sales orders. This may be exactly as intended, however, if the number of sales orders is larger than expected, but still within the capacity, then more than the actual cost hold will be allocated to the products. By using the capacity as basis this will not be the case, but then all costs may not be allocated to the products. If the number of sales orders is 2,000 as expected, then only DKK 800,000 will be allocated.

It is recommended, that the slaughterhouses use the principles in Activity Based Costing for their price and cost survey as this will reflect the actual costs of producing different products better than the traditional cost allocation methods.
Chapter 7

Conclusion

The main conclusion is that the Mixed Integer Programming models developed and described in the four different papers can be used by the slaughterhouses to improve their basis for making decisions for a number of important strategic decisions which are performed on a regular basis. Prices may change from one week to another, and consequently a price and cost study should be performed and more products and product alternatives included before using the models for actual decision making.

The first model developed and described in paper A [Kjærsgaard, N. (2008a)] regarding improved quality measurements, was a relatively simple model only considering one part of the pig at a time. The model can be used to compute the consequences of improved quality measurements and raw material use. Prices and costs used in the model are important for the computations, and it is found that the prices and costs can be split into four different contributions with advantage. The economic consequences found by using the model are considered conservative as they do not take into consideration the considerable strategic value of being able to deliver the exact quality to the customers as promised. This strategic value is very difficult to estimate but the slaughterhouses are convinced that it is of a very considerable size maybe even as important as the economic consequences computed.

The second model, described in paper B [Kjærsgaard, N. (2008b)] regarding a general increase in slaughter weight uses the entire pig, and the weight of different potential products is estimated based on real slaughter data as well as on a simulated measuring error. The third model, described in paper C [Kjærsgaard, N. (2008c)] regarding the logistic limitations in the production is a further improvement of the second model, by also taking into consideration some of the logistic limitations at the slaughterhouses. The model is more accurate than the models developed previously and can be used to support a number of strategic decisions regularly made at the slaughterhouses, such as:

- Decisions regarding establishing the optimal slaughter weight and determination of payments to the farmers. The basis for making decisions is improved significantly compared to the current ad-hoc analysis. Now the computations can also take into consideration changes in product yields as a consequence of changes in the slaughter weight.
Decisions regarding investments in new quality measurement systems, by improving the economic value of improved measurements.

Evaluation of different sorting strategies.

Decisions regarding new investments in logistic improvements e.g. in connection with the raw material flow. The models can be used to find the economic value of logistic changes.

The price and costs study can advantageously be based on the principles from activity based costing. These principles reflect the real costs of producing different products better by allocating part of the considerable fixed costs of the slaughterhouses to the individual products without doing so in a too arbitrary way.

The third model had another application in paper D [Kjærgaard, N. (2008d)], where it was used to evaluate different sorting criteria and sorting strategies. It was shown that the model can be used to evaluate and compare different sorting strategies by computing the profit when using different sorting strategies. Furthermore, an important tool has been developed which can be used in connection with designing new sorting strategies. It can provide a graphical view of the potential sorting strategies worth investigating further. See the matrix in Figure 18 below:

![Figure 18: Distribution of pigs on quality groups at the current measuring accuracy with improved sorting groups](image)

For each quality group the optimal product alternative is computed and has been coloured in accordance herewith. Four sorting groups have been indicated in Figure 18.
above and are marked by the red lines. It is an aim in the design process that each sorting group should mostly consist of quality groups with the same optimal product alternative (same colour).
Bibliography


Fertin, C. (1992). Validation of long term planning model for optimization of the raw material use at pig slaughterhouses (in Danish). The Royal Veterinary and Agricultural University, Copenhagen, Denmark.


Appendices
Appendix 1- Overall production flow at the slaughterhouses
Paper A
The Value of Improved Measurements in a Pig Slaughterhouse

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Abstract

The pig industry is an essential and important part of Danish economy with an export value in 2006 of more than DKK 28 billions [Danish Meat Association (2007)]. The competition is hard, and potential new competitors from low cost countries can be expected to enter the traditional Danish export markets. Therefore it is more important than ever to optimize all aspects of Danish pig production, slaughtering processes and delivery. This paper concerns the aspects of optimization at the slaughterhouses regarding estimation of the value of improved measurements.

The slaughterhouse industry differs from the traditional industry in a number of ways. There is a large natural variation in the raw materials regarding quality, weight, size, lean meat percentage, as a consequence of pigs being a biological material. The slaughterhouses handle this large variation by sorting the pigs into groups consisting of pigs with almost the same characteristics and thereby reducing the variation within the individual sorting groups substantially. The accuracy of the measurements is the most important limiting factor for how much the variation within each sorting group can actually be reduced. Substantial investments are expected to improve the quality of the measurements further. This paper concerns the use of Operations Research to solve a practical problem, which is of major importance for the industry, namely to improve the estimation of the economic effects of improved measurements. The benefit for the industry is obviously to be able to decide upon the level of measuring accuracy worth investing in.
The main conclusion is that even relatively simple optimization models can advantageously be used to improve the basis of the slaughterhouses for making decisions regarding improved measurements. The model is a Mixed Integer Programming (MIP) model and is used to compute the consequences of improved measurements and analyze different scenarios regarding restrictions in sales volume and quality restrictions.

The assumptions regarding pricing and cost are found to be very important to obtain a true and fair view of the size of the profit. For future (and improved) computations the net prices used can advantageously be split into 3-4 different contributions, which should be estimated separately for each product.

1 Background

The pig industry is essential for Danish economy and in 2006 more than 25 million pigs were produced in Denmark and approx. 90% of the meat was exported. The export value amounted to DKK 28.8 billion [Danish Meat Association (2007)].

Competition in the pork industry is substantial and the feeding costs have increased considerably. Therefore it is increasingly important that Danish farmers and slaughterhouses continuously optimize their production. This paper concerns the aspects of optimization at the slaughterhouses.

Even within our neighbouring countries the competition is hard and pressure is on the slaughterhouses to offer the best payments to the farmers. During the last couple of years, a substantial number of Danish farmers have started delivering part of their pigs to German slaughterhouses.

The slaughterhouse industry differs from the traditional industry in a number of ways: Pigs and meat are biological materials with a high degree of natural variation in quality, weight, size, meat content (lean meat percentage) etc. The way slaughterhouses handle this variation is by sorting the pigs into different sorting groups in which pigs with almost the same characteristics are placed. The sorting can be based on a number of factors, each describing some quality characteristics. However, one problem is that the measurement of those characteristics is not trivial at all nor is the current accuracy overwhelming:
The measurements often have quite an indirect nature. For instance, today the lean meat percentage is estimated based on a number of ultrasound measurements at different parts of the pig. Those measurements are mathematically transformed into a value, which is an estimation of the lean meat percentage. Due to variability in size and shapes as well as to the fixation of pigs during the measurements many noise factors interfere. The differences in the exact fixation might result in measurements being performed in slightly different places of the pigs.

There is only a limited time to perform the measurements, as the throughput of a slaughtering line is approx. 350 pigs per hour.

New and improved technology makes it possible to obtain better and more accurate measurements of the pigs. Using CT scanners instead of ultrasound for instance makes it possible to measure the lean meat percentage almost without noise and very close to the true/correct values. Today, these true values can only be found in relatively small experiments, by costly dissections of the pigs.

2 Literature survey

The amount of literature addressing improved or optimized raw material use in the food industry is substantial. However, the main part of the contributions is related to different aspects regarding either optimization of meat quality or different production processes. Examples of this are optimization of the industrial thermal sterilization of canned foods [Garcia, M. et. al. (2006)] and pigs stunning optimization [Dupuis, P. et. al. (2004)]. These types of optimizations are not relevant for this project as they are either based on statistical analysis without optimization of a mathematical model or the mathematical models are very different from the models, which are used in this Ph.D. project regarding optimization of the raw material use at the slaughterhouses.

Within the pork industry relatively few contributions have been found regarding optimization based on operations research methods. In the paper “Location of slaughterhouses under economies of scale” [Broek et. al. (2006)] optimization is used to investigate the savings potential of reducing the number of slaughterhouses in Norway and investing in additional capacity in the remaining facilities in order to obtain economies of scale. Another facility location problem is described in the paper “The impact of changes in livestock supply on the optimum number, size and location of slaughterhouses in East Macedonia” [Kamenidis, C. & Sorensen, V. (1978)]. In the
The above mentioned four examples of optimization problems within the pork industry are all somewhat different from the problem of optimizing the raw material use at the slaughterhouses. More similar problems have been found in the following contributions:

In 1990-1992 a project regarding optimization of the raw material use at the slaughterhouses was performed as a cooperation between Danish Meat Research Institute and the Royal Veterinary and Agricultural University (now the Faculty of Life Sciences at University of Copenhagen). Several reports were made:

A Linear Programming (LP) model for production planning and control for the hog slaughterhouses was developed and reported in [Rasmussen, S. & Thomsen, M. (1991)] and [Rasmussen, S. (1992)]. The model is a 2-stage model. First stage concerns a planning horizon of 3 months and the second stage one weeks day to day planning. In [Fertin, C. (1992)] the long term planning model (stage 1) is validated. In his Ph.D. thesis [Fertin [1995]] Fertin describes and further develops and validates the model.

There has been searched for literature in other food related industries, e.g. poultry and beef slaughterhouses and the fish industry, but no relevant literature has been found.

Other industries have similar problems as the slaughterhouses regarding its raw material use. An example is the refineries, but unlike the slaughterhouses the refineries have the option of blending different qualities in order to change the quality characteristics of the products. Another example is the lumber and wood industry. A few papers of the product mix problem within the wood industry have been identified. In the paper “An Optimization-Based Decision Support System for a Product Mix Problem” [Roy et. al. (1982)] an LP-model has been used to solve a plywood product mix problem for Ponderosa Industrial in Mexico.

Even though literature within food optimization is substantial, the main part of the contributions are related to optimization based on e.g. statistical analysis without optimization of a mathematical model. Other models are very different from the models
used in this Ph.D. project. Except for the contributions from the Royal Veterinary and Agricultural University and the Danish Meat Research Institute not much literature of relevance for the Ph.D. project has been identified.

3 Measurements

As mentioned before, the accuracy of the measurements is very important for how precise the pigs are placed in the respective sorting groups and thereby essential for how well the pigs fit the production chosen. This directly influences the net prices obtainable and the profit for the pigs being produced.

Even though Danish slaughterhouses on an international scale are in the forefront regarding classification and measuring of the quality of the pigs, the measuring is - as described - characterized by substantial uncertainties. With the current measuring accuracy it is unavoidable that some pigs are placed into wrong sorting groups. Consequently, some pigs are used for products, for which they are not well suited resulting in additional costs, lower obtainable prices and unsatisfied customers.

As an extreme, one could imagine the effect of having online CT scanners at the slaughtering lines. Depending on how fine the scanning is performed and whether all parts of the pigs are being scanned, measurements with an almost 100% accuracy can be obtained. Another important way to improve the measuring accuracy is to improve the physical fixing of the pigs to ensure that the measurements are performed at the exact intended place of the pig.

The measuring system’s ability to measure accurately is specified by its standard error of prediction (SEP). The standard error of prediction is found as the standard deviation of the differences between the measured values and the reference values (true values) using a test data set.

Some of the key issues in connection with measuring accuracy are shown in the following example, where the relations between the measured and true qualities (here determined by the fat layer in millimetres) are shown:
Figure 1. Relations between measured and true values of the fat layer.

The example regards backs sorted into four different sorting groups based on the measured fat layer.

In this example the four sorting groups are identified as follows:\textsuperscript{1}:

Sorting group 1 the 10% leanest backs (as measured)
Sorting group 2 the following 30% leanest backs (as measured)
Sorting group 3 the next 15% leanest backs (as measured)
Sorting group 4 the remaining 45% of the backs (as measured)

Sorting group 1 consists of the 10% leanest backs (as measured) and will be used for products, where lean qualities are preferred. In Figure 1 above, sorting group 1 is found in the areas marked 1 and 2. Area 1 consists of the products of the highest quality, which fully lives up to specifications and are sorted correctly. Products in area 2 do not fully live up to specifications for raw materials to be used for those products. It will immediate be profitable for the slaughterhouses to have some products in area 2 as long as the level is considered acceptable (maybe 5-15%) for the customers. The large degree of variation in meat has made customers within the industry familiar to this kind of

\textsuperscript{1} Only for illustrative purposes in this paper. The groups do not in any way reflect the sorting practice in Danish Crown or Tican.
problems and they accept a small part of the deliverances not living up to specifications. However, the strategic value of being able to deliver precise the quality promised, is considered to be very considerable.

The area marked 3 consists of products, which have been classified as not being part of sorting group 1 by mistake. This is probably the most costly of the four areas as the raw materials are used for products, which are sold too cheap compared to their true quality. The products in area 4 are the products, which have been correctly sorted off.

An improvement in measurements will make the ellipse “slimmer” as shown in Figure 2 below. The number of pigs not sorted correctly decreases significantly (area 2 and 3). Especially notable is the limited number of pigs within sorting group 1 not living up to specifications (area 2). Similar considerations can be made for sorting groups 2, 3 and 4.

![Figure 2. Relations between measured and true values of the fat layer when measurements are improved](image)

Alternatively, sorting group 1 can be widened to include pigs with a larger measured fat layer. In figure 2, this corresponds to a displacement of the vertical line to the right on the x-axis. By so doing more of the pigs will be placed in the leaner sorting groups without increasing the level of pigs not living up to specifications compared to the current situation.
4 The Model

The purpose of the model is to investigate the economic benefits of improved measuring accuracy. The model was the first model developed and is a relative simple model only using one product type at a time and was used to obtain knowledge of short comings and other requirements for future models. The measuring noise has been simulated at the current level of the standard error of prediction (SEP) and at an improved level where SEP has been halved. Benefits are then found by performing two optimizations with data at these two levels of measuring accuracy. The improvement can be found as the difference in profits of these two optimizations.

We have a set of backs from carcasses which are placed into different sorting groups $\mathcal{I} = \{1, \ldots, I\}$ based on their measured quality. Each carcass can be used to produce a set of different back product $\mathcal{J} = \{1, \ldots, J\}$. The decision variable $x_{i,j}$ indicate the number of pigs from sorting group i used to produce product j. The variable $y_{i,j}$ is a binary variable with the value 1 if sorting group i is used to produce product j and 0 otherwise and is used to control the number of products made of a quality without of specifications. The problem is to find the optimal utilization of each sorting group and the total profit for the optimal solution given the carcasses’ distribution on sorting groups (Distribution$_i$) and restrictions in the maximum sale of each product (MaxSale$_j$). Furthermore, there are restrictions that the number of products not living up to the quality specifications (QualityLow$_j$) cannot exceed the acceptable level (AcceptableQuality).

4.1 Mathematical formulation of the model

The objective function:

1) $\text{Max. } Z = \sum_{i,j} \text{Price}_{i,j} \cdot x_{i,j} \cdot \text{AverageWeight}$

Subject to:

2) $\sum_j x_{i,j} \leq \text{Distribution}_i \quad \forall \ i \in \mathcal{I}$

3) $\sum_i x_{i,j} / (N \cdot 100) \leq \text{MaxSale}_j \quad \forall \ j \in \mathcal{J}$

4) $y_{i,j} \leq x_{i,j} \quad \forall \ i \in \mathcal{I}, \ j \in \mathcal{J}$

5) $y_{i,j} \cdot M \geq x_{i,j} \quad \forall \ i \in \mathcal{I}, \ j \in \mathcal{J}$
Indices:
i: sorting group i based on the measured meat quality.
j: product j.

Variables:
$x_{i,j}$: Number of pigs from sorting group i used to produce product j.
y$_{i,j}$: 1 if carcass i is used to produce product j; else 0.

Parameters:
Distribution$_{i}$: Number of pigs available in sorting group i.
MaxSale$_{j}$: Maximum sale of products j in percentage of the total number of pigs.
QualityLow$_{i,j}$: Numbers of pigs from sorting group i not living up to the desired specifications for product j.
Price$_{i,j}$: Net price for product j if raw materials come from sorting group i.
QualityLow$_{i,j}$: Number of items in sorting group i not living up to specifications for producing product j.
SumQualityLow$_{j}$: Sum of items not living up to specifications for producing product j.
AcceptableQuality: Acceptable level (percentage) for items not living up to specifications for producing the products. The acceptable level is valid for each product.
AverageWeight: Average weight of back.
N: Number of pigs in the sample.

6) $\text{SumQualityLow}_j = \sum_i \text{QualityLow}_{i,j} \cdot y_{i,j} \quad \forall \ j \in J$

7) $\text{SumQualityLow}_j \leq \text{AcceptableQuality} \cdot \sum_i x_{i,j} \quad \forall \ j \in J$

8) $x_{i,j} \geq 0 \quad \forall \ i \in I, \ j \in J$

9) $y_{i,j} = \begin{cases} 1 & \text{if production} \\ 0 & \text{else} \end{cases} \quad \forall \ i \in I, \ j \in J$
4.2 Description of the Model:

The model has been set up using 5 different back products, but can easily be modified to include more products and also to cover the remaining parts of the pig, e.g. the fore-end and the ham.

In the experiments we use approximately 60 different sorting groups (quality groups) based on the measured quality values. Actual slaughtering data from approx. 61,000 pigs slaughtered in one of the Danish slaughterhouses are used as input for the model. Data has been collected during May/June 2005. The data has been used to estimate the distribution of pigs on different sorting groups as well as the percentage of pigs placed in each group not living up to specifications for producing different products.

The pigs are placed into different sorting groups depending on the estimated “measured values” of the quality. The placement into sorting groups is therefore depending on the level of measuring accuracy. Estimation of the model parameters is commented on in detail in the section Estimation of Model Parameters.

4.2.1 Pricing

In general, the five different products in question can all be produced from different sorting groups. Some sorting groups, however, are more suitable for some products than others, according to the need for trimming of the products to a certain fat layer or length. A high suitability results in less work when products are further processed and in a larger yield (the weight of the main product, where the best prices can be obtained, as a percentage of the total weight of the part).

The most suitable thickness of the fat layer for different products is given in Table 1 below:

<table>
<thead>
<tr>
<th>Product 1</th>
<th>Product 2</th>
<th>Product 3</th>
<th>Products 4</th>
<th>Product 5</th>
</tr>
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<tbody>
<tr>
<td>Most suitable layer of fat (thickness - mm)</td>
<td>0-9</td>
<td>0-11</td>
<td>0-15</td>
<td>0-18</td>
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</tbody>
</table>

Table 1. Most suitable layer of fat for the different products

When a product is made from pigs outside the intervals above it is designated to be outside the specifications.

In order to take the suitability into consideration an additional cost is added to the products made from raw materials outside the specifications. This results in the prices
shown in Appendix 1 based on the true value of the fat layer. These prices are then transformed into prices obtainable for different sorting groups and are calculated individually for each level of measuring accuracy.

The prices are transformed from being based on the true fat layer to be based on sorting groups. This is done by finding the average price per kg for each of the five products for the carcasses in each sorting group. By so doing, it is indirectly assumed, that for the individual product a price can be obtained that reflects its true fat layer no matter from which sorting group it comes. See the chapter “Discussion” for further information regarding the importance of assumptions in prices.

4.2.2 Quality

As long as there is measuring noise it is unavoidable that some pigs are placed into wrong sorting groups and that products are being produced from pigs which were not intended for these products.

The number of pigs placed into sorting groups not living up to specifications for producing different products is estimated through simulation. The model keeps track of the number of items not living up to specifications for different optimized solutions. Estimation of the model parameters is commented on in detail in the section Estimation of Model Parameters.

4.2.3 Max sale

The maximum allowed sale is established for each product as a percentage of the total number of items (here approx. 61,000), which can be used to produce the products in question. As default the maximum sale for each product is set to 100% i.e. there are no restrictions in the sales volumes for any of the products.

4.3 Estimation of Model Parameters:

4.3.1 Estimation of the measured values of the fat layer

The estimation is based on registered slaughtering data for approx. 61,000 pigs. In order to work with different sizes of measuring accuracy, the registered quality values are considered as true values. The measured values of the quality have been estimated at
two different levels of measuring noise by simulating some measuring noise for each pig and adding it to the registered quality values.

The two different levels of measuring noise chosen are the current level of the standard error of prediction (SEP) and an improved level in which SEP has been halved.

4.3.2 *Estimation of prices*

The estimation of prices is based on net prices from Appendix 1, given for the true fat layer after reductions for additional costs if the raw materials are not considered most suitable for the specific products.

5 **Results**

The economic consequences of improved measurements are found by performing two optimizations: One with data from the improved accuracy and one with the current measuring accuracy. The improvement can be found as the difference in profits between these two optimizations. The computations in this paper are for illustrative purposes and as prices vary over time a price and cost study should be performed before the computations are used for actual decision support.

The following three different scenarios are investigated:

1. No constraints in volume and quality
2. Constraints in volume
3. Constraints in quality

5.1 **Results without constraints in volume and quality**

In the first scenario there are no constraints in sales volume nor in quality. The individual sorting groups will be used to produce the product for which the highest net price can be obtained, given the distribution of the true fat layer.

Improved measuring accuracy increases the profit by DKK 75,296 for the approximately 61,000 pigs, being part of the experiment, equalling an increase in profits
by 0.83% for the products. For Danish slaughterhouses, which produce approx. 25 million pigs annually, it would amount to DKK 31 million per year.

<table>
<thead>
<tr>
<th>Measuring accuracy</th>
<th>Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>SEP 20.00</td>
<td>9,181,962</td>
</tr>
<tr>
<td>SEP 41.28</td>
<td>9,106,666</td>
</tr>
<tr>
<td>Improved profit</td>
<td>75,296</td>
</tr>
</tbody>
</table>

Table 2. Improved profit due to improved measurements

### 5.2 Results with constraints in sales volume

To make the estimations more realistic constraints have been added regarding the maximum sales for different products. Three different alternatives for the maximum sales of different products are used:

<table>
<thead>
<tr>
<th>Product 1</th>
<th>Product 2</th>
<th>Product 3</th>
<th>Product 4</th>
<th>Product 5</th>
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</thead>
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<td>Scenario 2</td>
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<td>Scenario 3</td>
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<td>25%</td>
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</table>

In all three alternatives it is assumed that there are no restrictions regarding the sale of Product 5.

<table>
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<tr>
<th>Measuring accuracy</th>
<th>Profit - sub scenario 1</th>
<th>Profit - sub scenario 2</th>
<th>Profit - sub scenario 3</th>
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<td>9,055,337</td>
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<td>SEP 41.28</td>
<td>9,019,039</td>
<td>8,962,523</td>
<td>8,990,778</td>
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<td>Improved profit</td>
<td>82,308</td>
<td>83,990</td>
<td>64,559</td>
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</table>

Table 3. Improved profit due to improved measurements with constraints in sales

Introducing constraints in the sales volumes decreases the level of profit, but improved measurements still results in an increased profit of between DKK 64,559 and DKK 83,990 for the three scenarios for the approx. 61,000 pigs. This is an increase in profits between 0.72% and 0.94%, and for the Danish slaughterhouses it amounts to between DKK 26 and DKK 31 million annually.

The share of products produced from raw materials not living up to specifications decreases substantially with the improved measurements and is reduced by 33%, 25% and 57% respectively for sub scenario 1, 2 and 3. The two leanest products (product 1 and 2) are, however, still at a relatively high level. For the three sub scenarios, 15% - 23% of the raw materials used to produce product 1 are not living up to specifications. For product 2, the same share of raw materials is between 7% - 22%.
5.3 Results with constraints in quality

In the third scenario, constraints regarding the percentage of items not living up to specifications are introduced. Three different levels of the percentage allowed not being within specifications, namely 15%, 10% and 5%, are investigated. It is noticed, that product 5 can be produced of all qualities, but the net prices obtainable are heavily influenced by the fat layer.

<table>
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<th>Profit - max 15% not within spec.</th>
<th>Profit - max 10% not within spec.</th>
<th>Profit - max 5% not within spec.</th>
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</thead>
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<td>Improved profit</td>
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<td>78,670</td>
<td>86,106</td>
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</tbody>
</table>

Table 4. Improved profit due to improved measurements with constraints re. quality

There will be no cost of introducing a constraint that allows a maximum of 15% of a product to be produced of raw materials not within specifications, as an optimal production even without limitations would only have a share of maximum 14% for one of the products. Improved measurements would result in an increase in profits by DKK 75,296.

Strengthening the constraints decreases the level of profit, but improved measurements still results in an increased profit of DKK 78,296 and DKK 86,106 for constraints allowing a maximum of 10% and 5% respectively to be outside specifications. This equals an improvement of 0.82% - 0.94% for the three levels investigated.

For the Danish slaughterhouses improved measurements would result in an increase in the profits by DKK 31-35 millions annually for the back products under the different quality scenarios.

The GAMS code used for the modelling can be seen in Appendix 2. The model has 731,938 constraints and 595 variables and was solved to optimality in less than 0.2 seconds, which can be considered very acceptable.

6 Discussion

The assumption chosen regarding pricing and cost is of key importance for obtaining a true and fair view of the size of the profit and thereby to calculate the economic consequences of improved measurements. By using the net prices as in this paper, it is
indirectly assumed that for the individual raw materials a net price, which reflect its true fat layer, can be obtained.

This assumption correctly takes into account that a product produced of raw materials outside specifications may have extra costs for trimming and waste. For products made of raw materials, which are better (leaner) than required for the given product the assumption is less than perfect. The customers would hardly pay a premium for receiving products, which are better than the ones they had actually ordered.

As correct net prices are so essential for obtaining reliable results, the prices can advantageously be split into the price itself and three different cost contributions:

Cost contribution 1 (cost of using raw material of too good a quality):

![Figure 3. Cost of using raw material of too good a quality](image)

Figure 3 above shows the cost of delivering products produced of a better quality than required. This is a marginal cost consideration, where profit lost by not receiving the highest price for raw materials is considered a cost.

Cost contribution 2 (additional cost for extra trimming and waste due to raw materials not living up to specifications):
Figure 4. Additional cost for extra trimming and waste due to raw materials not living up to specifications.

Figure 4 shows the additional costs for extra trimming and waste (lower yield) as a consequence of raw materials being outside specifications. The contribution from figure 4 can be split into two separate contributions: 1) additional cost for extra trimming and 2) additional costs for waste. For many products the first contribution is almost the same no matter if the products are trimmed by 2 or 8 mm.

Cost/profit contribution 3 (Cost/profit for delivering products produced of raw materials not living up to specifications):

Figure 5. Cost/profit for delivering products produced of raw materials not living up to specifications.

Figure 5 shows the immediate profit of delivering products produced of raw materials being outside specifications. This, however, is only true as long as it is at an acceptable level and as long as the difference between actual quality and specifications is not too
big. If the slaughterhouses could improve the measuring accuracy and deliver more homogenous products, the slaughterhouses might be able to obtain higher prices. This contribution is very hard to estimate, but it is believed to be considerably larger than the profit contribution from Figure 5. Central persons within the industry are convinced that the value is of a very considerable size, may be even as big as the computed profit of improved measurements. A study must be made tracing added value in the complete process from slaughterhouse to end user. Case studies should be performed, involving marketing and sales personnel as well as product and process specialists from Danish slaughterhouses and the Danish Meat Association. Scenario analysis can be used to analyse how the value added may be split between the slaughterhouses and their customers. Such a study involves both the slaughterhouses and its customers, and further planning and research should be made before initiating the study.

The three cost/profit contributions, i.e.: 1) Cost of using raw material of too good a quality; 2) Additional cost for extra trimming and waste due to raw materials not living up to specifications; 3) Cost/profit for delivering products produced of raw materials not living up to specifications (see Figure 3, 4 and 5) should be estimated separately for each product. A study can be performed at the slaughterhouses establishing how different products are distributed on different qualities (quality groups) before delivery. CT scanners, manual quality control and statistical testing can be used to establish the quality. The additional costs for extra trimming and cutting can be estimated by completing time and yield studies at the cutting departments. The cost of delivering a better quality than required and the immediate profit of delivering a quality not living up to specifications can be estimated when the distribution on different qualities are known and a price study has been performed, involving marketing and sales personnel.

7 Conclusion

The main conclusion is that even relatively simple optimization models can advantageously be used to improve the basis of the slaughterhouses for making decisions regarding improved measurements. The value of improved measurements has been estimated using Mixed Integer Programming under three different scenarios where different limitations in volume and quality were introduced. The consequences of improved measurements on quality and profit were investigated.

Improved measurements make it possible for the slaughterhouses to produce more lean products without receiving complaints from customers. Furthermore, the percentage is
reduced of products produced of a lower quality than desired. The improved sorting in the form of more correct placements into sorting groups results in a substantial improvement in profit. The same effect can be seen when constraints regarding volume and quality are introduced.

The assumptions chosen regarding pricing and costs are very important for obtaining a true and fair view of the profit. In order to make the prices reflect the actual cost even more realistically, the costs can advantageously be split into the following four different contributions:

- “Cost” of delivering a better quality than required. It is a marginal cost consideration, where “lost profit” by not obtaining the best price possible is considered a cost.

- Contribution from delivering products from raw materials outside specifications.

- Waste in connection with extra trimming as a consequence of raw materials being outside specifications. The waste both relates to the actual waste and the cut-off meat, which is sold at lower prices than the main product.

- Cost of extra cutting etc. as a consequence of raw materials being outside specifications. For many products, it is not necessary to take this contribution into account, as cutting costs will often be the same, no matter if the product is trimmed by 2 or 8 mm.

The different contributions should be estimated separately for each product.

In reality, the economic consequences of improved measurements would be considerably higher than calculated in this paper as the contribution from delivering a better quality than required is not taken into account, and the contribution from delivering a quality which is outside specifications is only partly considered. In order to take those effects into consideration new cost analysis should be performed at the Danish slaughterhouses estimating the above mentioned four cost contributions for each product.

Furthermore, improved measurement would provide a considerable strategic value as the slaughterhouses would be able exactly to (or at least closer to) deliver the quality, which the customers require. It is very difficult to quantify this strategic value, but the slaughterhouses are of the opinion that it is of a considerable size and maybe even more important than the calculations of the non strategic values.
Bibliography


Fertin, C. (1992). Validation of long term planning model for optimization of the raw material use at pig slaughterhouses (in Danish). The Royal Veterinary and Agricultural University, Copenhagen, Denmark.


Rasmussen, S. (1992), The use of a multi-period LP-model as the core of a Decision Support System for a hog slaughterhouse. The Royal Veterinary and Agricultural University, Copenhagen, Denmark.


### Appendix 1 – price per kg depending on quality (based on fat layer in mm)

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<th>fat layer (mm)</th>
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### Appendix 2 – GAMS code

* CanneryTransport.gms

* CanneryTransport.gms

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option iterlim=999999999;  // avoid limit on iterations
option reslim=300;          // timelimit for solver in sec.
option optcr=0.0;           // gap tolerance
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option limcol=100;          // limit number of columns in .lst file

---------------------------------------------------------

SETS
  i     kval2 værdi       / k30, k40, k50, k60, k70, k80, k90,
       k100, k110, k120, k130, k140, k150, k160, k170, k180, k190,
       k200, k210, k220, k230, k240, k250, k260, k270, k280, k290,
       k300, k310, k320, k330, k340, k350, k360, k370, k380, k390,
       k400, k410, k420, k430, k440, k450, k460, k470, k480, k490,
       k500, k510, k520, k530, k540, k550, k560, k570, k580, k590,
       k610, k630 /
  j     produkter        / Product1, Product2, Product3, Product4, Product5 /

PARAMETER
  Fordeling(i) Number of carcasses available in sorting group i /
    k30       2
    k40       11
    k50       23
    k60       87
    k70      135
    k80      274
    k90      486
    k100     687
    k110     995
    k120     1286
    k130     1681
    k140     2113
    k150     2319
    k160     2592
    k170     2845
    k180     3114
    k190     3294
    k200     3392
    k210     3410
    k220     3394
    k230     3293
    k240     3188
    k250     2946
    k260     2673
    k270     2455
    k280     2166
    k290     1962
    k300     1736
    k310     1447
    k320     1209
    k330     1052
    k340      857
    k350      745
    k360      580
    k370      480
    k380      379
Afsaet_max(j)  Max sale of product j (in percentage)
/  Product1  100
  Product2  100
  Product3  100
  Product4  100
  Product5  100  / ;

Table Kvalitet(i,j) Number of items in sorting group i not living up to specification when producing product j

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Variables
x(i,j) number of pigs from sorting group i used for product j
z total profit
kval_ringe_sum(j)
prod_sum(j) ;

Binary Variables
y(i,j) 1 if product j is produced from sorting group i, else 0 ;

Positive Variable x ;

Equations
profit objective function
distribution_con constraint re. number of pigs available in sorting group
sale_con constraint re. max sale
y1_con constraint re. production
y2_con constraint re. production
production_sum sum of production
quality_con constraint re. quality ;

profit .. z =e= sum((i,j), Pris(i,j) * x(i,j)*11);
distribution_con(i) .. sum(j, x(i,j)) =l= Fordeling(i);
sale_con(j) .. sum(i, x(i,j)/60993*100) =l= Afsaet_max(j);
y1_con(i,j) .. y(i,j) =l= x(i,j);
y2_con(i,j) .. y(i,j)*65000 =g= x(i,j);
production_sum(j) .. prod_sum(j) =e= sum(i, x(i,j)) ;
quality_con(j) .. sum(i, Kvalitet(i,j)*y(i,j)) =l= 0.15*sum(i, x(i,j)) ;

Model Slagteri /all/ ;
Solve Slagteri using mip maximizing z ;
Paper B
The pig industry is an important part of Danish economy with an export value in 2006 of more than DKK 28 billions [Danish Meat Association (2007)]. The competition is hard and increased feeding costs are affecting the farmer’s profitability. Therefore it is important to optimize all aspects of Danish pig production, slaughtering processes and delivery.

There have been varying opinions within the industry concerning an improved profit for the slaughterhouses and the farmers when increasing the slaughter weight. The argument for an increased slaughter weight is based on the fact that some of the costs at the slaughterhouses and at the farmers are unit costs variable to the number of pigs produced and for that reason it would be interesting if the slaughter weight was increased. If the slaughter weight was increased and the number of items produced almost unchanged, the costs per kg produced meat would decrease. The savings in costs should be compared to the possible decrease in the average sales price. These considerations are continuously taking place in the industry, but some more accurate tools to find the economic consequences are desired.

This paper concerns the aspects of optimization at the slaughterhouses and farmers, especially regarding estimation of the economic consequences of an increased slaughter weight.
Operations Research methods are used to solve this important practical problem for the industry.

The model is a Mixed Integer Programming (MIP) model applied on four different weight scenarios, namely the current slaughter weight as well as increases in the slaughter weight of 5, 10 and 15 kg respectively. The model set up in this paper consists of 17 different products and four alternative uses of each pig, but the model can easily be changed to include more products and alternative uses.

The model is illustrated in this paper using test data consisting of slaughtering data for 43,949 pigs slaughtered at one of the Danish slaughterhouses. Increased slaughter weight results in an increased turnover, but also in a decrease in the average price per kg. For a weight increase of 5 kg the average price per kg decreases by DKK 0.241. The similar decrease in average prices for weight increases of 10 and 15 kg is DKK 0.492 and DKK 0.706 respectively. This should be compared to savings at the slaughterhouses as well as at the farmers, which previously have been estimated to approximately DKK 0.25 for each increase in slaughter weight of 5 kg.

The main conclusion is that even relatively simple optimization models can be used to improve the basis of the slaughterhouses considerably for making decisions regarding the value of increased slaughter weight. Prices may vary over time as the market situation changes continuously. In order to make the results trustworthy and reliable for decision making, it is essential that prices, costs and product yields for different products are estimated carefully. Even though the computations are made for illustrative purposes, the figures indicate that other options than increased slaughter weight may be more profitable to pursue.

1 Background

The pig industry is important for Danish economy, with a production in 2006 of more than 25 million pigs in Denmark. Approximately 90% of the meat was exported and had an export value of DKK 28.8 billion [Danish Meat Association (2007)].

Competition is hard, and increased feeding costs have substantial impact on the farmers’ profitability. There is a substantial pressure on the slaughterhouses to provide increased payment to the farmers.

Therefore it is becoming more and more important that Danish farmers and slaughterhouses continue to optimize their production and slaughtering processes. This paper concerns the
aspects of the optimization at the slaughterhouses, especially the economic consequences of increased slaughter weight.

Danish slaughterhouses can quickly change important characteristics such as meat content and the weight of the pigs being slaughtered by changing the payments to the farmers.

There have been varying opinions within the industry concerning an improved profit for the slaughterhouses and the farmers when increasing the slaughter weight. The argument for an increased slaughter weight is based on the fact that some of the costs at the slaughterhouses are unit costs variable to the number of pigs slaughtered and for that reason it would be interesting if the slaughter weight was increased. If the slaughter weight was increased and the number of items produced almost unchanged, the costs per kg produced meat would decrease. The savings in costs should be compared to the possible decrease in the average sales price. These considerations are continuously taking place in the industry, but some more accurate tools to find the economic consequences are desired. This paper concerns the use of Operations Research to solve this practical problem of major importance for the industry.

2 Literature survey

The literature regarding optimized raw material use at the slaughterhouses has been addressed in the paper “The Value of Improved Measurements in a Pig Slaughterhouse” [Kjersgaard, N. (2008a)] but is repeated here for convenience.

The amount of literature addressing improved or optimized raw material use in the food industry is substantial. However, the main part of the contributions is related to different aspects regarding either optimization of meat quality or different production processes. Examples of this are optimization of the industrial thermal sterilization of canned foods [Garcia, M. et. al. (2006)] and pigs stunning optimization [Dupuis, P. et. al. (2004)]. These types of optimizations are not relevant for this project as they are either based on statistical analysis without optimization of a mathematical model or the mathematical models are very different from the models, which are used in this Ph.D. project regarding optimization of the raw material use at the slaughterhouses.

Within the pork industry relatively few contributions have been found regarding optimization based on operations research methods. In the paper “Location of slaughterhouses under economies of scale” [Broek et. al. (2006)] optimization is used to investigate the savings potential of reducing the number of slaughterhouses in Norway and investing in additional capacity in the remaining facilities in order to obtain economies of scale. Another facility
location problem is described in the paper “The impact of changes in livestock supply on the optimum number, size and location of slaughterhouses in East Macedonia” [Kamenidis, C. & Sorensen, V. (1978)]. In the paper ”Economic optimization of pork production – marketing chains. II. Modelling outcome” [Ouden et. al. (1996)] are using Dynamic Linear programming to evaluate the development of pork chain concepts that also takes animal welfare into consideration. Kure in his Ph.D. thesis “Marketing Management Support in Slaughter Pig Production” [Kure, H. (1997)] uses Dynamic Programming to solve parts of the “slaughter pig marketing management problem”, which regards how the farmers should select and market their pigs to the slaughterhouses.

The above mentioned four examples of optimization problems within the pork industry are all somewhat different from the problem of optimizing the raw material use at the slaughterhouses. More similar problems have been found in the following contributions:

In 1990-1992 a project regarding optimization of the raw material use at the slaughterhouses was performed as a cooperation between Danish Meat Research Institute and the Royal Veterinary and Agricultural University (now the Faculty of Life Sciences at University of Copenhagen). Several reports were made:

A Linear Programming (LP) model for production planning and control for the hog slaughterhouses was developed and reported in [Rasmussen, S. & Thomsen, M. (1991)] and [Rasmussen, S. (1992)]. The model is a 2-stage model. First stage concerns a planning horizon of 3 months and the second stage one weeks day to day planning. In [Fertin, C. (1992)] the long term planning model (stage 1) is validated.
In his Ph.D. thesis [Fertin (1995)] Fertin describes and further develops and validates the model.

There has been searched for literature in other food related industries, e.g. poultry and beef slaughterhouses and the fish industry, but no relevant literature has been found.

Other industries have similar problems as the slaughterhouses regarding its raw material use. An example is the refineries, but unlike the slaughterhouses the refineries have the option of blending different qualities in order to change the quality characteristics of the products. Another example is the lumber and wood industry. A few papers of the product mix problem within the wood industry have been identified. In the paper “An Optimization-Based Decision Support System for a Product Mix Problem” [Roy et. al. (1982)] an LP-model has been used to solve a plywood product mix problem for Ponderosa Industrial in Mexico.

Even though literature within food optimization is substantial, the main part of the contributions are related to optimization based on e.g. statistical analysis without optimization
of a mathematical model. Other models are very different from the models used in this Ph.D. project. Except for the contributions from the Royal Veterinary and Agricultural University and the Danish Meat Research Institute not much literature of relevance for the Ph.D. project has been identified.

3 The Model

The purpose of the model is to investigate the economic consequences of a general increase in slaughter weight. The model has been used on four different weight scenarios, namely for the current slaughter weight as well as for increases in the slaughter weight of 5, 10 and 15 kg respectively. Benefits are found by comparing the optimal solution for the current slaughter weight with the optimal solution at the increased levels.

3.1 Description of the Model

In the experiments we use the actual slaughtering data from 43,949 pigs slaughtered at one of the Danish slaughterhouses. For each pig the registered fat layer (in mm) and the actual slaughter weight are used. In the computations, the registered fat layer is considered the true value, and the effect of a general increase in slaughter weight is estimated for each of the four weight scenarios. In general there is coherence between the slaughter weight and the size of the fat layer. When the slaughter weight is increased by 1 kg, the fat layer is increased by approx. 0.16 mm. This coherence is found using regression analysis. See the chapter “Estimation of model parameters” for further information regarding this issue.

The model has its basis in different alternative uses of the pigs. Each alternative use consists of a “package” of products for the specific part (the fore end, the middle piece and the ham) and can be seen in Figure 1 below. In the experiments, the back and the ham have two alternative uses each and the fore end has one. In total there are four different alternative uses of each pig and 17 different main products. The weight of each product is estimated for each pig at each of the four weight scenarios used in the computations.
Figure 1. Alternative uses of the pigs.

Some raw materials (pigs) are better suited for some products than others. This is taken into consideration when increasing or decreasing the price for some products depending on the level of the fat layer. In the model, this is done by splitting the pricing in two different contributions:

1. A fixed price per kg for the given product

   and

2. A price coefficient, which stipulates how much the price will decrease if the fat layer increases by 1 mm.
For two of the products there are a few special conditions which have to apply for the raw materials to be used for these products:

- The ham product P_Ham can only be produced if the fat layer does not exceed 14 mm
- The breast product P_Breast2 can only be produced if it does not exceed a weight of 4 kg.

The products P_CutOff (1, 2, 3, 5) consist of meat cut-off in connection with production of the main products and products P_Sundry (1-5) consist of fat, bones, rind etc.

The price for each product is calculated at different weight scenarios based on its estimated weight and fat layer. The best alternative use of each pig is found by optimization, and the total obtainable price for the 43,949 pigs are summed. The same computations are obtained at each of the different weight scenarios. The value of a general increase in slaughter weight is then found as the difference between the profit for the optimal solutions after and before the weight increase.

3.2 Mathematical formulation of the Model

We have a set of carcasses \( \mathcal{I} = \{1, \ldots, I\} \). Each carcass can be used to produce a set of different products \( \mathcal{J} = \{1, \ldots, J\} \). Finally we have a set of different weight scenarios \( \mathcal{K} = \{1, \ldots, K\} \). The decision variable \( x_{i,j,k} \) is a binary variable with the value 1 if pig \( i \) is used to produce product \( j \) in weight scenario \( k \) and 0 otherwise. The problem is to find the optimal utilization of each carcass at each weight scenario and the total profit for the optimal solution for each weight scenario:

The objective function:

1) \[ \text{Maximize } Z = \sum_{i,j,k} \left( \text{Price}_{j} + \text{PriceCoeff}_{j} \cdot \text{FatLayer}_{i,k} \right) \cdot \text{ProdWeight}_{i,j,k} \cdot x_{i,j,k} \]
Subject to:

2) \[ x_{i,j,P\_Breast2,k} \cdot \text{ProdWeight}_{P\_Breast2,k} \leq 4 \]
3) \[ x_{i,j,P\_Ham,k} \cdot \text{FatLayer}_{P\_Ham,k} \leq 14 \]
4) \[ x_{i,j,P\_Schoulder,k} = x_{i,j,P\_Neck,k} \quad \forall i \in \mathcal{I}, k \in \mathcal{K} \]
5) \[ x_{i,j,P\_Neck,k} = x_{i,j,P\_CutOff_{1,k}} \quad \forall i \in \mathcal{I}, k \in \mathcal{K} \]
6) \[ x_{i,j,P\_CutOff_{1,k}} = x_{i,j,P\_Sundry_{1,k}} \quad \forall i \in \mathcal{I}, k \in \mathcal{K} \]
7) \[ x_{i,j,P\_Backs\ (with\ bones),k} = x_{i,j,P\_Breast1,k} \quad \forall i \in \mathcal{I}, k \in \mathcal{K} \]
8) \[ x_{i,j,P\_Breast1,k} = x_{i,j,P\_CutOff_{2,k}} \quad \forall i \in \mathcal{I}, k \in \mathcal{K} \]
9) \[ x_{i,j,P\_CutOff_{2,k}} = x_{i,j,P\_Sundry_{2,k}} \quad \forall i \in \mathcal{I}, k \in \mathcal{K} \]
10) \[ x_{i,j,P\_Backs\ (boneless),k} = x_{i,j,P\_Breast2,k} \quad \forall i \in \mathcal{I}, k \in \mathcal{K} \]
11) \[ x_{i,j,P\_Breast2,k} = x_{i,j,P\_CutOff_{3,k}} \quad \forall i \in \mathcal{I}, k \in \mathcal{K} \]
12) \[ x_{i,j,P\_CutOff_{3,k}} = x_{i,j,P\_Sundry_{3,k}} \quad \forall i \in \mathcal{I}, k \in \mathcal{K} \]
13) \[ x_{i,j,P\_Ham,k} = x_{i,j,P\_Sundry_{4,k}} \quad \forall i \in \mathcal{I}, k \in \mathcal{K} \]
14) \[ x_{i,j,P\_Ham\ (boneless),k} = x_{i,j,P\_CutOff_{5,k}} \quad \forall i \in \mathcal{I}, k \in \mathcal{K} \]
15) \[ x_{i,j,P\_CutOff_{5,k}} = x_{i,j,P\_Sundry_{5,k}} \quad \forall i \in \mathcal{I}, \ldots, k \in \mathcal{K} \]
16) \[ x_{i,j,P\_Backs\ (with\ bones),k} + x_{i,j,P\_Backs\ (boneless),k} = 1 \quad \forall i \in \mathcal{I}, k \in \mathcal{K} \]
17) \[ x_{i,j,P\_Ham\ (boneless),k} = 1 \quad \forall i \in \mathcal{I}, k \in \mathcal{K} \]
18) \[ x_{i,j,k} = \begin{cases} 1 & \text{if production} \\ 0 & \text{otherwise} \end{cases} \quad \forall i \in \mathcal{I}, j \in \mathcal{J}, k \in \mathcal{K} \]

Indices:
i: pig i (pig no. 1 to 43,949)
j: product j (17 different products, see figure 1)
k: weight increase (4 scenarios with current weight and weight increases of 5, 10 and 15 kg)

Decision variables:
x_{i,j,k}: decision variable with value 1 if product j is produced of pig i at weight scenario k, otherwise 0.

Parameters:
PigWeight (i): slaughter weight of pig i
ProdWeight (i, j): weight of product j produced from pig i
FatLayer (i): layer of fat (in mm) for pig i
Price(j): price per kg (in DKK) for product j
PriceCoeff(j): price coefficient for product j (price increase in DKK when layer decreases by 1 mm)

The objective function (1) is in fact a “dummy” function, which optimizes the sum of the optimal solutions for all four different weight scenarios. By finding the optimal solution for the dummy function, the optimal solution for each of the four weight scenarios are found at the same time, as there is no interconnection between the four scenarios.

Constraints no. 2 and 3 control that the product P_Breast2 only can be produced if the product does not exceed a weight of 4 kg and product P_Ham only if the fat layer does not exceed 14 mm.

The slaughterhouses have some constraints regarding the products produced. When the slaughterhouse decides to use a carcass to produce one product other specific products often have to be produced as well. Constraints 4-15 ensures that when an alternative use of the pig is chosen, a whole package of products is chosen. For instance constraint 4 controls that if P_Schoulder is produced then P_Neck is produced as well. If this is the case, constraints 5 and 6 control that also P_CutOff1 and P_Sundry1 are produced as well.

Constraints 16 and 17 ensure, that the back and the ham only can be used for one alternative at a time and equation 18 that \( x_{i,j,k} \) is a binary variable keeping track on whether there is production or not.

3.3 Estimation of model parameters

In the following, estimation of the different model parameters used as input to the model will be described briefly.

3.3.1 Fat layer coefficient

The fat layer coefficient stating how much the fat layer is increased when the slaughter weight is increased by 1 kg has been estimated to 0.1648 and has been found through linear regression analysis. See the chapter “Sensitivity analysis” for further information regarding the coefficients effect on the results.
3.3.2 Fat layer

The fat layer is estimated based on the actually registered fat layer for each pig with the addition of a contribution as a consequence of the increased weight for weight scenario k.

\[
\text{Fat layer (in mm)}_{i,k} = \text{registered fat layer}_{i,k=0} + 0.1648 \times \text{weight increase}_k
\]

By using the actually registered fat layer as a basis, the fat layer realistically reflects the measuring noise.

3.3.3 Slaughter weight

The actually registered slaughter weight of each pig is used and for the different weight scenarios, 5, 10 and 15 kg respectively have been added.

3.3.4 Weight of products

The weight of the 17 products is estimated at different weight scenarios based on the estimated fat layer and the slaughter weight.

The coefficients used are found by linear regression analysis with data from a previous project (project “Europe Pig” [Danish Meat Research Institute (1996)]), where the connection between increased slaughter weight and product yields was found.

4 Results

The economic consequences of a general increase in slaughter weight of pigs produced in Denmark are found by performing optimizations at different weight scenarios. The consequences are presented as the difference between the profit from different weight scenarios and the profit from the current level of slaughter weight.

The following four weight scenarios have been investigated:

- current level of slaughter weight
• general increase in slaughter weight by 5 kg
• general increase in slaughter weight by 10 kg
• general increase in slaughter weight by 15 kg

For each weight scenario the optimal solution is found and the profit hereof computed. The profit for each weight scenario and the effect of increased slaughter weight can be seen in Table 1 below:

<table>
<thead>
<tr>
<th>Current weight</th>
<th>Plus 5 kg</th>
<th>Plus 10 kg</th>
<th>Plus 15 kg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profit in DKK</td>
<td>38,243,300</td>
<td>39,678,630</td>
<td>40,967,470</td>
</tr>
<tr>
<td>Profit improvement</td>
<td>0.135</td>
<td>0.269</td>
<td>0.395</td>
</tr>
<tr>
<td>Production in kg</td>
<td>3,576,982</td>
<td>3,796,727</td>
<td>4,016,472</td>
</tr>
<tr>
<td>Average price per kg</td>
<td>10.692</td>
<td>10.451</td>
<td>10.200</td>
</tr>
<tr>
<td>Change in average net price</td>
<td>-0.241</td>
<td>-0.492</td>
<td>-0.706</td>
</tr>
</tbody>
</table>

Table 1. Profit and change in average net price at the different weight scenarios.

There is a natural variation in prices from one week to another. As the market situation changes continuously it has not always been possible to gain access to the latest prices, but information of typical prevailing prices was received from the slaughterhouses. Before using the computations as actual decision support a price and cost survey should be performed.

It can be seen that the profit increases by DKK 1,435,329 if the slaughter weight is increased by 5 kg. However, the production volume increases by 219,745 kg, so the average profit per kg decreases by DKK 0.241. For the scenarios with weight increases of 10 and 15 kg the average prices obtained per kg decreases by DKK 0.492 and DKK 0.706 respectively.

Some of the costs at the slaughterhouses as well as at the farmers are, however, either fixed or variable according to the number of pigs produced instead of to the production in terms of weight. Therefore, the decrease in average prices should be compared to the cost savings in the slaughterhouses and at the farmers. These savings have previously been estimated to approximately DKK 0.15 [Mønsted, K. (2006)] and DKK 0.10 per kg respectively for each increase in slaughter weight by 5 kg.

Based on the test data, the model shows that the industry could gain some, however small, net cost savings by increasing the slaughter weight. If the number of pigs produced is the same as today, it might to some extent require additional investments in warehouse facilities at the slaughterhouses as well as capacity increases at the farmers. Another problem is whether the customers will accept the heavier products or not. All in all, the figures indicate that other options than increased slaughter weight may be more profitable to pursue.
The model has 2,812,736 constraints and 3,515,920 variables and was solved to optimality in 545 seconds in total for all four weight scenarios, which is considered a satisfactory time. See chapter 4.2 [Kjærsgaard, N. (2008e)] in the thesis for more information regarding solution time.

5 Sensitivity Analysis

As mentioned previously, the fat layer will generally increase when the slaughter weight increases. In connection with the computations in this paper it has been assumed that the fat level in general will increase by 0.1648 mm when the slaughter weight increases by 1 kg. In Table 2 below the sensitivity of the change in average price can be seen regarding changes in this coefficient.

<table>
<thead>
<tr>
<th>Changes in fat layer coefficient</th>
<th>Plus 5 kg</th>
<th>Plus 10 kg</th>
<th>Plus 15 kg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base (0.1648)</td>
<td>- 0.241</td>
<td>- 0.492</td>
<td>- 0.706</td>
</tr>
<tr>
<td>Plus 0.01 (0.1748)</td>
<td>- 0.249</td>
<td>- 0.507</td>
<td>- 0.727</td>
</tr>
<tr>
<td>Minus 0.01 (0.1548)</td>
<td>- 0.232</td>
<td>- 0.476</td>
<td>- 0.686</td>
</tr>
<tr>
<td>Plus 0.05 (0.2148)</td>
<td>- 0.283</td>
<td>- 0.566</td>
<td>- 0.805</td>
</tr>
<tr>
<td>Minus 0.05 (0.1048)</td>
<td>- 0.197</td>
<td>- 0.413</td>
<td>- 0.602</td>
</tr>
</tbody>
</table>

Table 2. Changes in average net price depending on fat layer coefficient.

Table 2 above shows that a change in the coefficient by plus/minus 0.01 results in an additional change in the average net price of between DKK 0.008 – 0.009 per kg when the slaughter weight increases by 5 kg. For increases in the slaughter weight of 10 and 15 kg, the additional change in the average price is DKK 0.015 and DKK 0.020 per kg respectively. If the coefficient changes by plus/minus 0.05, the additional change in the average net price will instead be between DKK 0.024 – 0.027 for a weight increase of 5 kg, and DKK 0.040 – 0.044 and DKK 0.047 – 0.054 respectively for weight increases of 10 and 15 kg.

For larger changes in the fat layer coefficient a considerable effect in the changes of the average net price can be seen. It should, however, be noticed that a change in the fat layer coefficient by plus/minus 0.05 is considered quite extreme, and it should be possible to estimate it much more accurately. Further studies should be made to examine if there are even better ways than linear regression analysis to describe the connection between slaughter weight and the fat layer.

It is very important that correct net prices are used and that they truly reflect the prices and costs of different products depending on the quality of the raw materials. There are
considerable uncertainties regarding the PriceCoeff_j, which is the coefficient describing how much the price in DKK per kg changes when the fat layer of the pig increases by 1 mm. Table 3 below describes the changes in the average net price as a consequence of increased slaughter weight when different price coefficients are applied.

<table>
<thead>
<tr>
<th>Changes in price coefficient</th>
<th>Plus 5 kg</th>
<th>Plus 10 kg</th>
<th>Plus 15 kg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base - current price coefficients</td>
<td>- 0.241</td>
<td>- 0.492</td>
<td>- 0.706</td>
</tr>
<tr>
<td>Price coefficients halved</td>
<td>- 0.205</td>
<td>- 0.422</td>
<td>- 0.604</td>
</tr>
<tr>
<td>Price coefficients incr. 50%</td>
<td>- 0.277</td>
<td>- 0.562</td>
<td>- 0.810</td>
</tr>
<tr>
<td>Price coefficients incr. 100%</td>
<td>- 0.313</td>
<td>- 0.633</td>
<td>- 0.914</td>
</tr>
</tbody>
</table>

Table 3. Changes in average net price due to increased slaughter weight at different levels of price coefficients.

Table 3 shows that the changes in the average net prices are significant and might even influence on whether or not it will be profitable for the industry to make a general increase of the slaughter weight. It is therefore important that the net prices, including the price coefficients, represent a fair view of the actual costs and prices which the slaughterhouses can obtain for its products.

6 Discussion

The purpose of the current paper has been to illustrate how Operations Research can be applied to improve estimations of the economic consequences of a general increase in slaughter weight. Prices vary from one week to another and are here only used for illustration of the model. In order to use the model for actual decision making the prices and cost as well as product yields and how all these factors are influenced by changes in the slaughter weight should be further studied in cooperation with Danish slaughterhouses.

The model can be improved by:

- Using more products and product alternatives. The model should ideally use all important products, and all important product alternatives should be determined. The product alternatives can be determined for each part individually (front end, middle piece and ham) requiring fewer product alternatives. For the relatively few products where parting in three influences possible products, the possible product alternatives should be determined separately as product yields may differ.
• Estimation of product yields should be updated. The weight of each product should be estimated as well as how much the weight changes depending on the measured fat layer and slaughter weight. In these cases where the product yields are influenced by which product alternatives it is part of, separate products should be established.

• Different measures of the fat layer should be used depending on which part (i.e. fore end, middle piece or ham) of the pig is used for the specific product. The model uses only one general measure of the fat layer for the pig, but individual measures for different parts can be used instead.

• Prices and cost, i.e. net prices per kg as well as changes in the net prices according to increases/decreases in the fat layer should be updated for each product.

• Introducing constraints regarding minimum and maximum sale volumes of different products would make the model even more realistic.

Especially regarding net prices and product yields it is very important that the data input describes the actual value of different products as well as possible. For that reason further investigations should be made into price and yield functions to clarify whether linear functions are the best illustrations or better alternatives could be found.

When estimating the slaughter weight after a general weight increase it was assumed that the slaughter weight of each pig is increased by the same amount (Figure 2). The increase has been 5, 10 and 15 kg for the different weight scenarios. In real life this may not exactly be the case as the distribution to some degree may be extended, meaning that the heavy pigs in average increases with more than 5 kg and the smaller pigs with less than 5 kg. See figure 2 and 3 below for an illustration of this.

![Figure 2. Distribution of weight](image1)

![Figure 3. Distribution of weight](image2)
Figure 2 and 3 are for illustrative purposes only. The red curve represents the current distribution of pigs on slaughter weight and the blue curve the future state with increased slaughter weight. In real life the distribution is not normal, but skewed a bit to the left. Figure 2 describes the distribution of pigs when the slaughter weights of all pigs are increased by the same amount as assumed in the computations in this paper. Figure 3 describes how the distribution may look if the slaughter weight of the heavier pigs increases with more than 5 kg and with less for the smaller pigs. However, it is likely that the slaughterhouses will have a slimmer contribution, which can be obtained by changes in the payment structure to the farmers. This is the reason for using the assumption that all pigs will be increased with the same amount in the computations.

7 Conclusion

Determining the optimal slaughter weight for pigs for slaughter is one of the important decisions for Danish slaughterhouses and the farmers, and it is important that the decision is made on as solid a basis as possible. Therefore, development of tools to improve the accuracy of such calculations is considered an important area. The purpose of this paper is to provide a first step to such an improvement by establishing a model using only few products and product alternatives.

The model has four alternative uses for each pig and 17 different products in total, but the model can easily be changed to include more products and alternative uses. The computations in this paper are for illustrative purposes only, and before making decisions based on the model it should include all important products and product alternatives. Furthermore, it is essential that prices, costs and product yields for the different products are estimated carefully. It should be investigated whether or not there are even better alternatives to describe the price and yields functions than linear functions.

The model can be made even more realistic by introducing constraints for the minimum and maximum sale of different products.

The main conclusion is that operations research methods and even relatively simple models can be used to improve the slaughterhouses basis for decision making regarding increased slaughter weight. Prices may vary over time, and before using the model for actual decision support more products and product alternatives should be included and a price and cost study should be obtained.
An increased slaughter weight results in an increased turnover, but as the volume sold increases even more, the average price per kg falls. The average price decreases with DKK 0.241 for a weight increase of 5 kg and DKK 0.492 and DKK 0.706 for weight increases of 10 and 15 kg respectively. However, some of the costs at the slaughterhouses and at the farmers are fixed or variable according to the number of pigs being produced and not to the weight of the pigs. An increased slaughter weight results in savings at the slaughterhouses as well as at the farmers and has previously been estimated to approx. DKK 0.15 and DKK 0.10 respectively, totalling approx. DKK 0.25 for each increase in the slaughter weight of 5 kg. In these circumstances, potential savings are almost of the same size as the decrease in average prices, and the profits are consequently not improved significantly.

The sensitivity analysis shows that large changes in the fat layer coefficient have a considerable effect on the changes in the average net price and can easily change whether a decision is profitable or not. This is especially the case for decisions close to break-even. Further studies should be made to analyse how well linear functions describe the connection between slaughter weight and the fat layer.

These computations are based on the assumption that the number of pigs being slaughtered is not affected by the increased slaughter weight. To some extent, however, it might require additional investments at the slaughterhouses and at the farmers to handle larger pigs. Another problem is whether the customers will accept the heavier products or not. All in all the figures computed indicate that even though the prices received from the slaughterhouses are for illustrative purposes other options than increased slaughter weight may be more profitable to pursue.
Bibliography


Fertin, C. (1992). Validation of long term planning model for optimization of the raw material use at pig slaughterhouses (in Danish). The Royal Veterinary and Agricultural University, Copenhagen, Denmark.


Appendix 1 – GAMS code

* CanneryTransport.gms
*
* CanneryTransport.gms
*
$eolcom //
option iterlim=999999999;     // avoid limit on iterations
option reslim=300;            // timelimit for solver in sec.
option optcr=0.0;             // gap tolerance
option solprint=OFF;          // include solution print in .lst file
option limrow=100;            // limit number of rows in .lst file
option limcol=100;            // limit number of columns in .lst file
//--------------------------------------------------------------------

SETS   i   Pigs i     /p1*p43949/
        j   Products   /P_Schoulder, P_Neck, P_Backs (with bones), P_Breast1,
                   P_Backs (boneless), P_Breast2, P_Ham, P_Ham (boneless),
                   P_CutOff1, P_CutOff2, P_CutOff3, P_CutOff5, P_Sundry1,
                   P_Sundry2, P_Sundry3, P_Sundry4, P_Sundry5, P_Tenderloin,
                   P_head, H_8201 /
        s(j) Products sold    / P_Schoulder, P_Neck, P_Backs (with bones),
                                 P_Breast1, P_Backs (boneless), P_Breast2, P_Ham,
                                 P_Ham (boneless), P_CutOff1, P_CutOff2,
                                 P_CutOff3, P_CutOff5, P_Sundry1, P_Sundry2,
                                 P_Sundry3, P_Sundry4, P_Sundry5, P_Tenderloin,
                                 P_Head /
        k   weight increase  /  w_0, w_5, w_10, w_15 /;

PARAMETER
    Price(j) Price per kg for products j
    /
       P_Schoulder 12.00
       P_Neck 13.00
       P_Backs (with bones) 18.00
       P_Breast1 13.00
       P_Backs (boneless) 25.00
       P_Breast2 17.00
       P_Ham 15.00
       P_Ham (boneless) 18.00
       P_CutOff1 9.00
       P_CutOff2 9.00
       P_CutOff3 9.00
       P_CutOff5 9.00
       P_Sundry1 3.00
       P_Sundry2 3.00
       P_Sundry3 3.00
       P_Sundry4 3.00
       P_Sundry5 3.00
       P_Tenderloin 30.00
<table>
<thead>
<tr>
<th>Product</th>
<th>Price Coefficient (in DKK)</th>
<th>ProdWeightCon(j) Product weight constant for product j</th>
</tr>
</thead>
<tbody>
<tr>
<td>P_Head</td>
<td>3.00</td>
<td></td>
</tr>
<tr>
<td>H_8201</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>P_Schoulder</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>P_Neck</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>P_Backs (with bones)</td>
<td>-0.20</td>
<td></td>
</tr>
<tr>
<td>P_Breast1</td>
<td>-0.20</td>
<td></td>
</tr>
<tr>
<td>P_Breast2</td>
<td>-0.20</td>
<td></td>
</tr>
<tr>
<td>P_Ham</td>
<td>-0.20</td>
<td></td>
</tr>
<tr>
<td>P_Ham (boneless)</td>
<td>-0.20</td>
<td></td>
</tr>
<tr>
<td>P_CutOff1</td>
<td>-0.10</td>
<td></td>
</tr>
<tr>
<td>P_CutOff2</td>
<td>-0.10</td>
<td></td>
</tr>
<tr>
<td>P_CutOff3</td>
<td>-0.10</td>
<td></td>
</tr>
<tr>
<td>P_CutOff5</td>
<td>-0.10</td>
<td></td>
</tr>
<tr>
<td>P_Sundry1</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>P_Sundry2</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>P_Sundry3</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>P_Sundry4</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>P_Sundry5</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>P_Tenderloin</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>P_Head</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>H_8201</td>
<td>-1.58570</td>
<td></td>
</tr>
</tbody>
</table>

PriceCoeff(j) Price Coefficient (in DKK) for product j for an increase of layer of fat (in mm)

ProdWeightCon(j) Product weight constant for product j
ProdWeightFat(j) Product weight fat dependent coefficient for product j

/  P_Schoulder -0.06938
P_Neck -0.04096
P_Backs (with bones) -0.01662
P_Breast1 0.04284
P_Breast (boneless) -0.08124
P_Breast2 0.04284
P_Ham -0.10204
P_Ham (boneless) -0.19054
P_CutOff1 -0.00596
P_CutOff2 -0.00596
P_CutOff3 -0.00596
P_CutOff5 -0.00596
P_Sundry1 0.07922
P_Sundry2 0.11178
P_Sundry3 0.00000
P_Sundry4 0.00000
P_Sundry5 0.00000
P_Tenderloin 0.00000
P_Head 0.00000
H_8201 -0.10160
/

ProdWeightWeight(j) Product weight slaughtering weight dependent coefficient for product j

/  P_Schoulder 0.10726
P_Neck 0.07282
P_Backs (with bones) 0.01354
P_Breast1 0.06002
P_Breast (boneless) 0.08666
P_Breast2 0.06002
P_Ham 0.27632
P_Ham (boneless) 0.22874
P_CutOff1 0.00834
P_CutOff2 0.00834
P_CutOff3 0.00834
P_CutOff5 0.00834
P_Sundry1 0.13368
P_Sundry2 0.24410
P_Sundry3 0.00000
P_Sundry4 0.00000
P_Sundry5 0.00000
P_Tenderloin 0.00000
P_Head 0.00000
H_8201 0.29790
/

$Include FatLayer.txt
$Include PigWeight.txt

Parameter ProdWeight(j,i,k) Weight of product j from pig i at weight increase k ;
ProdWeight\((j,i,k)\) = ProdWeightCon\((j)\) + ProdWeightFat\((j)\)*FatLayer\((i,k)\) + ProdWeightWeight\((j)\)*PigWeight\((i,k)\) ;

ProdWeight\('P_{Sundry3}',i,k\) = ProdWeight\('P_{Bachs (with bones)}',i,k\) + ProdWeight\('P_{Breast1}',i,k\) + ProdWeight\('P_{CutOff2}',i,k\)
  + ProdWeight\('P_{Sundry2}',i,k\) - ProdWeight\('P_{Bachs (boneless)}',i,k\) - ProdWeight\('P_{Breast2}',i,k\) - ProdWeight\('P_{Cutoff3}',i,k\) ;

ProdWeight\('P_{Sundry4}',i,k\) = ProdWeight\('H_{8201},i,k\) - ProdWeight\('P_{Ham},i,k\) ;

ProdWeight\('P_{Sundry5}',i,k\) = ProdWeight\('H_{8201},i,k\) - ProdWeight\('P_{Ham (boneless)},i,k\) - ProdWeight\('P_{Cutoff5}',i,k\) ;

ProdWeight\('P_{Head},i,k\) = PigWeight\((i,k)\) - ProdWeight\('P_{Schoulder},i,k\) - ProdWeight\('P_{Neck},i,k\) - ProdWeight\('P_{Bachs (with bones)},i,k\) - ProdWeight\('P_{Breast1},i,k\) - ProdWeight\('P_{Ham (boneless)},i,k\) - ProdWeight\('P_{Cutoff1},i,k\) - ProdWeight\('P_{Cutoff2},i,k\) - ProdWeight\('P_{Sundry1},i,k\) - ProdWeight\('P_{Sundry2},i,k\) - ProdWeight\('P_{Sundry5},i,k\) - ProdWeight\('P_{Tenderloin},i,k\) ;

Parameter BackDeduction\((j,i,k)\) Deduction in price per kg at product weight in excess of 3.5 kg per back (7 kg per pig) ;

BackDeduction\('P_{Bachs (with bones)},i,k\) = 2\(\$\) (ProdWeight\('P_{Bachs (with bones)},i,k\) gt 7) + 0\(\$\) (ProdWeight\('P_{Bachs (with bones)},i,k\) le 7) ;

BackDeduction\('P_{Bachs (boneless)},i,k\) = 2\(\$\) (ProdWeight\('P_{Bachs (boneless)},i,k\) gt 7) + 0\(\$\) (ProdWeight\('P_{Bachs (boneless)},i,k\) le 7) ;

Variables
  \(z\)  total profit

Binary Variables
  \(x(i,j,k)\) 1 if product \(j\) to be produced from pig \(i\) at weight increase \(k\) else 0 ;
Equations

profit objective function
back_con maximum weight of back
ham_con ham maximum layer of fat 14 mm
x1_con constraint regarding fore-end
x2_con constraint regarding fore-end
x3_con constraint regarding fore-end
x4_con constraint regarding backs (middle piece) alternative 1
x5_con constraint regarding backs (middle piece) alternative 1
x6_con constraint regarding backs (middle piece) alternative 1
x7_con constraint regarding backs (middle piece) alternative 2
x8_con constraint regarding backs (middle piece) alternative 2
x9_con constraint regarding backs (middle piece) alternative 2
x10_con constraint regarding ham alternative 3
x11_con constraint regarding ham alternative 4
x12_con constraint regarding ham alternative 4
x13_con constraint re. always production of one tenderloin
x14_con constraint re. always production of one head
x20_con one back product only
x21_con one ham product only
x22_con newer production of help product H_8201

profit .. z =e= sum((i,j,k), (Price(j) - KamFradrag(j,i,k)
+ PriceCoeff(j)*(FatLayer(i,k)-15.9))
* ProdWeight(j,i,k) * x(i,j,k)));

back_con(k,i) .. x(i,'P_Breast2',k)*ProdWeight('P_Breast2',i,k) =l= 8;
ham_con(k,i) .. x(i,'P_Ham',k)*FatLayer(i,k) =l= 14;
x1_con(k,i) .. x(i,'P_Schoulder',k) =e= x(i,'P_Neck',k);
x2_con(k,i) .. x(i,'P_Neck',k) =e= x(i,'P_CutOff1',k);
x3_con(k,i) .. x(i,'P_CutOff1',k) =e= x(i,'P_Sundry1',k);
x4_con(k,i) .. x(i,'P_Backs (with bones)',k) =e= x(i,'P_Breast1',k);
x5_con(k,i) .. x(i,'P_Breast1',k) =e= x(i,'P_CutOff2',k);
x6_con(k,i) .. x(i,'P_CutOff2',k) =e= x(i,'P_Sundry2',k);
x7_con(k,i) .. x(i,'P_Backs (boneless)',k) =e= x(i,'P_Breast2',k);
x8_con(k,i) .. x(i,'P_Breast2',k) =e= x(i,'P_CutOff3',k);
x9_con(k,i) .. x(i,'P_CutOff3',k) =e= x(i,'P_Sundry3',k);
x10_con(k,i) .. x(i,'P_Ham',k) =e= x(i,'P_Sundry4',k);
x11_con(k,i) .. x(i,'P_Ham (boneless)',k) =e= x(i,'P_CutOff5',k);
x12_con(k,i) .. x(i,'P_CutOff5',k) =e= x(i,'P_Sundry5',k);
x13_con(k,i) .. x(i,'P_Tenderloin',k) =e= 1;
x14_con(k,i) .. x(i,'P_Head',k) =e= 1;
x20_con(k,i) .. x(i,'P_Backs (with bones)',k) + x(i,'P_Backs boneless)',k) =e= 1;
x21_con(k,i) .. x(i,'P_Ham',k) + x(i,'P_Ham (boneless)',k) =e= 1;
x22_con(k,i) .. x(i,'H_8201',k) =e= 0;

Model vaegt3d /all/ ;
Solve vaegt3d using mip maximizing z ;
Parameter Res(k) profit calculation at different weight scenarios;
   Res(k) = sum((i,j), (Price(j)-KamFradråg(j,i,k)
               + PriceCoeff(j)*(FatLayer(i,k)-15.9)) * ProdWeight(j,i,k) * x.l(i,j,k));

Parameter Resimp(k) increasement in profit by increasing weight;
   Resimp(k) = Res(k) - Res('w_0');

Parameter SlaughteringWeight(k) SlaughteringWeight in kg at weight scenario
   SlaughteringWeight(k) = sum((i), PigWeight(i,k));

Parameter Production(k) Production in kg at weight scenario k;
   Production(k) = sum((i,j), ProdWeight(j,i,k)*x.l(i,j,k));

Parameter AvgPrice(k) Average price at weight scenario k;
   AvgPrice(k) = Res(k)/Production(k);

Parameter AvgPriceChange(k) Change in average price at different weight increases;
   AvgPriceChange(k) = AvgPrice(k) - AvgPrice('w_0');

Parameter ProductionProduct(s,k) Production volume of sold product s (subset of j) at weight scenario k;
   ProductionProduct(s,k) = sum((i), ProdWeight(s,i,k)*x.l(i,s,k));

Parameter TurnoverProduct(j,k) Average price for product j at weight scenario k;
   TurnoverProduct(j,k) = sum((i), (Price(j) +
           PriceCoeff(j)*FatLayer(i,k))*ProdWeight(j,i,k)*x.l(i,j,k));

Parameter NumberProducts(s,k) Number of sold product s (subset of j) at weight scenario k;
   NumberProducts(s,k) = sum((i), x.l(i,s,k));

Display Res;
Display Resimp;
Display SlaughteringWeight;
Display Production;
Display AvgPrice;
Display AvgPriceChange;
Display ProductionProduct;
Display TurnoverProduct;
Display NumberProducts;
Paper C
Limitations in Production and Stocks and their Effect on the Profitability of the Slaughterhouses

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Abstract

The pig industry is important for the Danish economy with an export value of more than DKK 28 billions in 2006 [Danish Meat Association (2007)] and the competition is increasing. Therefore it is more important than ever to optimize all aspects of Danish pig production, slaughtering processes and delivery.

This paper concerns the aspects of optimization at the slaughterhouses and addresses the modelling of physical logistic conditions. The description of the production processes and the logistic flow in this paper is primarily based on the conditions in one of the largest Danish slaughterhouses, used as model slaughterhouse.

The model is based on the model described in the paper regarding the value of a general increase in the slaughter weight for pigs [Kjærgaard, N. (2008b)] but is more accurate by taking physical limitations at the slaughterhouses into consideration. The model is a Mixed Integer Programming (MIP) model and is used to estimate the costs of logistical limitations in the equalization room being the central cooling room where the carcasses should hang for a period of approx. 16 hours. Furthermore, the model is used to find economic effects of improved measurements as well as a general increase in the slaughter weight. This paper concerns the use of Operations Research to solve these practical problems, which is of major importance for the industry.
The cost of logistical limitations in the equalization room has been estimated to DKK 0.072 per kg or approximately DKK 145 million per year in total for the Danish slaughterhouses.

The main conclusion is that even relatively simple optimization models can be used to improve the basis of the slaughterhouses for decision making considerably, both regarding computing the costs of having limitations in the production as well as the value of improved measurements and increased slaughter weight. Prices vary from one week to another, and consequently a price and cost study should be performed before the computations are used for actual decision support and more products and product alternatives should be included.

Estimations of the economic consequences of improved measurements and of a general increase in the slaughter weight have been improved among other things by taking the logistic limitations into consideration. Compared to the previous model regarding improved measurements [Kjærsgaard, N. (2008a)], the new model uses the entire pig and not just the middle piece. Furthermore, the weights of different products are estimated in much more detail based on the actual registered fat layer and the slaughter weight for each pig. Regarding increased slaughter weight, the model is now improved by taking the important logistic limitations into consideration.

1 Background

The pig industry is important for Danish Economy and exports. More than 25 million pigs were produced in Denmark in 2006, and approx. 90% of the meat was exported. The export value amounted to DKK 28.8 billion [Danish Meat Association (2007)].

Competition in the pork industry is substantial, and European farmers are pressed by increased feeding costs without having access to genetically modified crops as their American competitors. It is therefore more important than ever to optimize all aspects of Danish pig production and slaughtering processes.

Even from our neighbouring countries there is a substantial competition for the slaughterhouses to offer the best payments to the farmers, and during the last couple of years a substantial number of Danish farmers have started delivering part of their pigs to German slaughterhouses. It is therefore more important than ever to optimize all aspects of Danish pig production and slaughtering processes.
This paper concerns the aspects of optimization at the slaughterhouses, especially regarding the economic consequences of limitations in the production. Taking these limitations into consideration improves the estimation of the consequences of improved measurements and sorting, as well as the value of a general increase in the slaughter weight [Kjærsgaard, N (2008b)]. Operations Research methods are used to improve the decision support within these topics, which are of major importance for the slaughterhouse industry.

2 Literature survey

The literature regarding optimized raw material use at the slaughterhouses has been addressed in the paper “The Value of Improved Measurements in a Pig Slaughterhouse” [Kjærsgaard, N. (2008a)]. For convenience it is repeated here as this paper should be readable independently.

The amount of literature addressing improved or optimized raw material use in the food industry is substantial. However, the main part of the contributions is related to different aspects regarding either optimization of meat quality or different production processes. Examples of this are optimization of the industrial thermal sterilization of canned foods [Garcia, M. et. al. (2006)] and pigs stunning optimization [Dupuis, P. et. al. (2004)]. These types of optimizations are not relevant for this project as they are either based on statistical analysis without optimization of a mathematical model or the mathematical models are very different from the models, which are used in this Ph.D. project regarding optimization of the raw material use at the slaughterhouses.

Within the pork industry relatively few contributions have been found regarding optimization based on operations research methods. In the paper “Location of slaughterhouses under economies of scale” [Broek et. al. (2006)] optimization is used to investigate the savings potential of reducing the number of slaughterhouses in Norway and investing in additional capacity in the remaining facilities in order to obtain economies of scale. Another facility location problem is described in the paper “The impact of changes in livestock supply on the optimum number, size and location of slaughterhouses in East Macedonia” [Kamenidis, C. & Sorensen, V. (1978)]. In the paper ”Economic optimization of pork production – marketing chains. II. Modelling outcome” [Ouden et. al. (1996)] are using Dynamic Linear programming to evaluate the development of pork chain concepts that also takes animal welfare into consideration. Kure in his Ph.D. thesis “Marketing Management Support in Slaughter Pig Production” [Kure, H. (1997)] uses Dynamic Programming to solve parts of the “slaughter pig
marketing management problem”, which regards how the farmers should select and market their pigs to the slaughterhouses.

The above mentioned four examples of optimization problems within the pork industry are all somewhat different from the problem of optimizing the raw material use at the slaughterhouses. More similar problems have been found in the following contributions:

In 1990-1992 a project regarding optimization of the raw material use at the slaughterhouses was performed as a cooperation between Danish Meat Research Institute and the Royal Veterinary and Agricultural University (now the Faculty of Life Sciences at University of Copenhagen). Several reports were made:

A Linear Programming (LP) model for production planning and control for the hog slaughterhouses was developed and reported in [Rasmussen, S. & Thomsen, M. (1991)] and [Rasmussen, S. (1992)]. The model is a 2-stage model. First stage concerns a planning horizon of 3 months and the second stage one weeks day to day planning. In [Fertin, C. (1992)] the long term planning model (stage 1) is validated.
In his Ph.D. thesis [Fertin [1995)] Fertin describes and further develops and validates the model.

There has been searched for literature in other food related industries, e.g. poultry and beef slaughterhouses and the fish industry, but no relevant literature has been found.

Other industries have similar problems as the slaughterhouses regarding its raw material use. An example is the refineries, but unlike the slaughterhouses the refineries have the option of blending different qualities in order to change the quality characteristics of the products. Another example is the lumber and wood industry. A few papers of the product mix problem within the wood industry have been identified. In the paper “An Optimization-Based Decision Support System for a Product Mix Problem” [Roy et. al. (1982)] an LP-model has been used to solve a plywood product mix problem for Ponderosa Industrial in Mexico.

Even though literature within food optimization is substantial, the main part of the contributions are related to optimization based on e.g. statistical analysis without optimization of a mathematical model. Other models are very different from the models used in this Ph.D. project. Except for the contributions from the Royal Veterinary and Agricultural University and the Danish Meat Research Institute not much literature of relevance for the Ph.D. project has been identified.
3 Limitations in production and stock

The slaughterhouse industry is characterized by the fact that the raw material (the pigs) in the short term exists in a given volume with a relatively varied quality and has a limited shelf-life. The raw material can be used for several different products, but to a large extent yields, prices and costs depend on how well the raw material in question fits the final products.

The slaughterhouses deal with the natural variation in quality, weight, size, fat layer, lean meat percentage, etc. by sorting the pigs into different sorting groups, in which pigs with almost the same characteristics are placed. However, the variation within each sorting group is still substantial due to the considerable measuring error in the current measuring systems as well as the limited number of sorting groups which the slaughterhouses are able to handle. Ideally, the slaughterhouses should be able to measure the quality without error and be able to handle an extensive number of sorting groups.

For the years to come it is expected that the measuring accuracy will be improved substantially. This will result in more pigs being placed in the correct sorting groups with less variation within each group as a consequence. If the full economic effect of improved sorting should be reached, more sorting groups are required. In general, the advantage of additional sorting groups increases when the measuring accuracy is improved.

As mentioned before a number of physical limitations influence the flow of raw materials in the slaughterhouses. Some of the most important are:

- The actual design of the equalization room.
- The number and design of the subsequent sorting places in the production.
- The design of the cold storage room used as buffer before cutting departments.

3.1 Production flow at the slaughterhouse

The production flow is almost the same at the different Danish slaughterhouses, but varies regarding capacities, e.g. the number of slaughter lines and bars in the equalization room. The description and the modelling are primarily based on the conditions at one of the largest Danish slaughterhouses, which is used as the model slaughterhouse.
The overall production flow at the model slaughterhouse can be seen in Figure 1 below and in a larger version in Appendix 2:

![Figure 1. Production flow at the model slaughterhouse](image)

The pigs are driven from the lairage area to one of the stunning centres with subsequent sticking and debleeding and continue at one of the four slaughtering lines to scalding, de-hairing and singeing.

The carcasses are cut open and heart, liver, kidney and intestines etc. are taken out. The carcasses are split in halves, but are still kept together by their jawbone. The carcass as well as heart, liver, kidney and intestines are inspected for deceases etc. If deceases are found, the carcass can be rejected or utilized for other purposes (e.g. as heat-treated products). The carcasses are cleaned and then weighted and graded in the classification centre. Here sundry measurements are performed, such as lean meat percentage and fat layer for the total carcass as well as for the individual main parts (fore end, middle piece...
and ham). The weight, and other measurements as well as information about whether the pigs are special production pigs (e.g. Antonius or “welfare pigs” to the UK market) are of the utmost importance for the subsequent sorting of the carcasses. Each of the four slaughtering lines at the model slaughterhouse has a capacity of 350 pigs per hour, i.e. a total of 1,400 pigs per hour. The model slaughterhouse is only using 1 shift of approx. 8 hours at the slaughter lines.

After slaughtering, the carcasses are led through a freezing tunnel which quickly reduces temperature of the carcasses surfaces. Depending on predefined sorting groups the carcasses are placed on bars in the equalization room (cold storage), where they are hang for a period of minimum 16 hours to ensure a uniform temperature throughout the entire carcass. In general, these 16 hours will pass if the pigs are used for further processing the day after they have been slaughtered and placed at the equalization room.

The equalization room consists of 180 bars each with a capacity of 80 pigs. Each bar on which the carcasses are hung can only be emptied from the opposite side of the filling side, and consequently it has to be emptied in the same order as it was filled. See Figure 2 below:

![Figure 2. Limitations in the equalization room.](image)

For practical reasons whole bars are emptied at a time making it possible to dispose freely of the bars once more. The physical limitations in the equalization room (the number and capacity of the bars) are of the utmost importance for the number of active sorting groups that can be handled effectively. In practice, the slaughterhouses base the sorting on the middle piece and the ham, which are the most valuable parts of the pigs and hence the largest sorting potential. As the main parts (fore end, middle piece and ham) are not yet separated it is only possible to sort by one criterion which is primarily the middle piece. To some extent the sorting can be based on both the middle piece and
the ham at the same time, but the number of combinations (and the need for further sorting groups) increases rapidly thereby.

Due to logistic reasons the bars numbered 1 to 108 are primarily used for cutting line 1 and the bars from 109 to 180 for cutting line 2. To some extent, however, it is possible to relocate the carcasses at other bars, but that is a time consuming procedure which is not much used.

The placement in the equalization room is based on forecasts for how the pigs being slaughtered that day will be placed in sorting groups. The planning task here is to decide which bars should be allocated for different sorting groups. If possible, the carcasses are placed so that one bar consists of carcasses from only one sorting group and preferably from the same slaughtering day so the entire bar can be used in production at the same time for the same production batch. Sorting groups consisting of only few carcasses, for instance carcasses taken out for tests, damaged carcasses etc. may be mixed at the same bar due to space considerations. Due to quality and yield considerations it the carcasses should preferable be used for further processing the day after slaughtering and not stocked further at the equalization room.

During the stay in the equalization room, each carcass has not yet been parted into the three main parts. This limits the possible number of sorting groups as a combinatorial “explosion” takes place. If there is e.g. 10 sorting groups for the middle piece, 10 for the ham and 3 for the fore end 300 sorting groups are required if any of these combinations should be possible. Therefore, in practice, a limited number of sorting groups can be handled and primarily based on the quality of the middle piece and only to some extent on the ham.

The number of active sorting groups can vary over time, but there are typically 15-20 main groups. In addition to these, there are approximately 30 smaller sorting groups for pigs with salmonella, diseases, pigs used in sundry experiments, special production pigs as well as damaged pigs or pigs with missing identification etc.

After 16 hours of placement in the equalization room, the carcasses are taken to one of the two cutting lines. Here the tenderloin and head are cut off and each half of the carcass is parted into three main parts: Fore end, middle piece and ham. The middle piece can further be split in two pieces and the middle piece and the ham can be sorted further by weight. The sorting by weight is automated, and the middle piece and the ham can be sorted separately into a maximum of 3 sorting groups each at a time.
At the end of the cutting line the parts are placed on stands each consisting of 20 items. The stands are placed in a buffer storage before they are taken to different cutting departments for further processing. After further processing the products are packed and delivered to the customers.

4 The Model

The purpose of the model is to estimate the costs of having certain limitations in the production as well as improving previously developed models [Kjærsgaard, N. (2008a, b)] used to estimate the economic effects of improved measurements and increased slaughter weight. The costs or benefits are found by performing two optimizations, one under current conditions and one under improved conditions. The improvement can then be found as the difference between the profits of the two optimizations.

4.1 Description of the Model

As mentioned before there are a number of logistic limitations at the slaughterhouses. The most important concerns the equalization room, where the temperature of the pigs is equalized through the entire carcass. In the slaughterhouse used as basis for the modelling the carcasses are placed in the equalization room on bars containing 80 carcasses each. Each bar can only be emptied from the opposite side of the filling side. In principle, carcasses placed on the same bar are therefore used to produce the same production orders (the same package of products). This limitation is considered to be the most important logistic limitation, but the model can be extended to include more of the limitations described in Chapter 3.

The model is based on the model described in paper B [Kjærsgaard, N. (2008b)] but is repeated here in order to make this paper readable independently. In the experiments we use the actual slaughtering data from 43,949 pigs slaughtered at one of the Danish slaughterhouses. For each pig the registered fat layer and the actual slaughter weight are used. In the computations the registered fat layer is considered the true value. When performing computations regarding improved measurements and sorting, a simulated measuring error is added to the registered fat layer and then considered the measured fat layer. For computations regarding increased slaughter weight, in general there is coherence between slaughter weight and the size of the fat layer. When the slaughter weight is increased by 1 kg, the fat layer is increased with approx. 0.16 mm [Kjærsgaard, N. (2008b)].
The model has its basis in different alternative uses of the pigs. Each alternative use consists of a “package” of products for the specific part. The back and the ham have two alternative uses each and the fore end has one. In total there are four different alternative uses for each pig. In total, 17 different main products are used, but the model can easily be modified to include more products and alternative uses.

The different alternative uses for different parts of the pigs can be seen in Figure 3 below:

![Diagram of a pig showing different parts and alternative uses](image)

<table>
<thead>
<tr>
<th>Fore-end</th>
<th>Middle piece</th>
<th>Ham</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alternative 1</td>
<td>Alternative 1</td>
<td>Alternative 1</td>
</tr>
<tr>
<td>P_Schoulder</td>
<td>P_Backs (with bones)</td>
<td>P_Ham</td>
</tr>
<tr>
<td>P_Neck</td>
<td>P_Breast1</td>
<td>P_Sundry4</td>
</tr>
<tr>
<td>P_CutOff1</td>
<td>P_CutOff2</td>
<td>P_Sundry2</td>
</tr>
<tr>
<td>P_Sundry1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Alternative 2</th>
<th>Alternative 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>P_Backs (boneless)</td>
<td>P_Ham (boneless)</td>
</tr>
<tr>
<td>P_Breast2</td>
<td>P_CutOff5</td>
</tr>
<tr>
<td>P_CutOff3</td>
<td>P_Sundry5</td>
</tr>
<tr>
<td>P_Sundry3</td>
<td></td>
</tr>
</tbody>
</table>

Figure 3. Alternative uses of the pigs.

Some raw materials (pigs) are better suited for some products than others. This is taken into consideration when increasing or decreasing the price for some products depending on
the level of the fat layer. In the model, this is done by splitting the pricing in two different contributions:

1. A fixed price per kg for the given product

and

2. A price coefficient, which stipulates how much the price will decrease if the fat layer increases by 1 mm.

For two of the products there are a few special additional conditions which have to apply in order for the raw materials to be used for these products:

- The ham product P_Ham can only be produced if the fat layer does not exceed 14 mm

and

- The breast product P_Breast2 can only be produced if it does not exceed a weight of 4 kg.

If these conditions are not met, a penalty is introduced in the prices. The penalty cover additional handling costs if these raw materials should be used for other products instead.

The products P_CutOff (1, 2, 3, 5) consist of meat cut-offs in connection with production of the main products, and products P_Sundry (1-5) consist of fat, bones, rind etc.

### 4.2 Mathematical Formulation of the Model

We have a set of carcasses $i = \{1, \ldots, I\}$. Each carcass can be used to produce a set of different product alternatives $m = \{1, \ldots, M\}$ and each product alternative consists of a set of different products $j = \{1, \ldots, J\}$. Finally the carcasses are hung on bars $k = \{1, \ldots, K\}$ in the equalization room. The decision variable $y_{k,n}$ is a binary variable with the value 1 if the pigs placed on bar $k$ are used to produce product alternative $n$ and otherwise 0. The problem is to find the optimal utilization (product alternatives) of the carcasses placed at each bar and the total profit for the optimal solution:
The objective function:

1) \[ \text{Maximize } Z = \sum_{k,n} \text{ValueBar}_{k,n} \cdot y_{k,n} \]

Subject to:

2) \[ \sum_{n} y_{k,n} = 1 \quad \forall \ k \in \mathcal{K} \]

3) \[ y_{k,n} = \begin{cases} 1 & \text{if product alternative } n \text{ is produced by pigs placed on bar } k, \text{otherwise } 0 \end{cases} \]

4) \[ \text{ValuePig}_{i,n} = \sum_{j,n} (\text{Price}_j + \text{PriceCoeff}_j \cdot \text{FatLayerDeviation}_i - \text{QualityDeduction}_{i,j}) \cdot \text{ProdWeight}_{i,j} \cdot \text{AltUse}_{j,n} \]

5) \[ \text{ValueBar}_{k,n} = \sum_{k} \text{ValuePig}_{k,n} \]

Indices:
\[ i: \text{ pig } i \quad k: \text{ bar } k \quad j: \text{ product } j \quad n: \text{ alternative use } n \]

Decision variables:
\[ y_{k,n}: \text{ Decision variable with value 1 if the carcasses placed on bar } k \text{ are used for product alternative } n, \text{otherwise 0.} \]

Parameters:
\[ \text{ValueBar}_{k,n}: \text{ Value of the carcasses placed on bar } k \text{ when used to produce alternative } n. \]
\[ \text{ValuePig}_{i,n}: \text{ Value of carcass } i \text{, when producing alternative } n. \]
\[ \text{Price}_j: \text{ Fixed net price per kg for producing product } j. \]
\[ \text{PriceCoeff}_j: \text{ Change in net price per kg for product } j \text{ when the fat layer increases by 1 mm.} \]
\[ \text{FatLayerDeviation}_i: \text{ Deviation in the fat layer of carcass } i \text{ compared to the average fat layer.} \]
\[ \text{QualityDeduction}_{i,j}: \text{ Price deduction per kg if quality demands are not being met when carcass } i \text{ is used for production of product } j. \]
\[ \text{ProdWeight}_{i,j}: \text{ Estimated weight of product } j \text{, when produced from carcass } i. \]
\[ \text{AltUse}_{j,n}: \text{ Alternative use (product package) with value 1 if product } j \text{ is part of product alternative } n, \text{otherwise 0.} \]

The objective function (1) maximizes the sum of the value of carcasses at each bar by finding the best alternative use for each bar when all carcasses placed on the same bar are used for the same product alternative. The constraint (2) controls that the carcasses placed at each bar are only used once. The model uses a number of different parameters. The most important ones are shown in (4) and (5) and are either directly or indirectly used in the
objective function. The parameter ValuePig_{i,n} (4) finds the value of each carcass i, when producing product alternative n. The value is based on a price per kg for each potential product, a price coefficient depending on the fat layer and a deduction in price if certain quality measurements are not met. This net price for different products is multiplied with the estimated weight of the products. The parameter ValueBar_{k,n} finds the total value of carcasses placed at bar k, when producing product alternative n.

5 Results

In this chapter, the model is used to compute the economic consequences of having limitations in the production, improving the measurements at the slaughterhouses and a general increase in the slaughter weight.

5.1 Limitations in the Production

The economic consequences of having limitations in the equalization room are found by performing two optimizations and comparing the results. The model described in the previous chapter is used to find the optimal alternative use (product package) of the carcasses placed on each bar and the profit hereof. The computation takes the limitations in the equalization room into consideration. Without these limitations it corresponds to that the cooling room is a random access cooling room from where each carcass can be taken out for the production in any order required. When finding the profit without limitations in the equalization room the following slightly modified mathematical model is used:

The objective function:

1a) \[ \text{Maximize } Z = \sum_{i,n} \text{ValuePig}_{i,n} \cdot y_{i,n} \]

Subject to:

2a) \[ \sum_{n} y_{i,n} = 1 \quad \forall \ i \in J \]

3a) \[ y_{i,n} = \{1 \text{ if product alternative } n \text{ is produced of pig } i, \text{ otherwise } 0\} \]
The parameter $ValueBar_{k,n}$ (5) used in the model described in the previous chapter is not necessary in this optimization and the parameter $ValuePig_{i,n}$ (constraint 4) remains unchanged.

$$4) \quad ValuePig_{i,n} = \sum_{i,n} (Price_j + PriceCoeff_j \cdot FatLayerDeviation_i - QualityDeduction_{i,j}) \cdot ProdWeight_{i,j} \cdot AltUse_{i,n}$$

Index $k$ for bars is no longer necessary, but the other indices $i$, $j$ and $n$ remain unchanged.

For the 43,949 pigs being part of the experiment, the costs of having limitations are computed and can be seen in Table 1 below:

<table>
<thead>
<tr>
<th></th>
<th>DKK</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profit with no limitations</td>
<td>38,243,284</td>
</tr>
<tr>
<td>Profit with limitations</td>
<td>37,987,026</td>
</tr>
<tr>
<td>Cost of limitations</td>
<td>256,258</td>
</tr>
<tr>
<td>Cost of limitations per kg</td>
<td>0,072</td>
</tr>
</tbody>
</table>

Table 1. Cost of having limitations in the equalization room (in DKK)

In the experiment, the pigs have been placed in 65 different sorting groups based on the estimated measured fat layer in intervals of 0.5 mm. The number of sorting groups is considerably larger than used by the slaughterhouses today.

If a much smaller number of sorting groups were used in the experiment, the computed costs of the limitations would be increased. The computed profit with limitations would decrease and the profit with no limitations would remain unchanged (see Table 1 above). However, by designing the sorting strategies in a more intelligent way (see [Kjersgaard, N. (2008d)]) the increase in costs can be limited.

The model can also be used to find economic consequences of improved measurements as well as a general increase in slaughter weight.

5.2 Improved measurements

In Table 2 below, the economic consequences of improving the measuring accuracy by 20% can be seen. This means that the standard error of prediction (SEP) of the fat layer is decreased by 20% from its current level of 1.28 mm to 1.02 mm.
Table 2. Improvement in the profit due to increased measurements (SEP decreased by 20%)

<table>
<thead>
<tr>
<th>Measuring accuracy</th>
<th>Profit (DKK)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Improved with 20% (SEP 1.02)</td>
<td>37,909,314</td>
</tr>
<tr>
<td>Current (SEP 1.28)</td>
<td>37,875,591</td>
</tr>
<tr>
<td>Improvement</td>
<td>33,723</td>
</tr>
</tbody>
</table>

As it can be seen, the profit increases by DKK 33,723 for the 43,949 pigs used in the experiment. This equals DKK 0.77 per pig or approx. DKK 19 million per year for the Danish slaughterhouses. Similar computations have been carried out for other sizes of improvements and can be seen in Figure 4 below:

Figure 4. Increase in profit due to improved measurements (for the 43,949 pigs)

As can be seen in figure 4 above, the effect is almost linear up to an improvement in the measuring accuracy of 80% after which the improvement declines.

5.3 Increased slaughter weight

As mentioned before, the model can also be used to compute the economic consequences of increased slaughter weight. The result of the experiment can be seen in Table 3 below:
<table>
<thead>
<tr>
<th></th>
<th>Current weight</th>
<th>Plus 5 kg</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Profit in DKK</strong></td>
<td>37,875,592</td>
<td>39,270,690</td>
</tr>
<tr>
<td><strong>Profit improvement (DKK)</strong></td>
<td>1,395,098</td>
<td></td>
</tr>
<tr>
<td><strong>Production in kg</strong></td>
<td>3,576,982</td>
<td>3,796,727</td>
</tr>
<tr>
<td><strong>Average price per kg</strong></td>
<td>10.589</td>
<td>10.343</td>
</tr>
<tr>
<td><strong>Change in average price (DKK)</strong></td>
<td>-0.245</td>
<td></td>
</tr>
</tbody>
</table>

Table 3. Improvement in profit due to a general increase in slaughter weight by 5 kg.

It can be seen that the profit for the 43,949 pigs used in the experiments increases by 3.7% (DKK 1,395,098) when the slaughter weight is increased by 5 kg. This increase in slaughter weight is equivalent to a 6.1% increase in the production volume and the average price per kg will decrease by 2.3% or DKK 0.245. Some of the costs at the slaughterhouses as well as at the farmers are, however, fixed or variable according to the number of pigs produced instead of according to the production in terms of weight. Therefore, the decrease in average prices should be compared to the cost savings in the slaughterhouses and at the farmers. These savings have previously been estimated to approx. DKK 0.15 and DKK 0.10 respectively for an increase in the slaughter weight of 5 kg [Mønsted, K. (2006)]. Taking these savings into account, the computations show that a minor increase in the total profits of the slaughterhouses and the farmers can be expected if the slaughter weight is increased. However, the increase in profits is small and the net prices used are for illustrative purposes. There is a natural variation in prices over time. As the market situation changes continuously it has not been possible to have access to the latest prices, but information of typical prevailing prices have been received from the Danish slaughterhouses. Therefore a price study should be performed before the computation should be used for actual decision making. In the next chapter potential improvements of the model and in the data used are discussed further. See the paper [Kjærsgaard, N. (2008b)] for further information regarding the consequences of increased slaughter weight.

The GAMS code for the model can be seen in Appendix 1. The model has 178,696 constraints and 3,980 variables and was solved to optimality in just 18 seconds which is considered very acceptable. See chapter 4.2 in the thesis [Kjærsgaard, N. (2008e)] for further information regarding solution times.
6 Discussions

The models presented in this paper have been used to illustrate how Operations Research can be applied to improve the slaughterhouses basis for decisions. The models can be used to estimate the costs of having logistic limitations at the slaughterhouses and is also an improvement compared to the previous models described in papers regarding improved measurements and increased slaughter weight [Kjaersgaard, N. (2008a, b)]. Prices vary over time and consequently a price and cost study should be performed before the computations are used for actual decision support.

In order to be able to use the model for actual decision making the prices and cost as well as product yields should be studied further. The study should include how all these factors are influenced by changes in slaughter weight and other quality characteristics, such as fat layer, lean meat percentage etc.

The model can be improved in a number of ways:

- Increasing the number of products and product alternatives. In this paper the model is illustrated by using 17 main products and four product alternatives. In order to receive as reliable results as possible, all essential products and product alternatives should be used. See the Ph.D. thesis [Kjaersgaard, N. (2008e)] for further information regarding the complexity of such an increase in the number of products and products alternatives.

- Extending the model with more logistic limitations, e.g. the automatic sorting by weight of the middle piece and the ham and the limitations at the buffer storage. This requires that the number of product alternatives is much larger than the current four.

- Extending the model with maximum and minimum sales volumes of different products. For some of the products the slaughterhouses can only sell a certain volume. This is especially the case in the short and medium term where the slaughterhouses have entered into contracts to deliver a certain volume. These situations can be modelled by having constraints regarding the sales of each product.

- Performing a study of the prices and costs of producing and selling different products. The study should be performed by the slaughterhouses, as the information is considered very sensitive. See paper regarding improved measurements for further information regarding prices and costs [Kjaersgaard, N. (2008a)].

- Using the actual sorting groups and criteria, which the slaughterhouses use today, to estimate the current level of profit (and use this as a basis to compare any
improvements/changes). This requires the number of products and product alternatives to be much larger than the current numbers used in the computations.

7 Conclusion

The costs of having logistic limitations in the equalization room have been estimated using Mixed Integer Programming.

Even though only four different product alternatives are used (two for the middle pieces and two for the ham) the computations show that the cost of logistic limitations in the equalization room is considerable. The cost of having these logistic limitations has been estimated to DKK 256,258 for the 43,949 pigs used in the experiment. This equals DKK 0.072 per kg or approximately DKK 145 million annually for Danish slaughterhouses. The fact that the computations are only based on 17 main products and four alternative uses is likely to underestimate the profit. When taking the constraints regarding placement on bars into consideration more products and product alternatives will increase the share of pigs which are no longer used optimally. This will increase the computed costs. Furthermore, the average costs for each pig not being used optimally may increase too as the spread between optimal and poor utilization of the pig is likely to increase. On the other hand, some of these additional costs can be saved by sorting the middle piece and ham by weight later in the production process.

The economic consequence of improving the measuring accuracy by e.g. 20% is an increase in profit by DKK 33,723 for the 43,949 pigs used in the experiments. This equals DKK 0.77 per pig or approximately DKK 19 million annually for the Danish slaughterhouses. It can be seen in figure 5 below that the effect is almost linear up to an improvement in the measuring accuracy by 80% after which the increase in profits decline:
The economic consequences of increased slaughter weight have been computed. The average price per kg produced meat decreases with DKK 0.245 when the slaughter weight increases by 5 kg. This should be compared to the cost saving in the slaughterhouse and at the farmers. These savings has previously been estimated to DKK 0.15 and DKK 0.10 respectively for an increase in the slaughter weight of 5 kg [Mønsted, K. (2006)].

Compared to the previous models described in the two papers [Kjærgaard, N. (2008a, b)] regarding the value of improved measurements and a general increase in the slaughter weight, the accuracy of the model has been improved by taking the logistic limitations in the equalization room into consideration.

The main conclusion is that even relatively simple optimization models can improve the basis of the slaughterhouses for decision making considerably in connection with finding the value of logistic changes in the production, improved measurements and increased slaughter weight. Before using the model as actual decision support, the accuracy of the model can be further improved by:

- Increasing the number of products and product alternatives, so that most of the products produced by the slaughterhouses are covered.
- Extending the model with more logistic limitations, e.g. the automatic sorting by weight of the middle piece and ham and the buffer storage (requires that the number of product alternatives is much larger than the current four).
• Extending the model with maximum and minimum sales volumes of different products.

• Performing a study of the prices and costs of selling and producing different products. The study should be performed by the slaughterhouses, as the information is considered very sensitive.

• Using the sorting groups and criteria, used by the slaughterhouses today, to estimate the current level of profit (and use this as a basis of comparing new computations).
Bibliography


Fertin, C. (1992). Validation of long term planning model for optimization of the raw material use at pig slaughterhouses (in Danish). The Royal Veterinary and Agricultural University, Copenhagen, Denmark.


Rasmussen, S. (1992), The use of a multi-period LP-model as the core of a Decision Support System for a hog slaughterhouse. The Royal Veterinary and Agricultural University, Copenhagen, Denmark.


Appendix 1 – GAMS code

* CanneryTransport.gms
*
* CanneryTransport.gms
*
$eolcom //
option iterlim=999999999; // avoid limit on iterations
option reslim=300; // timelimit for solver in sec.
option optcr=0.0; // gap tolerance
option solprint=OFF; // include solution print in .lst file
option limrow=100; // limit number of rows in .lst file
option limcol=100; // limit number of columns in .lst file
//-------------------------------

SETS
  i   Pigs i           / p1*p43949 /
  j   Products        / P_Schoulder, P_Neck, P_Backs (with bones), P_Breast1, P_Backs (boneless),
                   P_Breast2, P_Ham, P_Ham (boneless), P_CutOff1, P_CutOff2,
                   P_CutOff3, P_CutOff5, P_Sundry1, P_Sundry2, P_Sundry3,
                   P_Sundry4, P_Sundry5, P_moerbrad, P_hoved, H_8201 /
  s(j) Products sold / P_Schoulder, P_Neck, P_Backs (with bones), P_Breast1, P_Backs (boneless),
                    P_Breast2, P_Ham, P_Ham (boneless), P_CutOff1, P_CutOff2,
                    P_CutOff3, P_CutOff5, P_Sundry1, P_Sundry2, P_Sundry3,
                    P_Sundry4, P_Sundry5, P_Tenderloin, P_Head /
  l   Bar l           / Bar1*Bar585 /
  n   Product Alternative n / Alt1*Alt4 /

$Include weight_5_SG100_fordeling_stænger_65SG.txt

PARAMETER
  Price(j) Price per kg for products j /
                   P_Schoulder        12.00
                   P_Neck             13.00
                   P_Backs (with bones)     18.00
                   P_Breast1          13.00
                   P_Breast2          17.00
                   P_Ham             15.00
                   P_Ham (boneless)    18.00
                   P_CutOff1          9.00
                   P_CutOff2          9.00
                   P_CutOff3          9.00
<table>
<thead>
<tr>
<th>Product</th>
<th>Price Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>P_CutOff5</td>
<td>9.00</td>
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<td>P_Sundry1</td>
<td>3.00</td>
</tr>
<tr>
<td>P_Sundry2</td>
<td>3.00</td>
</tr>
<tr>
<td>P_Sundry3</td>
<td>3.00</td>
</tr>
<tr>
<td>P_Sundry4</td>
<td>3.00</td>
</tr>
<tr>
<td>P_Sundry5</td>
<td>3.00</td>
</tr>
<tr>
<td>P_Tenderloin</td>
<td>30.00</td>
</tr>
<tr>
<td>P_Head</td>
<td>3.00</td>
</tr>
<tr>
<td>H_8201</td>
<td>0.00</td>
</tr>
</tbody>
</table>

PriceCoeff(j) Price Coefficient (in DKK) for product j for an increase of layer of fat (in mm)

<table>
<thead>
<tr>
<th>Product</th>
<th>Price Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>P_Schoulder</td>
<td>0.00</td>
</tr>
<tr>
<td>P_Neck</td>
<td>0.00</td>
</tr>
<tr>
<td>P_Backs (with bones)</td>
<td>-0.20</td>
</tr>
<tr>
<td>P_Breast1</td>
<td>-0.20</td>
</tr>
<tr>
<td>P_Backs (boneless)</td>
<td>-0.20</td>
</tr>
<tr>
<td>P_Breast2</td>
<td>-0.20</td>
</tr>
<tr>
<td>P_Ham</td>
<td>-0.20</td>
</tr>
<tr>
<td>P_Ham (boneless)</td>
<td>-0.20</td>
</tr>
<tr>
<td>P_CutOff1</td>
<td>-0.10</td>
</tr>
<tr>
<td>P_CutOff2</td>
<td>-0.10</td>
</tr>
<tr>
<td>P_CutOff3</td>
<td>-0.10</td>
</tr>
<tr>
<td>P_CutOff5</td>
<td>-0.10</td>
</tr>
<tr>
<td>P_Sundry1</td>
<td>-1.95414</td>
</tr>
<tr>
<td>P_Sundry2</td>
<td>0.00</td>
</tr>
<tr>
<td>P_Sundry3</td>
<td>0.00</td>
</tr>
<tr>
<td>P_Sundry4</td>
<td>0.00</td>
</tr>
<tr>
<td>P_Sundry5</td>
<td>0.00</td>
</tr>
<tr>
<td>H_8201</td>
<td>0.00</td>
</tr>
</tbody>
</table>

ProdWeightCon(j) Product weight constant for product j

<table>
<thead>
<tr>
<th>Product</th>
<th>Product weight constant</th>
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</thead>
<tbody>
<tr>
<td>P_Schoulder</td>
<td>0.00000</td>
</tr>
<tr>
<td>P_Neck</td>
<td>0.00000</td>
</tr>
<tr>
<td>P_Backs (with bones)</td>
<td>10.77058</td>
</tr>
<tr>
<td>P_Breast1</td>
<td>2.00642</td>
</tr>
<tr>
<td>P_Breast2</td>
<td>2.00642</td>
</tr>
<tr>
<td>P_Ham</td>
<td>0.00000</td>
</tr>
<tr>
<td>P_Ham (boneless)</td>
<td>-1.11490</td>
</tr>
<tr>
<td>P_CutOff1</td>
<td>0.00000</td>
</tr>
<tr>
<td>P_CutOff2</td>
<td>0.00000</td>
</tr>
<tr>
<td>P_CutOff3</td>
<td>0.00000</td>
</tr>
<tr>
<td>P_CutOff5</td>
<td>0.00000</td>
</tr>
<tr>
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</table>

24
<table>
<thead>
<tr>
<th>Product</th>
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<tbody>
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<td>P_Sundry2</td>
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<tr>
<td>P_Sundry3</td>
<td>0.00000</td>
</tr>
<tr>
<td>P_Sundry4</td>
<td>0.00000</td>
</tr>
<tr>
<td>P_Sundry5</td>
<td>0.00000</td>
</tr>
<tr>
<td>P_Tenderloin</td>
<td>1.20000</td>
</tr>
<tr>
<td>P_Head</td>
<td>0.00000</td>
</tr>
<tr>
<td>H_8201</td>
<td>-1.58570</td>
</tr>
</tbody>
</table>

ProdWeightFat(j) Product weight fat dependent coefficient for product j

/ P_Schoulder -0.06938
  P_Neck -0.04096
  P_Backs (with bones) -0.01662
  P_Breast1 0.04284
  P_Backs (boneless) -0.08124
  P_Breast2 0.04284
  P_Ham -0.10204
  P_Ham (boneless) -0.19054
  P_CutOff1 -0.00596
  P_CutOff2 -0.00596
  P_CutOff3 -0.00596
  P_CutOff5 -0.00596
  P_Sundry1 0.07922
  P_Sundry2 0.11178
  P_Sundry3 0.00000
  P_Sundry4 0.00000
  P_Sundry5 0.00000
  P_Tenderloin 0.00000
  P_Head 0.00000
  H_8201 -0.10160

$Include FatLayer_beg.txt
$Include PigWeight_beg.txt

ProdWeightWeight(j) Product weight slaughtering weight dependent coefficient for product j

/ P_Schoulder 0.10726
  P_Neck 0.07282
  P_Backs (with bones) 0.01354
  P_Breast1 0.06002
  P_Backs (boneless) 0.08666
  P_Breast2 0.06002
  P_Ham 0.27632
  P_Ham (boneless) 0.22874
  P_CutOff1 0.00834
  P_CutOff2 0.00834
Table Anvendelse(j,n)  Product alternative n in which product j is part of

<table>
<thead>
<tr>
<th></th>
<th>Alt1</th>
<th>Alt2</th>
<th>Alt3</th>
<th>Alt4</th>
</tr>
</thead>
<tbody>
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<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>P_Neck</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>P_Backs (with bones)</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>P_Breast1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>P_Breast2</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>P_Ham</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>P_Ham (boneless)</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>P_CutOff1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
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<td>1</td>
</tr>
<tr>
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<td>1</td>
</tr>
<tr>
<td>P_Sundry1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>P_Sundry2</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>P_Sundry3</td>
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<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>P_Sundry4</td>
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<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>P_Tenderloin</td>
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<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>H_8201</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Parameter ProdWeight(j,i) Weight of product j from pig i :

ProdWeight(j,i) = ProdWeightCon(j) + ProdWeightFat(j)*FatLayer(i) +
ProdWeightWeight(j)*PigWeight(i) ;

ProdWeight('P_Sundry3',i) = ProdWeight('P_Backs (with bones)',i) + ProdWeight('P_Breast1',i) +
ProdWeight('P_CutOff2',i) + ProdWeight('P_Sundry2',i) - ProdWeight('P_Backs (boneless)',i) -
ProdWeight('P_Breast2',i) - ProdWeight('P_CutOff3',i) ;
ProdWeight('P_Sundry4',i) = ProdWeight('H_8201',i) - ProdWeight('P_Ham',i) ;

ProdWeight('P_Sundry5',i) = ProdWeight('H_8201',i) - ProdWeight('P_Ham (boneless)',i)
- ProdWeight('P_CutOff5',i) ;

ProdWeight('P_hoved',i) = PigWeight(i) - ProdWeight('P_Soulder',i) - ProdWeight('P_Neck',i)
- ProdWeight('P_Backs (with bones)',i) - ProdWeight('P_Breast1',i)
- ProdWeight('P_Ham (boneless)',i) - ProdWeight('P_CutOff1',i)
- ProdWeight('P_CutOff2',i) - ProdWeight('P_CutOff5',i)
- ProdWeight('P_Sundry1',i) - ProdWeight('P_Sundry2',i)
- ProdWeight('P_Sundry5',i) - ProdWeight('P_Tenderloin',i) ;

Parameter QualityDeduction(j,i) Deduction in price at product weight above 3.5 kg per back (7 kg per pig) and requirement for breast and ham ;

Fradrag('P_Backs (with bones)',i) = 2$(ProdWeight('P_Backs (with bones)',i) gt 7) + 0$(ProdWeight('P_Backs (with bones)',i) le 7) ;

Fradrag('P_Backs (boneless)',i) = 2$(ProdWeight('P_Backs (boneless)',i) gt 7) + 0$(ProdWeight('P_Backs (boneless)',i) le 7) ;

Fradrag('P_Breast2',i) = 6$(ProdWeight('P_Breast2',i) gt 8) + 0$(ProdWeight('P_Breast2',i) le 8) ;

Fradrag('P_Ham',i) = 4$(FatLayer(i) gt 14) + 0$(FatLayer(i) le 14) ;

Parameter ValueGris(i,n) Price for pig i at alternative use n ;

ValueGris(i,n) = sum(j, (Price(j)-Fradrag(j,i)+PriceCoeff(j)*(FatLayer(i)-15.9)) * ProdWeight(j,i) * Anvendelse(j,n)) ;

Parameter ValueStang(l,n) ;

$Include ValueStang_43949.txt

Variables
  z               total profit

  Binary Variables
    y(l,n)   1 if alternative n is chosen to be produced of pig i with bars;
    // y(i,n)   1 if alternative n is chosen to be produced of pig i without bars;

Equations
  profit definering af objekt funktion
con(l)  * with bars
// con1(i)  * without bars
;

profit .. z =e= sum((l,n), ValueBar(l,n)*y(l,n)) ;
// profit .. z =e= sum((i,n), ValuePig(i,n)*y(i,n)) ;

con(l) .. sum((n),y(l,n)) =e= 1 ;
// con1(i) .. sum(n,y(i,n)) =e= 1 ;

Model begraensninger_v11 /all/ ;
Solve begraensninger_v11 using mip maximizing z ;

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Appendix 2 – The overall production flow at the slaughterhouse
Paper D
Evaluation Different Sorting Criteria and Strategies Using Mathematical Programming

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Technical University of Denmark
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Denmark

March 31, 2008

Abstract

The pig industry is an essential and important part of Danish economy with an export value in 2006 of more than DKK 28 billions [Danish Meat Association (2007)]. The competition increases, and potential new competitors from low cost countries can be expected to enter the traditional Danish export markets. Therefore it is more important than ever to optimize all aspects of Danish pig production, slaughtering processes and delivery.

The raw material (the pigs) used by the slaughterhouses is a biological material, with a large variation in weight, size, fat layer and other quality characteristics. The slaughterhouses deal with this variation by sorting the pigs into different sorting groups, whereby variation within each sorting group is reduced substantially.

Deciding on the sorting criteria and sorting limits used to define the sorting groups has substantial influence on the economy of the slaughterhouses. In principle, sorting can be based on every kind of quality characteristics, e.g. fat layer, slaughter weight, lean meat percentage or whether the pigs are special production pigs (welfare pigs etc.). Each slaughterhouse defines its sorting groups according to customer demands (and whether or not a premium for that quality can be obtained) and the production costs (more uniform raw materials can make the production easier and less expensive).
The sorting parameters can be combined in different ways, and the sorting limits can have a numerous number of values. In this paper, the different aspects of sorting will be illustrated by using the following two sorting parameters:

- Fat layer (in mm) and
- Slaughter weight (in kg).

Figure 1 below is an example of sorting groups and sorting limits based on the fat layer.

![Sorting groups and limits based on fat layer](image)

The paper is concerned with tools for evaluation of different sorting strategies by the use of operations research methods. Evaluation of sorting strategies is a practical problem of major importance for the slaughterhouse industry. The model used is the same Mixed Integer Programming (MIP) model as described in the paper regarding limitations in production and stocks [Kjærgaard, N. (2008c)].

The model is illustrated by performing experiments using slaughter data from 43,949 pigs slaughtered at one of the Danish slaughterhouses. In the computations, 17 different products and four alternative uses of each pig are used, but the model can easily be modified to include more products and alternative uses.

The main conclusion of the experiments is that even relatively simple optimization models can be used to improve the basis of the slaughterhouses for making decisions considerably. The graphical tool based on the optimization model provides an overview of the sorting criteria and limits which can be used to develop good sorting strategies, and the optimization model can be used to evaluate these different strategies further.

1 Background

The pig industry is essential for Danish economy and exports. In 2006, more than 25 million pigs were produced in Denmark, and approx. 90% of the meat was exported. The export value amounted to DKK 28.8 billion [Danish Meat Association (2007)].
Competition in the pig industry is substantial, and farmers from a number of countries can produce pigs at considerably lower costs than European farmers. Farmers from e.g. Brazil, USA and Canada are able to produce pigs at approximately 80% of the costs in Denmark, Spain and The Netherlands [Udesen, F. & Rasmussen, J. (2006)].

Even within our neighbouring countries there is a fierce competition for the slaughterhouses to offer the best payments to the farmers, and during the last couple of years a substantial number of Danish farmers have delivered part of their pigs to German slaughterhouses.

Therefore it is becoming more and more important that Danish farmers and slaughterhouses continue to optimize their production and slaughtering processes.

The slaughterhouses are characterized by the fact that the raw material (the pigs) is a biological material with a relative large variation in quality, size and shape. The slaughterhouses deal with this variation by sorting the pigs into different sorting groups. By so doing, the slaughterhouses are able to reduce the variation within each sorting group substantially.

This paper is concerned with the aspects of optimization at the slaughterhouses, specifically regarding computing of the value of improved sorting and evaluation of different sorting strategies. This practical problem is of major importance for the slaughterhouse industry and is solved by operations research methods.

2 Literature survey

The literature regarding optimized raw material use at the slaughterhouses has been addressed in the paper “The Value of Improved Measurements in a Pig Slaughterhouse” [Kjærsgaard, N. (2008a)] but is repeated here as it should be possible to read this paper independently.

The amount of literature addressing improved or optimized raw material use in the food industry is substantial. However, the main part of the contributions is related to different aspects regarding either optimization of meat quality or different production processes. Examples of this are optimization of the industrial thermal sterilization of canned foods [Garcia, M. et. al. (2006)] and pigs stunning optimization [Dupuis, P. et. al. (2004)]. These types of optimizations are not relevant for this project as they are either based on
statistical analysis without optimization of a mathematical model or the mathematical models are very different from the models, which are used in this Ph.D. project regarding optimization of the raw material use at the slaughterhouses.

Within the pork industry relatively few contributions have been found regarding optimization based on operations research methods. In the paper “Location of slaughterhouses under economies of scale” [Broek et. al. (2006)] optimization is used to investigate the savings potential of reducing the number of slaughterhouses in Norway and investing in additional capacity in the remaining facilities in order to obtain economies of scale. Another facility location problem is described in the paper “The impact of changes in livestock supply on the optimum number, size and location of slaughterhouses in East Macedonia” [Kamenidis, C. & Sorensen, V. (1978)]. In the paper ”Economic optimization of pork production – marketing chains. II. Modelling outcome” [Ouden et. al. (1996)] are using Dynamic Linear programming to evaluate the development of pork chain concepts that also takes animal welfare into consideration. Kure in his Ph.D. thesis “Marketing Management Support in Slaughter Pig Production” [Kure, H. (1997)] uses Dynamic Programming to solve parts of the “slaughter pig marketing management problem”, which regards how the farmers should select and market their pigs to the slaughterhouses.

The above mentioned four examples of optimization problems within the pork industry are all somewhat different from the problem of optimizing the raw material use at the slaughterhouses. More similar problems have been found in the following contributions:

In 1990-1992 a project regarding optimization of the raw material use at the slaughterhouses was performed as a cooperation between Danish Meat Research Institute and the Royal Veterinary and Agricultural University (now the Faculty of Life Sciences at University of Copenhagen). Several reports were made:

A Linear Programming (LP) model for production planning and control for the hog slaughterhouses was developed and reported in [Rasmussen, S. & Thomsen, M. (1991)] and [Rasmussen, S. (1992)]. The model is a 2-stage model. First stage concerns a planning horizon of 3 months and the second stage one weeks day to day planning. In [Fertin, C. (1992)] the long term planning model (stage 1) is validated. In his Ph.D. thesis [Fertin [1995]] Fertin describes and further develops and validates the model.

There has been searched for literature in other food related industries, e.g. poultry and beef slaughterhouses and the fish industry, but no relevant literature has been found.
Other industries have similar problems as the slaughterhouses regarding its raw material use. An example is the refineries, but unlike the slaughterhouses the refineries have the option of blending different qualities in order to change the quality characteristics of the products. Another example is the lumber and wood industry. A few papers of the product mix problem within the wood industry have been identified. In the paper “An Optimization-Based Decision Support System for a Product Mix Problem” [Roy et. al. (1982)] an LP-model has been used to solve a plywood product mix problem for Ponderosa Industrial in Mexico.

Even though literature within food optimization is substantial, the main part of the contributions are related to optimization based on e.g. statistical analysis without optimization of a mathematical model. Other models are very different from the models used in this Ph.D. project. Except for the contributions from the Royal Veterinary and Agricultural University and the Danish Meat Research Institute not much literature of relevance for the Ph.D. project has been identified.

3 Sorting at the Slaughterhouses

The slaughterhouse industry is characterized by the fact that the raw material (the pigs) in the short term exists in a given volume with a relatively varied quality and has a limited shelf-life. The raw material can be used for several different products, but to a large extent yields, prices and costs depend on how well the raw material in question fits the final products.

The slaughterhouses deal with the natural variation in quality, weight, size, fat layer, lean meat percentage, etc. by sorting the pigs into different sorting groups, in which pigs with almost the same characteristics are placed. However, the variation within each sorting group is still substantial due to the considerable measuring error in the current measuring systems as well as the limited number of sorting groups which the slaughterhouses are able to handle.

For the years to come it is expected that the measuring accuracy will be improved substantially. This will result in more pigs being placed in correct sorting groups with less variation within each group as a consequence. If the full economic effect of improved sorting should be reached, more sorting groups are required.

In principle, sorting can be based on every kind of quality characteristics, such as the fat layer, lean meat percentage, slaughter weight, weight of a specific part, colour, pH-
value of the meat as well as whether the pigs are special production pigs (welfare pigs etc.). The sorting parameters chosen will depend on:

- Customer demands and whether a premium for the quality in question can be obtained and
- Production costs (more uniform raw materials can make production easier and less expensive).

Obviously, it is also important that the slaughterhouse can perform the measurements and that the logistics at the slaughterhouse are able to handle additional sorting groups.

In this paper, sorting aspects will be illustrated by using two quality characteristics: The fat layer and the slaughter weight. The possible sorting strategies investigated in this paper are:

1. Sorting based on slaughter weight (Figure 2a)
2. Sorting based on fat layer (Figure 2b)
3. Sorting based on fat layer and slaughter weight (Figure 2c)

The different strategies are illustrated in figure 2a-c.

Compared to other industries, the value added at the slaughterhouses is relatively limited. The Profit and Loss Account for 2006/07 for Tican a.m.b.a. [Tican (2007)] has been analysed in order to investigate the cost structure at the slaughterhouses. It was
found that approx. 65\% \(^1\) of the turnover is used for direct payments to the farmers for the pigs received and that only 35\% are left to cover all the value adding activities taking place at the slaughterhouses. By investigating the cost structure it was found that even a small improvement in raw material use has a significant impact on profitability. If the raw material use for instance can be improved to increase the total yield by just 1\%, this will increase the profit corresponding to:

- A decrease in administrative costs by 64\%, or
- A decrease in the production wages by 7\%.

These examples illustrate that one of the most important ways for the slaughterhouses to improve the earning power is to improve the raw material use. See [Kjærsgaard, N. (2008e)] chapter 2.1 for further information regarding the cost structure and value added at the slaughterhouses.

4 The Model

The model used is the same Mixed Integer Programming (MIP) model as described in the paper [Kjærsgaard N. (2008c)] regarding limitations in production and stocks which is based on the model developed in the paper regarding improved measurements [Kjærsgaard, N. (2008b)]. The model was described in these two papers, but as this paper should be readable independently it is repeated here. The benefits or costs are found by performing two optimizations, one under the current conditions and one under improved conditions. The improvement can then be found as the difference between the profits of the two optimizations.

4.1 Description of the Model

There are a number of logistic limitations at the slaughterhouses [Kjærsgaard, N. (2008c)]. The most important concerns the equalization room, where the temperature is equalized through the entire carcass. In the slaughterhouse used as basis for the modelling, the carcasses are placed in the equalization room on bars containing 80 carcasses each. Each bar can only be emptied from the opposite side of the filling side. In principle, carcasses

\(^{1}\) Based on annual accounts for the parent company which include both the primary industry (slaughtering etc.) and the secondary industry (processed meat, sausages etc.). If only the primary industry is considered, the raw materials share of the turnover will increase further.
placed on the same bar are therefore used for the same production order (the same package of products).

In the experiments we use actual slaughtering data from 43,949 pigs slaughtered at one of the Danish slaughterhouses. For each pig the registered fat layer and the actual slaughter weight are used. In the computations the registered fat layer is considered to be the true value. When performing computations regarding improved measurements and sorting, a simulated measuring error is added to the registered fat layer and is then considered the measured fat layer.

The model has its basis in different alternative uses of the pigs. Each alternative use consists of a “package” of products for the specific part. The back and the ham have two alternative uses each and the fore end has one. In total there are four different alternative uses for each pig. In total, 17 different main products are used, but the model can easily be augmented with more products and alternative uses.

The different alternative uses for different parts of the pigs can be seen in Figure 3 below:
Figure 3. Alternative uses of the pig.

Some raw materials (pigs) are better suited for some products than others. This is taken into consideration when increasing or decreasing the price for some products depending on the level of the fat layer. In the model, this is done by splitting the pricing in two different contributions:

1. A fixed price per kg for the given product

and
2. A price coefficient, which stipulates how much the price will decrease if the fat layer increases by 1 mm.

For two of the products there are a few special conditions which have to apply for the raw materials to be used for these products:

- The ham product P_Ham can only be produced if the fat layer does not exceed 14 mm
  
  and

- The breast product P_Breast2 can only be produced if it does not exceed a weight of 4 kg.

If these conditions are not met, a penalty is introduced in the prices. The penalty covers additional handling costs if these raw materials should be used for other products instead.

The products P_CutOff (1, 2, 3, 5) consist of meat cut-offs in connection with production of the main products, and products P_Sundry (1-5) consist of fat, bones, rind etc.

### 4.2 Mathematical Formulation of the Model

We have a set of carcasses \( J = \{1, \ldots, I\} \). Each carcass can be used to produce a set of different product alternatives \( M = \{1, \ldots, N\} \) and each product alternative consists of a set of different products \( J = \{1, \ldots, J\} \). Finally the carcasses are hung on a set of bars \( K = \{1, \ldots, K\} \) in the equalization room. The decision variable \( y_{k,n} \) is a binary variable with the value 1 if the pigs placed on bar \( k \) are used to produce product alternative \( n \) and otherwise 0. The problem is to find the optimal utilization (product alternatives) of the carcasses placed at each bar and the total profit for the optimal solution:

The objective function:

1) \[ \text{Maximize } Z = \sum_{k,n} \text{ValueBar}_{k,n} \cdot y_{k,n} \]

Subject to:

2) \[ \sum_n y_{k,n} = 1 \quad \forall \ k \in K \]
3) \( y_{k,n} = \{1 \text{ if product alternative n is produced by pigs placed on bar k, otherwise 0}\} \)

4) \( \text{ValuePig}_{i,n} = \sum_{i,n} (\text{Price}_j + \text{PriceCoeff}_j \cdot \text{FatLayerDeviation}_i - \text{QualityDeduction}_{i,j}) \cdot \text{ProdWeight}_{i,j} \cdot \text{AltUse}_{j,n} \quad \forall \ i \in \mathcal{I}, \ n \in \mathcal{N} \)

5) \( \text{ValueBar}_{k,n} = \sum_{k,n} \text{ValuePig}_{k,n} \quad \forall \ k \in \mathcal{K}, \ n \in \mathcal{N} \)

Indices:
- \( i \): pig
- \( k \): bar
- \( j \): product
- \( n \): alternative use

Decision variables:
- \( y_{k,n} \): Decision variable with value 1 if the carcasses placed on bar k are used for product alternative n, otherwise 0.

Parameters:
- \( \text{ValueBar}_{k,n} \): Value of the carcasses placed on bar k when used to produce alternative n.
- \( \text{ValuePig}_{i,n} \): Value of carcass i, when producing alternative n.
- \( \text{Price}_j \): Fixed net price per kg for producing product j.
- \( \text{PriceCoeff}_j \): Change in net price per kg for product j when the fat layer increases by 1 mm.
- \( \text{FatLayerDeviation}_i \): Deviation in the fat layer of carcass i compared to the average fat layer.
- \( \text{QualityDeduction}_{i,j} \): Price deduction per kg if quality demands are not being met when carcass i is used for production of product j.
- \( \text{ProdWeight}_{i,j} \): Estimated weight of product j, when produced from carcass i.
- \( \text{AltUse}_{j,n} \): Alternative use (product package) with value 1 if product j is part of product alternative n, otherwise 0.

The objective function (1) maximizes the sum of the value of carcasses at each bar by finding the best alternative use for each bar when all pigs placed on the same bar are used for the same product alternative. The constraint (2) controls that the carcasses placed at each bar are only used once. The model uses a number of different parameters. The most important ones are shown in (4) and (5) and are either directly or indirectly used in the objective function. The parameter \( \text{ValuePig}_{i,n} \) (4) finds the value of each carcass i, when producing product alternative n. The value is based on a price per kg for each potential product, a price coefficient depending on the fat layer and a deduction in price if certain quality measurements are not met. This net price for different products is multiplied with the estimated weight of the products. The parameter \( \text{ValueBar}_{k,n} \) finds the total value of carcasses placed at bar k, when producing product alternative n.
5 Results

As mentioned before, the sorting strategies illustrated in this chapter are relatively simple strategies as they are using only two sorting criteria: Slaughter weight and fat layer. This provides the following three main principles for sorting:

1. Sorting based on slaughter weight
2. Sorting based on fat layer
3. Sorting based on both fat layer and slaughter weight

5.1 Sorting based on slaughter weight

The 43,949 pigs used in the experiments have been sorted into four different sorting groups based on the registered slaughter weight. The four sorting groups contain almost the same number of pigs and are used to place the carcasses on bars. The distribution of pigs on slaughter weight and sorting groups can be seen in Figure 4 below.

![Figure 4. Distribution of pigs on slaughter weight and sorting groups.](image-url)
The optimal use of the pigs has been found provided that all pigs placed on the same bar are used for the same product alternative. The profit of the 43,949 pigs used in the experiment is computed to:

<table>
<thead>
<tr>
<th></th>
<th>DKK</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Profit</strong></td>
<td>37,810,962</td>
</tr>
</tbody>
</table>

Table 1. Result of experiment with four sorting groups based on slaughter weight

Compared to no sorting at all, where pigs are placed on bars without taking any quality measurements into consideration this is an improvement in profit by DKK 80,167.

The sorting strategy could be slightly changed by using other limits for slaughter weights used to define each of the four sorting groups. Furthermore, the number of sorting groups could be increased as well.

### 5.2 Sorting based on fat layer

Now the fat layer of the 43,949 pigs used in the experiments has been simulated, and the pigs are sorted based on these simulated values of the measured fat layer. All four sorting groups are of approximately the same size and are used when placing the carcasses on bars. The distribution of pigs on measured fat layer at the current level of measuring accuracy and on the sorting groups can be seen in Figure 5 below.
When using the bars optimally the profit of the 43,949 pigs is computed to:

<table>
<thead>
<tr>
<th>Profit</th>
<th>DKK</th>
</tr>
</thead>
<tbody>
<tr>
<td>37,827,885</td>
<td></td>
</tr>
</tbody>
</table>

Table 2. Profit with sorting based on fat layer at the current level of measuring accuracy.

Compared to sorting based on slaughter weight alone, the profit increases by an additional DKK 16,924 for the 43,949 pigs being part of the experiment when both computations are based on four sorting groups of approximately the same size.

The measuring system’s ability to measure accurately is specified by its standard error of prediction (SEP). The standard error of prediction is found as the standard deviation of the differences between the measured values and the reference values (true values) using a test data set. For each level of measuring accuracy a similar distribution as the one in Figure 5 is estimated and sorting limits for the sorting groups are established. The carcasses are placed on bars based on sorting groups. Computations of the optimal use of carcasses are made, and the profit for different levels of measuring accuracy is found and can be seen in Table 3 below:

<table>
<thead>
<tr>
<th>Profit</th>
<th>Improved profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>37,827,885</td>
<td>16,924</td>
</tr>
<tr>
<td>37,870,426</td>
<td>59,464</td>
</tr>
<tr>
<td>37,916,962</td>
<td>106,000</td>
</tr>
<tr>
<td>37,941,708</td>
<td>130,746</td>
</tr>
<tr>
<td>37,966,898</td>
<td>155,936</td>
</tr>
<tr>
<td>38,027,786</td>
<td>216,824</td>
</tr>
<tr>
<td>38,066,957</td>
<td>255,996</td>
</tr>
</tbody>
</table>

Table 3. Improved profit with sorting based on measured fat layer compared to sorting by weight.

It can be seen that when the measuring accuracy is improved, the profit increases substantially. The improvement in profits is almost linear with approximate DKK 2,400 for each percentage the measuring accuracy (SEP) is improved.
5.3 Sorting based on both slaughter weight and fat layer

Now the sorting is based on both the slaughter weight and the fat layer. The sorting is still based on the same sorting limits, which were used previously, but now it requires 16 sorting groups instead of four (see Figure 6 below).

<table>
<thead>
<tr>
<th>Fat layer (mm)</th>
<th>0-6</th>
<th>6-7</th>
<th>7-8</th>
<th>8-9</th>
<th>9-10</th>
<th>10-11</th>
</tr>
</thead>
<tbody>
<tr>
<td>SG 1</td>
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<td>SG 6</td>
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<td>SG 10</td>
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<td>SG 13</td>
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<td>SG 14</td>
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<td>SG 15</td>
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<tr>
<td>SG 16</td>
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</tbody>
</table>

Figure 6. Sorting groups and sorting limits.

The profit increases substantially when sorting is based on both fat layer and slaughter weight. Compared to the scenario where sorting is based on the fat layer alone, the profit at the current measuring accuracy increases by DKK 104,064 for the 43,949 pigs being part of the experiment. This equals DKK 59 million for the Danish slaughterhouses on an annually basis. The profit improvements at different levels of the measuring error can be seen in table 4 below.

<table>
<thead>
<tr>
<th>Current measuring error (SEP)</th>
<th>Profit (DKK)</th>
<th>Improved profit (DKK)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current measuring error (SEP)</td>
<td>37,931,949</td>
<td>104,064</td>
</tr>
<tr>
<td>Current measuring error (SEP) - 20%</td>
<td>37,964,185</td>
<td>136,300</td>
</tr>
<tr>
<td>Current measuring error (SEP) - 40%</td>
<td>38,001,568</td>
<td>173,683</td>
</tr>
<tr>
<td>Current measuring error (SEP) - 50%</td>
<td>38,024,909</td>
<td>197,024</td>
</tr>
<tr>
<td>Current measuring error (SEP) - 60%</td>
<td>38,048,667</td>
<td>220,782</td>
</tr>
<tr>
<td>Current measuring error (SEP) - 80%</td>
<td>38,099,793</td>
<td>271,908</td>
</tr>
<tr>
<td>Current measuring error (SEP) - 100%</td>
<td>38,148,722</td>
<td>320,837</td>
</tr>
</tbody>
</table>

Table 4. Profit with sorting based on both slaughter weight and fat layer compared to sorting based on fat layer alone.

Based on the figures in Table 4, which stems from computations for the 43,949 pigs used in the experiment, the consequences of improved measurements have been calculated. For the Danish slaughterhouses, which produce approximately 25 million pigs per year, the equivalent improvement in profits can be seen in Figure 7 below:
Figure 7. Profit increase for the Danish slaughterhouses due to improved measurements with sorting based on both slaughter weight and fat layer.

Figure 7 shows that improved measurements are valuable for Danish slaughterhouses. If the measurements were perfect (current measuring error reduced 100%), the increased profits for Danish slaughterhouses is estimated to more than DKK 120 million per year.

Even though the profit increases substantially by using the 16 sorting groups indicated in Figure 6, it is possible to increase the profit further by defining the sorting limits in a more intelligent way than just requiring them to be of approximately the same size. This may even be possible with much fewer sorting groups.

The GAMS code can be seen in Appendix 1. The solution time for solving the model to optimality was just 17 seconds and this is considered very acceptable. See chapter 4.2 in the thesis [Kjærsgaard, N. (2008e)] for further information regarding solution times when increasing the number of products and product alternatives.

### 5.4 Definition of sorting groups

The 43,949 pigs used in the experiments are divided into 20 quality groups according to the fat layer and 37 groups according to slaughter weight. This in total gives 740 combinations or quality groups. The distribution of pigs in different quality groups can be seen in the matrix in Figure 8 below:
Figure 8. Distribution of pigs on quality groups at the current level of measuring accuracy.

For each quality group the optimal product alternative is found and has been coloured in accordance herewith. In order to evaluate the previous sorting strategy, the 16 different sorting groups used in the computations have been indicated in Figure 8 above and are marked by the red lines. It can be seen that the sorting groups contain different optimal product alternatives and consequently the sorting is far from optimal. Even with four sorting groups much better sorting criteria and limits can be obtained:

**Sorting group A:**
- Slaughter weight < 73
- Fat layer < 14

**Sorting group B:**
- 73 ≤ Slaughter weight < 90
- Fat layer < 13

**Sorting group C:**
- 73 ≤ Slaughter weight < 88
- Fat layer < 24

**Sorting group D:**
- Rest

The four more “intelligent” sorting groups can be seen in Figure 9 below.
Figure 9 indicates that this new sorting strategy is a clear improvement. Sorting group A (in the upper left corner) mostly consists of quality groups where the optimal production is product alternative 1 (marked yellow). Sorting group B seems quite good too, but it might be possible to improve sorting group C and D further. The profit using the sorting groups, which can be seen in Figure 9 above, is computed to:

<table>
<thead>
<tr>
<th>Fat layer (mm)</th>
<th>Slaughter weight (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>63</td>
</tr>
<tr>
<td>2</td>
<td>64</td>
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<tr>
<td>3</td>
<td>65</td>
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<tr>
<td>37</td>
<td>99</td>
</tr>
</tbody>
</table>

Table 5. Profit using new sorting criteria.

| Profit         | 38.013.205 |

This is a further improvement in profits by DKK 81,256 for the 43,949 pigs used and equals DKK 46 million for the Danish slaughterhouses on an annually basis. If each of the 740 quality groups of which the matrix in Figure 10 consist were used optimally, the profit can be computed to DKK 38,039,658 for the 43,949 pigs used.

When using the four sorting groups A-D the profit should be compared to the computed profit using the 740 quality groups, as this can be considered an upper bound for the size of the profit. The profit is only improved by DKK 26,453, equalling DKK 15 million annually for the Danish slaughterhouse when using 740 sorting groups instead of four (A-D).

As suggested, the sorting can be improved even further by using the following five sorting groups instead:
Sorting group A1:
slaughter weight < 73
fat layer < 14

Sorting group B:
73 ≤ slaughter weight < 90
fat layer ≤ 13

Sorting group C:
73 ≤ slaughter weight < 90
13 ≤ fat layer
if fat layer < 26:
slaughter weight + fat layer ≤ 104
if fat layer ≥ 26:
slaughter weight < 80

Sorting group D:
90 ≤ slaughter weight < 97
10 ≤ fat layer < 12

Sorting group E:
Rest

Sorting group C is a bit more complex as there are different criteria whether or not the fat layer is less than 26 mm. The five sorting groups can be seen in Figure 10 below but only result in minor improvements. The number of carcasses without similarity in the optimal use of sorting group and quality group make up 5.0% using the new sorting groups indicated in Figure 10 and 6.4% using the previous sorting groups from Figure 9.

Figure 10. Distribution of pigs on quality groups using 5 sorting groups.
6 Conclusion

Even the very simple sorting strategy using only four sorting groups of approximately the same size and pigs sorted based only on slaughter weight, the computed profit is increased by DKK 80,167 compared to no sorting for the 43,949 pigs used in the experiment. This equals DKK 46 million for Danish slaughterhouses on an annually basis.

When a similar sorting in four sorting groups of approximately the same size is used, but this time based on the fat layer instead of the slaughter weight, the profit increases by an additional DKK 16,924 with the current level of measuring accuracy. For the Danish slaughterhouses this equals an additional DKK 10 million annually.

When combining sorting based on slaughter weight and on fat layer the profit improves substantially. When using the same sorting criteria as when the sorting was based on slaughter weight and on fat layer individually, this 2 dimensional sorting strategy requires 16 sorting groups. Using this strategy, the profit improves with an additional DKK 104,064 which equals DKK 59 million on an annually basis for Danish slaughterhouses.

The profit increases substantially when the accuracy of the measurements improves. Computations have been performed; still using sorting based on the 16 sorting groups mentioned in Figure 6 above, and gives the following increase in profits for the Danish slaughterhouses on an annually basis:

![Figure 11. Increased profit due to improved measurements for Danish slaughterhouses.](image-url)
If the measurements were perfect (current measuring error reduced 100%), the increased profits for Danish slaughterhouses is estimated to more than DKK 120 million per year.

It has been shown that the model can be used to evaluate and compare different sorting strategies. The matrix which can be seen in Figure 12 below is an important tool in connection with designing new sorting strategies, as it provides a graphical view of the potential sorting groups worth investigating further.

![Figure 12. Distribution of pigs on quality groups at the current level of measuring accuracy with improved sorting groups.](image)

The main conclusion is that even relatively simple optimization models can advantageously be used to improve the basis of the slaughterhouses for making decisions considerably. The graphical tool based on the optimization model provides an overview of the sorting criteria and limits which result in good sorting strategies, and the optimization model can be used to evaluate these different strategies further.

### 6.1 Future work

Before the slaughterhouses can rely on the model for actual decision making, several products and product alternatives should be included as input in the model, and a price and cost study should be obtained.

In this paper, the simulation of which carcasses are placed at different bars has been made in Excel outside the optimization environment GAMS. If a sub program is made to perform this task within GAMS or a similar optimization environment new simulations can be performed fast.

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2 GAMS (The General Algebraic Modeling System) is a high-level modelling system for mathematical programming problems.
A graphical interface based on the matrix in Figure 13 where sorting groups etc. are defined graphically could be a very interesting tool for the slaughterhouse. Further work should be made to find out how best to represent more dimensions than the current two: Slaughter weight and fat layer.
Bibliography


Fertin, C. (1992). Validation of long term planning model for optimization of the raw material use at pig slaughterhouses (in Danish). The Royal Veterinary and Agricultural University, Copenhagen, Denmark.


Appendix 1 – GAMS code

* CanneryTransport.gms
*
* CanneryTransport.gms
*
$eolcom //
option iterlim=999999999;     // avoid limit on iterations
option reslim=300;            // timelimit for solver in sec.
option optcr=0.0;             // gap tolerance
option solprint=OFF;          // include solution print in .lst file
option limrow=100;            // limit number of rows in .lst file
option limcol=100;            // limit number of columns in .lst file
//------------------------------------------------------------------------------

SETS
  i        Pigs i                / p1*p43949/
  j        Products           /  P_Schoulder, P_Neck, P_Backs (with bones), P_Breast1, P_Backs (boneless),
                                P_Breast2, P_Ham, P_Ham (boneless), P_CutOff1, P_CutOff2,
                                P_CutOff3, P_CutOff5, P_Sundry1, P_Sundry2, P_Sundry3,
                                P_Sundry4, P_Sundry5, P_moerbrad, P_hoved, H_8201 /
  s(j)     Products sold  /  P_Schoulder, P_Neck, P_Backs (with bones), P_Breast1, P_Backs (boneless),
                                P_Breast2, P_Ham, P_Ham (boneless), P_CutOff1, P_CutOff2,
                                P_CutOff3, P_CutOff5, P_Sundry1, P_Sundry2, P_Sundry3,
                                P_Sundry4, P_Sundry5, P_Tenderloin, P_Head /
  l        Bar l                   /  Bar1*Bar585 /
  n        Product Alternative n   /  Alt1*Alt4 /

$Include weight_5_SG100_fordeling_stænger_65SG.txt

PARAMETER
  Price(j) Price per kg for products j
    /  
    P_Schoulder 12.00
    P_Neck 13.00
    P_Backs (with bones) 18.00
    P_Breast1 13.00
    P_Breast2 17.00
    P_Ham 15.00
    P_Ham (boneless) 18.00
    P_CutOff1 9.00
    P_CutOff2 9.00
    P_CutOff3 9.00

PriceCoeff(j) Price Coefficient (in DKK) for product j for an increase of layer of fat (in mm)

/ P_Soulder 0.00
    P_Neck 0.00
    P_Backs (with bones) -0.20
    P_Breast1 -0.20
    P_Backs (boneless) -0.20
    P_Breast2 -0.20
    P_Ham -0.20
    P_Ham (boneless) -0.20
    P_CutOff1 -0.10
    P_CutOff2 -0.10
    P_CutOff3 -0.10
    P_CutOff5 -0.10
    P_Sundry1 0.00
    P_Sundry2 0.00
    P_Sundry3 0.00
    P_Sundry4 0.00
    P_Sundry5 0.00
    P_Tenderloin 0.00
    P_Head 0.00
    H_8201 0.00
/

ProdWeightCon(j) Product weight constant for product j

/ P_Soulder 0.00000
    P_Neck 0.00000
    P_Backs (with bones) 10.77058
    P_Breast1 2.00642
    P_Backs (boneless) 0.46036
    P_Breast2 2.00642
    P_Ham 0.00000
    P_Ham (boneless) -1.11490
    P_CutOff1 0.00000
    P_CutOff2 0.00000
    P_CutOff3 0.00000
    P_CutOff5 0.00000
    P_Sundry1 -1.95414
P_Sundry2            -14.54192
P_Sundry3               0.00000
P_Sundry4                0.00000
P_Sundry5               0.00000
P_Tenderloin           1.20000
P_Head            0.00000
H_8201            -1.58570
/
ProdWeightFat(j) Product weight fat dependend coefficient for product j
/  P_Schoulder             -0.06938
    P_Neck             -0.04096
    P_Backs (with bones)  -0.01662
    P_Breast1             0.04284
    P_Backs (boneless)    -0.08124
    P_Breast2             0.04284
    P_Ham             -0.10204
    P_Ham (boneless)     -0.19054
    P_CutOff1            -0.00596
    P_CutOff2             -0.00596
    P_CutOff3             -0.00596
    P_CutOff5             -0.00596
    P_Sundry1            0.07922
    P_Sundry2             0.11178
    P_Sundry3             0.00000
    P_Sundry4             0.00000
    P_Sundry5             0.00000
    P_Tenderloin         0.00000
    P_Head             0.00000
    H_8201             -0.10160
/
$Include FatLayer_beg.txt
$Include PigWeight_beg.txt
ProdWeightWeight(j) Product weight slaughtering weight dependent coefficient for product j
/  P_Schoulder             0.10726
    P_Neck             0.07282
    P_Backs (with bones)  0.01354
    P_Breast1             0.06002
    P_Backs (boneless)   0.08666
    P_Breast2             0.06002
    P_Ham             0.27632
    P_Ham (boneless)     0.22874
    P_CutOff1            0.00834
    P_CutOff2             0.00834
27
Table Anvendelse(j,n)  Product alternative n in which product j is part of

<table>
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<tr>
<th></th>
<th>Alt1</th>
<th>Alt2</th>
<th>Alt3</th>
<th>Alt4</th>
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<td>P_Breast1</td>
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<tr>
<td>P_Backs (boneless)</td>
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<tr>
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<tr>
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<tr>
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<td>H_8201</td>
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</tr>
</tbody>
</table>

Parameter ProdWeight(j,i) Weight of product j from pig i ;

ProdWeight(j,i) = ProdWeightCon(j) + ProdWeightFat(j)*FatLayer(i) +
                  ProdWeightWeight(j)*PigWeight(i) ;

ProdWeight(’P_Sundry3’,i) = ProdWeight(’P_Backs (with bones)’,i) + ProdWeight(’P_Breast1’,i) +
                        ProdWeight(’P_CutOff2’,i) +
                        ProdWeight(’P_Sundry2’,i) - ProdWeight(’P_Backs (boneless)’,i) -
                        ProdWeight(’P_Breast2’,i) - ProdWeight(’P_CutOff3’,i) ;
ProdWeight(’P_Sundry4’,i) = ProdWeight(’H_8201’,i) - ProdWeight(’P_Ham’,i) ;
ProdWeight(’P_Sundry5’,i) = ProdWeight(’H_8201’,i) - ProdWeight(’P_Ham (boneless)’,i)  
- ProdWeight(’P_CutOff5’,i) ;
ProdWeight(’P_hoved’,i) = PigWeight(i) - ProdWeight(’P_Schoulder’,i) - ProdWeight(’P_Neck’,i)  
- ProdWeight(’P_Backs (with bones)’,i) - ProdWeight(’P_Breast1’,i)  
- ProdWeight(’P_Ham (boneless)’,i) - ProdWeight(’P_CutOff1’,i)  
- ProdWeight(’P_CutOff2’,i) - ProdWeight(’P_CutOff5’,i)  
- ProdWeight(’P_Sundry1’,i) - ProdWeight(’P_Sundry2’,i)  
- ProdWeight(’P_Sundry5’,i) - ProdWeight(’P_Tenderloin’,i) ;

Parameter QualityDeduction(j,i) Deduction in price at product weight above 3.5 kg per back (7 kg per pig) and requirement for breast and ham ;
Fradrag(’P_Backs (with bones)’,i) = 2$(ProdWeight(’P_Backs (with bones)’,i) gt 7) 
+ 0$(ProdWeight(’P_Backs (with bones)’,i) le 7) ;
Fradrag(’P_Backs (boneless)’,i) = 2$(ProdWeight(’P_Backs (boneless)’,i) gt 7) 
+ 0$(ProdWeight(’P_Backs (boneless)’,i) le 7) ;
Fradrag(’P_Breast2’,i) = 6$(ProdWeight(’P_Breast2’,i) gt 8) + 0$(ProdWeight(’P_Breast2’,i) le 8) ;
Fradrag(’P_Ham’,i) = 4$(FatLayer(i) gt 14) + 0$(FatLayer(i) le 14) ;

Parameter ValueGris(i,n) Price for pig i at alternative use n ;
ValueGris(i,n) = sum(j, (Price(j)-Fradrag(j,i)+PriceCoeff(j)*(FatLayer(i)-15.9)) * ProdWeight(j,i) *  
Anvendelse(j,n)) ;

Parameter ValueStang(l,n) ;
$Include ValueStang_43949.txt

Variables
 z total profit

Binary Variables
 y(l,n) 1 if alternative n is chosen to be produced of pig i with bars;
// y(i,n) 1 if alternative n is chosen to be produced of pig i without bars;

Equations
 profit definering af objekt funktion
con(l) * with bars
// con1(i) * without bars

profit ..   
  z =e= sum((l,n), ValueBar(l,n)*y(l,n)) ;
  // profit ..   
  z =e= sum((i,n), ValuePig(i,n)*y(i,n)) ;

con(l) ..   
  sum((n),y(l,n)) =e= 1 ;
// con1(i) ..   
  sum(n,y(i,n)) =e= 1 ;

Model begraensninger_v11 /all/ ;
Solve begraensninger_v11 using mip maximizing z ;