Conductance enhancement in quantum-point-contact semiconductor-superconductor devices

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I. INTRODUCTION

Charge transport through a normal-conductor–superconductor (NS) interface is accompanied by a conversion of quasiparticle current to a supercurrent. In the Andreev reflection, by which the conversion occurs, an electronlike quasiparticle in the normal conductor (with an excitation energy lower than the energy gap of the superconductor) incident on the NS interface is retroreflected into a hololelike quasiparticle (with reversal of its momentum and its energy relative to the Fermi level) and a Cooper pair is added to the condensate of the superconductor.1 For an ideal NS interface, the signature of Cooper pair transport and the Andreev scattering is a doubling of the conductance compared to the normal state conductance.

A theoretical framework for studies of the scattering at NS interfaces is provided by the Bogoliubov–de Gennes formalism,2 which was studied within the time-dependent Bogoliubov–de Gennes formalism. We find that the factor-of-2 enhancement of the conductance $G_{NS}$ compared to the normal state conductance $G_N$ for ideal interfaces may be suppressed for interfaces with a quantum point contact with only a few propagating modes. The suppression is found to depend strongly on the position of the Fermi level. We also study the suppression due to a barrier, and find an anomalous behavior caused by quasiparticle interference. Finally, we consider the limit of sequential tunneling and find a suppression of the factor-of-2 enhancement which may explain the absence of conductance enhancement in experiments on metal-superconductor structures.

The linear-response conducting properties of QPC 2DEG-S devices have been studied by several groups. In the analytical work of Beenakker13 a ballistic normal region with a QPC modeled by a saddle-point potential was considered. The effect of elastic impurity scattering was considered numerically by Takagaki and Takayanagi14 who considered a disordered region between a narrow-wide (NW) constriction and the superconductor. Both of these studies of the conductance were based on a scattering matrix ($S$-matrix) approach and the BdG formalism.

One of the properties of the QPC is that most transmission eigenvalues are either close to zero or unity. For an ideal QPC, the Andreev reflection will therefore give rise to a factor-of-2 enhancement of the conductance $G_{NS}$ compared to the normal state conductance $G_N$, which is quantized in units of $2e^2/h$.16 However, as pointed
out by van Houten and Beenakker, deviations from the simple factor-of-2 enhancement should be expected when the position of the Fermi level does not correspond to a conductance plateau. The presence of impurity scattering in the normal region and/or interface roughness will also suppress the doubling of the conductance.\textsuperscript{13,17}

Using an S-matrix approach, we study the linear-response regime of a phase-coherent ballistic QPC 2DEG-S system where the QPC is modeled by a WNW constrictions with a hard-wall confining potential, see Fig. 1. We report new results for the device studied by De Raedt et al.,\textsuperscript{15} which had a relative width \( W/W' = 1.7 \) μm/(1.6×335 Å)=31.72, an aspect ratio \( L_1/W' = 5/1.6 \), and a relative length \( L_2/W' = 20/1.6 \). Applying the S-matrix formalism instead of the computationally more complicated time-dependent BdG formalism, we are able to study a larger part of the parameter space where we also consider a barrier (with a normalized barrier strength \( Z \)) at the NS interface. We focus on the regime with only a few propagating modes in the QPC. In this regime the transmission eigenvalues of the QPC depend strongly on the actual position of the Fermi level. Even for an ideal interface this gives rise to a strong suppression of the conductance for certain positions of the Fermi level as predicted by van Houten and Beenakker\textsuperscript{10} and subsequently seen in the work of Beenakker.\textsuperscript{13} In the presence of a barrier at the interface, the QPC gives rise to an enhanced tunneling through the barrier (compared to the case without a QPC) as in the case of reflectionless tunneling effect of diffusive junctions.\textsuperscript{13,17}

In the sequential tunneling limit the conductance can be found by considering the QPC and the interface as two series-connected resistive regions and in the limit \( W \gg W' \) the enhancement of the conductance compared to the normal state conductance vanishes even for ideal NS interfaces. This may be an explanation for the unexpectedly low conductance enhancement in the experimental results of Benistant et al.,\textsuperscript{18} on Ag-Pb interfaces where the current is injected into the Ag crystal through a point contact.

The text is organized as follows. In Sec. II the S-matrix formalism is introduced, in Sec. III we formulate our model, in Sec. IV the scattering scheme of the considered geometry is presented, and in Sec. V we present results of several applications of our scattering scheme. Finally, in Sec. VI discussion and conclusions are given.

II. SCATTERING MATRIX FORMALISM

The scattering approach to coherent dc transport in superconducting hybrids follows closely the scattering theory developed for nonsuperconducting mesoscopic structures, see, e.g., the textbook by Datta.\textsuperscript{19} For an ideal NS interface, the interface acts as a phase-conjugating mirror within the Andreev approximation\textsuperscript{1} and the rigid boundary condition for the pairing potential

\[
\Delta(r) = \Delta_0 e^{i \varphi} \Theta(x - L),
\]

where \( \Delta_0 \) is the BCS energy gap,\textsuperscript{20} \( \varphi \) is the phase of the pairing potential, \( \Theta(x) \) is a Heaviside function, and \( L = L_1 + L_2 \) is the length of the normal region (see Fig. 1).

In the linear-response regime in zero magnetic field, Beenakker\textsuperscript{13} found that the conductance \( G = \partial I / \partial V \) is given by

\[
G_{NS} = \frac{4e^2}{h} \text{Tr}(tt^\dagger[2 \mathbf{1} - tt^\dagger]^{-1})^2,
\]

\[
= \frac{4e^2}{h} \sum_{n=1}^{N} \frac{T_n^2}{(2 - T_n)^2},
\]

which, in contrast to the Landauer formula,\textsuperscript{21}

\[
G_N = \frac{2e^2}{h} \text{Tr} tt^\dagger = \frac{2e^2}{h} \sum_{n=1}^{N} T_n,
\]

is a nonlinear function of the transmission eigenvalues \( T_n \) \((n = 1, 2, \ldots, N)\) of \( tt^\dagger \). Here \( t \) is the \( N \times N \) transmission matrix of the normal region, \( N \) being the number of propagating modes.

Equation (3) holds for an arbitrary disorder potential and is a multichannel generalization of a conductance formula first obtained by Blonder, Tinkham, and Klapwijk,\textsuperscript{22} who considered a delta function potential as a model for the interface barrier potential. The computational advantage of Eq. (3) over the time-dependent BdG approach of De Raedt et al.\textsuperscript{15} is that we only need to consider the time-independent Schrödinger equation with a potential which describes the disorder in the normal region, so that we can use the techniques developed for quantum transport in normal conducting mesoscopic structures. For more details on, e.g., finite bias and/or temperature, see Lesovik, Fauchère, and Blatter,\textsuperscript{23} Lesovik and Blatter,\textsuperscript{24} and the reviews of Beenakker,\textsuperscript{6} and Lambert and Raimondi.\textsuperscript{7}

III. MODEL

We describe the geometry of Fig. 1 by the Hamiltonian

\[
\hat{H}(\mathbf{r}) = - \frac{\hbar^2}{2m} \nabla + V_\beta(\mathbf{r}) + V_\gamma(\mathbf{r}) - \mu,
\]

where \( \mu \) is the chemical potential. The barrier potential is given by a Dirac delta function potential with strength \( H \),\textsuperscript{22}

\[
V_\beta(\mathbf{r}) = H \delta(x - L),
\]

and the transverse motion is limited by a hard-wall confining potential.
FIG. 2. Scattering scheme appropriate for the normal region of the geometry shown in Fig. 1.

\[ V_{\ell}(r) = \begin{cases} 0, & |y| < \mathcal{W}(x)/2, \\ \infty, & |y| \geq \mathcal{W}(x)/2, \end{cases} \]

where the width \( \mathcal{W}(x) \) defines the WNW constriction and is given by

\[ \mathcal{W}(x) = \begin{cases} W, & x < 0, \\ W', & 0 \leq x \leq L_1, \\ W, & x > L_1. \end{cases} \]

The scattering states can be constructed as linear combinations of the eigenstates of the Schrödinger equation.

IV. SCATTERING SCHEME

In the following subsections, we consider the \( S \)-matrices of a system with the Hamiltonian in Eq. (5) relevant for the geometry shown in Fig. 1. The \( S \)-matrix \( S \) of a scattering region relates the incident current amplitudes \( a_{\delta} \) to the outgoing current amplitudes \( b_{\delta} \). For a scattering region with two leads, \( S \) is a 2x2 block-matrix with submatrices \( S_{11}, S_{12}, S_{21}, \) and \( S_{22} \), where the diagonal and off-diagonal submatrices are reflection and transmission matrices, respectively. The appropriate scattering scheme for three scattering regions (the WN and the NW constrictions, and the interface barrier potential, respectively) connected by ballistic conductors is shown in Fig. 2.

The WN constriction is described by the \( S \)-matrix \( S^{WN} \)

\[ \begin{pmatrix} a^+_{S_1} \\ a^-_{S_1} \\ b^+_{S_1} \\ b^-_{S_1} \\ a^+_{S_2} \\ a^-_{S_2} \\ b^+_{S_2} \\ b^-_{S_2} \\ a^+_{S_3} \\ a^-_{S_3} \\ b^+_{S_3} \\ b^-_{S_3} \end{pmatrix} = S^{WN} \begin{pmatrix} a^+_{S_1} \\ a^-_{S_1} \\ b^+_{S_1} \\ b^-_{S_1} \end{pmatrix} , \tag{9} \]

the narrow region of length \( L_1 \) by the propagation-matrix \( U^N \)

\[ \begin{pmatrix} a^+_{S_1} \\ a^-_{S_1} \\ b^+_{S_2} \\ b^-_{S_2} \end{pmatrix} = U^N \begin{pmatrix} a^+_{S_1} \\ a^-_{S_1} \\ b^+_{S_2} \\ b^-_{S_2} \end{pmatrix} , \tag{10} \]

the NW constriction by the \( S \)-matrix \( S^{NW} \)

\[ \begin{pmatrix} b^+_{S_1} \\ b^-_{S_1} \\ a^+_{S_2} \\ a^-_{S_2} \end{pmatrix} = S^{NW} \begin{pmatrix} a^+_{S_1} \\ a^-_{S_1} \\ b^+_{S_2} \\ b^-_{S_2} \end{pmatrix} , \tag{11} \]

the wide region of length \( L_2 \) by the propagation-matrix \( U^W \)

\[ \begin{pmatrix} a^+_{S_2} \\ a^-_{S_2} \end{pmatrix} = U^W \begin{pmatrix} b^+_{S_1} \\ b^-_{S_1} \end{pmatrix} , \tag{12} \]

and the delta function barrier by the \( S \)-matrix \( S^\delta \)

\[ \begin{pmatrix} b^+_{S_3} \\ b^-_{S_3} \end{pmatrix} = S^\delta \begin{pmatrix} a^+_{S_3} \\ a^-_{S_3} \end{pmatrix} . \tag{13} \]

To apply Eqs. (3) and (4) we need to calculate the composite transmission matrix \( t = S_{21} \) which is a submatrix of the composite \( S \)-matrix \( S = S^{WN} \otimes U^N \otimes S^{NW} \otimes U^W \otimes S^\delta \) relating the outgoing current amplitudes to the incoming current amplitudes

\[ \begin{pmatrix} b^+_{S_1} \\ b^-_{S_1} \end{pmatrix} = S \begin{pmatrix} a^+_{S_1} \\ a^-_{S_1} \end{pmatrix} = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix} . \tag{14} \]

The meaning of the symbol \( \otimes \) is found by eliminating the internal current amplitudes.\(^{19}\) As a final result we find the transmission matrix

\[ t = S_{21} \left[ I - U_{21}^W S_{21}^{NW} [ I - U_{21}^N S_{21}^{WN} U_{21}^N S_{21}^{NW} ]^{-1} U_{21}^N S_{21}^{NW} U_{21}^N S_{12}^{NW} ] U_{21}^N S_{11} \right]^{-1} U_{21}^W S_{21}^{WN} . \tag{15} \]

A. Quantum point contact

We consider a QPC which we model by a WNW constriction defined by a hard-wall confining potential, see Fig. 1. This geometry has been considered by Szafer and Stone\(^ {25} \) and Weisshaar, Lary, Goodnick, and Tripathi\(^ {26} \) in the context of conductance quantization of the QPC in a 2DEG, and recently by Kassubek, Stafford, and Grabert\(^ {27} \) in the context of conducting and mechanical properties of ideal two and three-dimensional metallic nanowires. We follow Kassubek \( et al. \)^{27} and calculate the composite \( S \)-matrix \( S^{WN} = S^{WN} \otimes U^N \otimes S^{NW} \). In zero magnetic field, where all \( S \)-matrices satisfy \( S = S^T \), the individual \( S \)-matrices are given by

\[ S^{WN} = \begin{pmatrix} r_{WN} & t_{WN} \\ t_{WN} & r_{WN} \end{pmatrix} , \tag{16} \]

\[ U^N = \begin{pmatrix} \delta & X^N \\ X^N & \delta \end{pmatrix} , \tag{17} \]

\[ S^{NW} = \begin{pmatrix} r_{NW} & t_{NW}^T \\ t_{NW} & r_{NW} \end{pmatrix} , \tag{18} \]

where \( X^N_{nn'} = \delta_{nn'} \exp (ik_nL_1) \) describes the narrow region with free propagation of propagating modes and an exponen-
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where the normalized barrier strength is given by $Z = k_F W'/\pi$. The conformation has an aspect ratio $L_1/W' = 1$ and a relative width $W/W' = 31.72$. The full lines are for a numerical calculation of $t_{\text{NNW}}$ and the dotted lines are results within the mean-field approximation of Szafer and Stone (Ref. 22). The upper inset shows the normalized conductance $g = G_{\text{NS}} / G_N$ also as function of $k_F W'/\pi$.

C. Interface barrier potential

We consider an NS interface of width $W$ with a barrier which we model by a Dirac delta function potential, following Blonder, Tinkham, and Klapiwijk. The $S$-matrix elements for the delta-function potential is found from a matching of scattering states which are eigenstates of the Schrödinger equation with the Hamiltonian in Eq. (5) where we only consider the part of the potential consisting of the barrier at the interface. In zero magnetic field one finds the symmetric result

$$S = \begin{pmatrix} r_\delta & t_\delta \\ t_\delta^* & r_\delta^* \end{pmatrix},$$  

with

$$t_{\delta}^{NW} = \frac{1}{1 + iZ/\cos \theta_w},$$  

$$r_{\delta}^{NW} = \frac{-iZ/\cos \theta_w}{1 + iZ/\cos \theta_w},$$

where the normalized barrier strength is given by $Z = \hbar / \sqrt{\varepsilon}$, and $\cos \theta_w = K_w / k_F = \sqrt{1 - (w \pi/k_F W)^2}$. The results differ from those of a one-dimensional calculation since we have taken the parallel degree of freedom into account. However, if we introduce an angle dependent effective barrier strength $Z_{\text{eff}}(\theta_w) = Z / \cos \theta_w$, the transmission and reflection amplitudes can formally be written in the onedimensional form of Ref. 22. The transmission eigenvalues of $t_{\delta,\delta}^{NW}$ are given by $T_{\delta}^{NW} = [1 + Z_{\text{eff}}^2(\theta_w)]^{-1}$ in contrast to the mode-independent result $T_{\delta} = (1 + Z^2)^{-1}$ of a one-dimensional calculation.

V. RESULTS

A. Phase-coherent junction with ideal interface

For the case of coherent transport through an ideal 2DEG-S interface with a WNW constriction in the normal region, see lower inset of Fig. 3, the conductance $G_{\text{NS}}$ and the normal state conductance $G_N$ can be found from Eqs. (3) and (4) with the transmission matrix $t = t_{\text{NNW}}$. Fig. 3 shows the conductance as a function of $k_F W'/\pi$ based on a numerical calculation of $t_{\text{NNW}}$ (full lines) and the mean-field approximation (dashed line) for a WNW constriction with an aspect ratio $L_1/W' = 1$ and a relative width $W/W' = 31.72$. The conductance $G_{\text{NS}}$ is seen to be approximately quantized in units of $4e^2/h$ which is twice the unit of conductance for the normal state conductance $G_N$. However, just above the thresholds $(k_F W'/\pi = 1, 2, 3, \ldots)$, oscillations due to resonances in the narrow region of the constriction are observed. In the normal state result, these resonances are small but in contrast to the Landauer formula, $G_{\text{NS}}$ is not linear in the transmission eigenvalues and this makes the resonances much more pronounced compared to those in the normal state conductance. Another signature of the nonlinearity of $G_{\text{NS}}$ and the importance of off-diagonal transmission, is that the mean-field approximation is in good agreement with the numerical calculation for $G_N$ whereas it has difficulties in accounting for $G_{\text{NS}}$. The sharpness of the resonances is to a certain extent due to the wide-narrow-wide constriction, and is suppressed in experiments with split-gate-defined constrictions. However, as shown by the simulations of Maao, Zozenenko, and Hauge resonance effects do persist even for smoother connections of the narrow region to the 2DEG reservoirs.

The normalized conductance $g = G_{\text{NS}} / G_N$, shown in the upper insert, is two on the conductance plateaus but for certain “mode fillings” of the constriction it is strongly suppressed and for $k_F W'/\pi = \pi - 2$ (two propagating modes) we get $g \sim 1.5$. This effect, which occurs at the onset of new modes, was also seen in the calculations of Beenakker. As the number of modes increases, these dips vanish and the normalized conductance approaches its ideal value of 2. The reason is simple: suppose the constriction has $N$ propagating modes, then the $N - 1$ of them will have a transmission of order unity and only a single mode (corresponding to the mode with the highest transverse energy) will have transmission different from unity. As $N$ increases, the effect of the single mode with transmission different from unity becomes negligible for the normalized conductance and from Eqs. (3) and (4) it follows that $\lim_{N \to \infty} g = 2$.

Since the quasiparticle propagation is coherent and the Andreev scattering is the only back-scattering mechanism, the phase conjugation between electronlike and holelike quasiparticles makes the conductance $G_{\text{NS}}$ independent of the separation $L_2$ of the constriction and the interface. If evanescent modes in this region were also taken into account the results would depend weakly on $L_2$ as it was found in the simulations of De Raedt et al. and our results should be compared with their results in the large $L_2$ limit. As we shall see below, interfaces with a finite barrier (and thereby nor-
normal scattering at the interface) lead to size quantization and thereby resonances which will depend on $L_2$.

The back-focusing phenomenon of the Andreev reflected quasiparticles and the lowering of the normalized conductance due to a QPC in the normal region was studied by De Raedt et al.\cite{4} for the device considered by De Raedt et al.\cite{15} For the position of the Fermi level (○) considered by De Raedt et al.,\cite{15} the normalized conductance is only weakly suppressed (compared to a system without a QPC, see, e.g., Ref. 29 for low barrier scattering ($Z<1$) and only for a very high barrier strength ($Z>2$) the normalized conductance approaches the crossover from an excess conductance ($g>1$) to a deficit conductance ($g<1$). The effect of the barrier for $Z<1$ is very similar to the reflectionless tunneling behavior in diffusively disordered junctions\cite{13,17} where the net result is as if tunneling through the barrier is reflectionless. In the case of a QPC instead of a diffusive region there is a weak dependence on the barrier strength and the tunneling is not perfectly reflectionless.

An interesting feature is the nonmonotonic behavior of $g$ as a function of $Z$. For $Z \rightarrow \infty$, the normalized conductance of course vanishes, but in some regions it increases with an increasing barrier strength [curve (○)] and for $Z \approx 1$ it has the same value for as for $Z=0$. This is purely an effect of size quantization in the cavity between the QPC and the barrier which enters the conducting properties because of the fully coherent propagation of electrons and holes. However, changing, e.g., the position of the Fermi level slightly [curves (△) and (□)], changes the quantitative behavior although the overall suppression of $g$ with increasing $Z$ is maintained.

\section*{B. Phase-coherent junction with barrier at interface}

We next consider coherent transport through an NS interface with a barrier at the interface and a WNW constriction at a distance $L_2$ from the interface, see lower right inset of Fig. 5. The conductance $G_{NS}$ and the normal state conductance $G_N$ are found from Eqs. (3) and (4) with the transmission matrix in Eq. (15).

In Fig. 5 we present a calculation of the normalized conductance $g$ as a function of the normalized barrier strength $Z$ for the device considered by De Raedt et al.\cite{15} For the position of the Fermi level (○) considered by De Raedt et al.,\cite{15} the normalized conductance is only weakly suppressed (compared to a system without a QPC, see, e.g., Ref. 29) for low barrier scattering ($Z<1$) and only for a very high barrier strength ($Z>2$) the normalized conductance approaches the crossover from an excess conductance ($g>1$) to a deficit conductance ($g<1$). The effect of the barrier for $Z<1$ is very similar to the reflectionless tunneling behavior in diffusively disordered junctions\cite{13,17} where the net result is as if tunneling through the barrier is reflectionless. In the case of a QPC instead of a diffusive region there is a weak dependence on the barrier strength and the tunneling is not perfectly reflectionless.

\section*{C. Incoherent junction}

In junctions where the propagation in the cavity between the QPC and the NS interface is incoherent, the so-called
sequential tunneling regime, the QPC and the NS interface can be considered as two series-connected resistive regions.\(^{31}\) This means that

\[
G_{\text{QPC-NS}} = (G_{\text{QPC}}^{-1} + G_{\text{NS}}^{-1})^{-1},
\]

where \(G_{\text{QPC}}\) and \(G_{\text{NS}}\) are found from Eq. (4) with \(t = t_{\text{QPC}}\) and \(t = t_{\delta}\), respectively, and \(G_{\text{NS}}\) from Eq. (3) with \(t = t_{\delta}\). The normalized conductance can be written as

\[
g = \frac{G_{\text{QPC-NS}}}{G_{\text{QPC}}} = \frac{G_{\text{NS}}}{G_{\text{N}}} \times \frac{G_{\text{QPC}} + G_{\text{NS}}}{G_{\text{QPC}}^{-1} + G_{\text{NS}}^{-1}},
\]

and for \(W \gg W'\) the major contribution to the resistance comes from the QPC, i.e., \(G_{\text{QPC}} \ll (G_{\text{N}}, G_{\text{NS}})\). This means that the enhancement of \(G_{\text{NS}}\) compared to \(G_{\text{N}}\) has a negligible effect on the total conductance so that the normalized conductance approaches \(g \sim 1\).

For an ideal QPC and an ideal interface we have \(G_{\text{QPC}} = (2e^2/h)N\), \(G_{\text{NS}} = (4e^2/h)M\), and \(G_{\text{N}} = (2e^2/h)M\), where \(N\) is the number of modes in the QPC and \(M\) is the number of modes at the NS interface. The corresponding normalized conductance is shown in Fig. 6.

The sequential tunneling behavior may provide an explanation for the unexpectedly small conductance enhancement seen in the experiments of Benistant et al.\(^ {18}\) on Ag-Pb interfaces with injection of quasiparticles into an Ag crystal through a point contact. The condition for the electronic transport to be incoherent is that the distance between the point contact and the NS interface is longer than the correlation length \(L_c = \min(L_T, L_{\text{th}})\), \(L_T\) being the inelastic scattering length and \(L_{\text{th}}\) the Thouless length.\(^ {4,6}\) For the ballistic device studied by Benistant et al. \(L_c = L_T = h\nu_e/k_BT \sim 9\ \mu\text{m}\) (at \(T = 1.2\ \text{K}\)) which is much shorter than the distance between the point contact and the NS interface \((\sim 200\ \mu\text{m})\). Lowering the temperature will increase the correlation length and for sufficiently low temperatures \((T \sim 0.05\ \text{K})\) we expect a crossover from the sequential tunneling regime to the phase-coherent regime where the Andreev mediated conductance enhancement should become observable.

**VI. DISCUSSION AND CONCLUSION**

For an ideal 2DEG-S interface with a QPC in the normal region, the normalized conductance \(g = G_{\text{NS}}/G_{\text{N}}\) depends strongly on the position of the Fermi level and only when the Fermi level corresponds to a conductance plateau a doubling of the conductance is found. The deviations from the factor-of-2 enhancement, when the Fermi level does not correspond to a plateau, can be significant and for a particular example of the WNW constriction we find that the normalized conductance can be suppressed to \(g \sim 1.5\) in a system with only two propagating modes in the constriction. In the presence of a barrier at the 2DEG-S interface, the normalized conductance depends strongly on the longitudinal quantization in the cavity set up by the QPC and the barrier. Depending on the barrier strength, the length of the cavity and the position of the Fermi level, this longitudinal quantization may give rise to both constructive and destructive inferences in the transmission and thus also in the conductance. Perhaps surprisingly, the effect of the barrier is very much suppressed (compared to a system without a QPC, see, e.g., Ref. 29) due to a very strong back scattering at the return of the quasiparticles to the normal probe. The localization of quasiparticles in the cavity gives rise to an almost reflectionless tunneling through the barrier as it is also found in systems with a diffusive normal region.\(^ {13,17}\) The interferences due to localization in the cavity will be smeared by a finite temperature and they are also expected to be suppressed by a finite inelastic scattering length compared to the length of the cavity.\(^ {32}\)

For the sequential tunneling regime we find that the conductance enhancement vanishes as the number of modes at the interface becomes much larger than the number of modes in the QPC. Our calculations show that the S-matrix approach provides a powerful alternative to the time-dependent Bogoliubov–de Gennes approach of De Raedt et al.\(^ {15}\) in detailed studies of the conducting properties of nanoscale 2DEG-S devices. Even though the back-focusing phenomenon of the Andreev reflection is robust against changes in the geometry,\(^ {15}\) the electron-hole conversion efficiency itself is not. Finally, we stress that for a quantitative comparison to experimental systems, it is crucial to take different Fermi wave vectors and effective masses of the 2DEG and the superconductor into account.\(^ {29}\)

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32. For a recent approach including possible interactions, see, e.g., Q. Sun, J. Wang, and T. Lin, Phys. Rev. B 59, 3831 (1999).