Slow light in quantum dot photonic crystal waveguides

Nielsen, Torben Roland; Lavrinenko, Andrei; Mørk, Jesper

Published in:
Applied Physics Letters

Link to article, DOI:
10.1063/1.3103286

Publication date:
2009

Document Version
Publisher's PDF, also known as Version of record

Link back to DTU Orbit

Citation (APA):
Slow light in quantum dot photonic crystal waveguides

Torben Roland Nielsen, Andrei Lavrinenko, and Jesper Mørk

Department of Photonics Engineering, DTU Fotonik, Technical University of Denmark, Orsted Plads 343, DK-2800 Kgs. Lyngby, Denmark

(Received 5 February 2009; accepted 27 February 2009; published online 18 March 2009)

A theoretical analysis of pulse propagation in a semiconductor quantum dot photonic crystal waveguide in the regime of electromagnetically induced transparency is presented. The slow light mechanism considered here is based on both material and waveguide dispersion. The group index $n_g$ for the combined system is significantly enhanced relative to slow light based on purely material or waveguide dispersion. © 2009 American Institute of Physics. [DOI: 10.1063/1.3103286]

Since the early experimental reports on slowing down the speed of light beams in atomic vapors, the study of slow light (SL) phenomena based on actively changing the material dispersion has become a topic of growing interest. Much of this attention is due to potential applications, e.g., as an optical buffer or a phase shifter, applications which are feasible if an efficient SL mechanism can be realized in a compact semiconductor waveguide. For a typical ridge waveguide (RWG) configuration, SL in quantum dot (QD) structures has theoretically been investigated in the regime of electromagnetically induced transparency (EIT). By applying an externally controlled light source, a variable SL buffer can in principle be realized. Very recently, experimental studies of EIT in QD systems have been performed, where a coherent absorption dip in a pump-probe experiment has been observed for an optically thin structure. SL based on waveguide dispersion such as in photonic crystal (PhC) waveguides has lately been intensely studied and recent theoretical and experimental studies have shown that group indices higher than 230/40 can be obtained for a narrow/large bandwidth. Since the dispersion properties of slab PhC waveguides are predetermined by the PhC structure and the bulk material properties, variable slow down factors cannot easily be obtained for fixed frequencies. However, introducing optical nonlinearities into the PhC are predicted to have significant technological impact on signal processing. SL QD medium, thereby treating the electromagnetic fields and the microscopic polarization on an equal footing. The polarization density which enters Ampere’s law is evaluated from the expectation value of the macroscopic polarization via the density matrix. The numerical implementation follows the method outlined in Ref. 10.

For the numerical results presented in this paper we consider a homogenous InGaAs QD system with parameters similar to Ref. 2. The inhomogeneity of the QD ensemble is neglected for illustration purposes. The electron-hole ground state transition wavelength is $\lambda_{31} = 1.38 \mu m$ while the electron-electron transition wavelength is $\lambda_{32} = 12.8 \mu m$. The corresponding dipole moments are $\mu_{21} = 0.68 e nm$ and $\mu_{32} = 2.46 e nm$. For simplicity we use the same dephasing rate $\gamma = 80 \mu e V$ for all QD coherences. The optical con-

![Diagram](https://example.com/diagram.png)

**FIG. 1.** (Color online) Left: Schematic of the semiconductor QD PhC waveguide system. The QDs are located in the waveguide as indicated by the dark yellow shading. Right: Level scheme of the QDs used for obtaining EIT in a ladder configuration. A weak probe beam is applied between the electron-hole ground state transition $(|2\rangle \leftrightarrow |1\rangle)$, while a strong pump beam is resonant with two electronic states $(|3\rangle \leftrightarrow |2\rangle)$.
The group index is then determined by a time of flight method\(^5\) through the waveguide, with only a small temporal widening of the pulse. The pulse is seen to propagate fairly undistorted through a passive PhC waveguide, with only a small temporal widening of the pulse. Thus in this regime a time delay between the input and output pulses can be attributed and hence a group index can be extracted. Furthermore, a drop in the transmittance is also observed. The actual total delay and the reduction of the transmittance depend on the length of the PhC waveguide, giving larger delay and lower transmittance for longer waveguides. Moreover, compared to the RWG EIT system we observed an enhanced drop in the transmission. A detailed analysis of the absorption properties is however beyond the scope of this letter, but a similar effect has recently been reported in Ref. 13. Short probe pulses on the other hand, will in general, experience strong dispersion and the probe pulse is broadened or even broken up\(^6,14,15\). In this regime the dynamics becomes nonlinear and a clear identification of, e.g., slow-down factors becomes impossible [see Fig. 2(b)].

In Fig. 3 we plot the group index for pulses propagating through a passive PhC waveguide, RWG embedded with QDs, and the QD PhC waveguide system as a function of the applied pump power density of the control beam \(b_{p} = \text{c} r_{P} \left| \mathbf{E}_{c} \right|^{2}\). The probe pulse duration is fixed to \(\tau_{p} = 10\) ps. The passive PhC waveguide group index is of course independent of the pump power density. The group index for the RWG EIT system is determined from the analytic model\(^5\) and decreases with increasing pump power density and reaches a plateau equal to the back ground index \(n_{b}\) at high densities. As the pump power density is lowered the group index increases dramatically, as also discussed in Refs. 2, 6,14,15. In this regime the dynamics becomes nonlinear and a clear identification of, e.g., slow-down factors becomes impossible [see Fig. 2(b)].

In Fig. 3 we plot the group index for pulses propagating through a passive PhC waveguide, RWG embedded with QDs, and the QD PhC waveguide system as a function of the applied pump power density of the control beam \(b_{p} = \text{c} r_{P} \left| \mathbf{E}_{c} \right|^{2}\). The probe pulse duration is fixed to \(\tau_{p} = 10\) ps. The passive PhC waveguide group index is of course independent of the pump power density. The group index for the RWG EIT system is determined from the analytic model\(^5\) and decreases with increasing pump power density and reaches a plateau equal to the back ground index \(n_{b}\) at high densities. As the pump power density is lowered the group index increases dramatically, as also discussed in Refs. 2, 6,14,15. In this regime the dynamics becomes nonlinear and a clear identification of, e.g., slow-down factors becomes impossible [see Fig. 2(b)].

In Fig. 3 we plot the group index for pulses propagating through a passive PhC waveguide, RWG embedded with QDs, and the QD PhC waveguide system as a function of the applied pump power density of the control beam \(b_{p} = \text{c} r_{P} \left| \mathbf{E}_{c} \right|^{2}\). The probe pulse duration is fixed to \(\tau_{p} = 10\) ps. The passive PhC waveguide group index is of course independent of the pump power density. The group index for the RWG EIT system is determined from the analytic model\(^5\) and decreases with increasing pump power density and reaches a plateau equal to the back ground index \(n_{b}\) at high densities. As the pump power density is lowered the group index increases dramatically, as also discussed in Refs. 2, 6,14,15. In this regime the dynamics becomes nonlinear and a clear identification of, e.g., slow-down factors becomes impossible [see Fig. 2(b)].

In Fig. 3 we plot the group index for pulses propagating through a passive PhC waveguide, RWG embedded with QDs, and the QD PhC waveguide system as a function of the applied pump power density of the control beam \(b_{p} = \text{c} r_{P} \left| \mathbf{E}_{c} \right|^{2}\). The probe pulse duration is fixed to \(\tau_{p} = 10\) ps. The passive PhC waveguide group index is of course independent of the pump power density. The group index for the RWG EIT system is determined from the analytic model\(^5\) and decreases with increasing pump power density and reaches a plateau equal to the back ground index \(n_{b}\) at high densities. As the pump power density is lowered the group index increases dramatically, as also discussed in Refs. 2, 6,14,15. In this regime the dynamics becomes nonlinear and a clear identification of, e.g., slow-down factors becomes impossible [see Fig. 2(b)].
with linear constitutive material relations. The harmonic magnetic eigenmodes are the solutions to the equation \( \omega^2/\epsilon = \langle H, \Theta H \rangle/(H, H) \) where \( \Theta = \nabla \times \epsilon(r, \omega)^{-1} \nabla \times \). Solving this equation must be performed in a self-consistent way if the dielectric function \( \epsilon \) is frequency dependent. For a propagating mode with propagation constant \( \beta \) the group velocity is then evaluated as \( v_g = \omega \partial / \partial \beta \). The total derivative of the eigenvalue \( \omega \) with respect to \( \beta \) may be evaluated as \( 2\omega/c^2 v_g = \frac{d}{d\beta} \langle \hat{H}, \hat{\Theta} \hat{H} \rangle/(\hat{H}, \hat{H}) = \frac{\partial}{\partial \beta} \langle \hat{H}, \hat{\Theta} \hat{H} \rangle + v_g \frac{\partial \epsilon}{\partial \omega} \frac{\partial}{\partial \omega} \langle \hat{H}, \hat{\Theta} \hat{H} \rangle \).

For low-loss dielectrics, with negligible imaginary dielectric function, the group velocity then reads \( 2v_g = \omega \partial \langle \hat{H}, \hat{\Theta} \hat{H} \rangle/(\hat{H}, \hat{H}) \), where \( v_g^{\text{PhC}} \) is group velocity for the PhC waveguide without any material dispersion, and \( E_d = \langle \hat{D}, \hat{E} \rangle_{QD}/\langle \hat{D}, \hat{E} \rangle \) is the fraction of the electric energy inside the dielectric having a frequency dependent dispersion. Using \( \epsilon = n^2 \), the partial derivative of \( \ln(\epsilon) \) may be expressed in terms of the dispersive group index, and the total group index can be written as

\[
\frac{\partial \ln(\epsilon)}{\partial \omega} = \frac{1}{n_b^2} \frac{\partial E_d}{\partial \omega} = \frac{1}{n_b^2} \frac{\partial}{\partial \omega} \frac{\partial}{\partial \epsilon} \langle \hat{H}, \hat{\Theta} \hat{H} \rangle.
\]

This result shows that the group index has a contribution that scales linearly with the product of \( n_b^{\text{PhC}} \) and \( n_b^{\text{EIT}} \). The total group index in a system in which both material and waveguide dispersion may then be evaluated from the passive PhC waveguide group index \( n_b^{\text{PhC}} \), filling factor \( E_d \), and from the QD ensemble group index \( n_b^{\text{EIT}} \) alone. In Fig. 3 we have also plotted the prediction of Eq. (1). The electric energy in side the QD region is evaluated from the numerical simulations as follows: \( E_d = \int d\hat{r} \langle \hat{D}, \hat{E} \rangle_{QD} / \int d\hat{r} \langle \hat{D}, \hat{E} \rangle \). From our calculations we obtain \( E_d = 0.535 \) and \( n_b^{\text{PhC}} = 38.5 \). We observed good agreement between numerical simulations and theory.

In conclusion, we have investigated light slow-down in a PhC structure including QDs. The slow-down factor may be significantly enhanced compared to systems relying only on material or waveguide dispersion. This could be important in achieving efficient and variable control over pulse propagation in compact semiconductor waveguides.

This work has been partially financed by the European Commission through the IST Project No. 29283, QPhoton, the project QUEST financed by the Danish Research Councils, and the project NATEC financed by the VILLUM KANN RASMUSSEN FOUNDATION. The authors also acknowledge access to the Danish Center for Scientific Computing (DCSC) at the Technical University of Denmark under Grant No. DCSC CPU-0107-12.

11. Our numerical implementation is equivalent to Eqs. (1)–(4b) of Ref. 10 with the obvious changes needed in describing a semiconductor system. Notice that the Eq. (4a) of Ref. 10 is the equivalent of Eq. (1) in Ref. 2.