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Published in:
Applied Physics Letters

Link to article, DOI:
10.1063/1.3103286

Publication date:
2009

Document Version
Publisher's PDF, also known as Version of record

Link back to DTU Orbit

Citation (APA):
Slow light in quantum dot photonic crystal waveguides

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(Received 5 February 2009; accepted 27 February 2009; published online 18 March 2009)

A theoretical analysis of pulse propagation in a semiconductor quantum dot photonic crystal waveguide in the regime of electromagnetically induced transparency is presented. The slow light mechanism considered here is based on both material and waveguide dispersion. The group index $n_g$ for the combined system is significantly enhanced relative to slow light based on purely material dispersion and the QD EIT medium. In the following we will just focus on the three QD levels coupled by the optical fields, and neglect any interaction with other energetically close (confined QD or delocalized) states, in which case the QD system can be modeled as a simple three level system. The transition between $|3\rangle$ and $|1\rangle$ is dipole disallowed, while the two remaining transitions are dipole allowed. A full vectorial analysis of the electromagnetic fields is needed in order to give a proper description of pulse propagation in these strongly diffractive PhC structures. Thus, we use a semiclassical description of the light-matter interaction by combining the finite-difference-time-domain (FDTD) (Ref. 9) and the density matrix methods for studying pulse propagation in a SL QD medium, thereby treating the electromagnetic fields and the microscopic polarization on an equal footing. The light-matter interaction is treated within the dipole approximation. When implemented on a FDTD grid, the Liouville equation is solved numerically at each grid point for the QD carrier dynamics without any further approximations. The polarization density which enters Ampere’s law is evaluated from the expectation value of the macroscopic polarization via the density matrix. The numerical implementation follows the method outlined in Ref. 10.

For the numerical results presented in this paper we consider a homogenous InGaAs QD system with parameters similar to Ref. 2. The inhomogeneity of the QD ensemble is neglected for illustration purposes. The electron-hole ground state transition wavelength is $\lambda_{31} = 1.38 \mu m$ while the electron-electron transition wavelength is $\lambda_{32} = 12.8 \mu m$. The corresponding dipole moments are $\mu_{31} = 0.68 e \text{ nm}$ and $\mu_{32} = 2.46 e \text{ nm}$. For simplicity we use the same dephasing rate $\gamma = 80 \mu eV$ for all QD coherences. The optical con-
The group index is then determined by a time of flight method and output pulses can be attributed and hence a group index through the waveguide, with only a small temporal widening waveguide. The pulse is seen to propagate fairly undistorted total PhC waveguide length is approximately 22 μm (75 periods). The refractive background index reads $n_g = 3.5$.

To study the group index $n_g$ for the combined system based on both material and waveguide dispersion, we compare three different scenarios: (1) Pulse propagation in a passive PhC WG waveguide structure, (2) a RWG with QDs excited in the ladder configuration, and (3) final pulse propagation in the active QD PhC WG waveguide structure. Throughout, we describe the QD ensemble as an effective medium smeared out over the waveguide region, thus implying the same QD properties at each FDTD lattice site. The three level ladder QD system depicted in Fig. 1 is driven with the probe beam which is resonant with electron-hole ground state transition and polarized along the $\hat{y}$ axis. The continuous wave control beam is resonant with the electron-electron transition, polarized along the $\hat{z}$ axis and propagates in the direction perpendicular to the PhC membrane. Throughout the simulations presented here, we assume that the control beam Rabi energy $\hbar \Omega_c = \mu E_c/2$ is much larger than the probe Rabi energy $\hbar \Omega_p = \mu E_p/2$, although this is not a limitation of the numerical scheme. The incoming probe beam has a temporal Gaussian envelope with full width half maximum (FWHM) $\tau_p$ and a peak amplitude $E_p$. For the numerical results related to the group index, we consider probe pulses whose spectral width is well located within the two EIT absorption peaks, which arise due to the dressing of the system by the control beam, such that $\tau_p = 10 \, \text{ps}$. The group index is then determined by a time of flight method giving the delay time between the peak of the pulse just before the entrance and just after the exit of the PhC WG. In order to remove the effects of reflection from the PhC waveguide, the pulse peak time at the entrance is determined by first performing a simulation with a simple straight waveguide without any PhC present. Figure 2(a) shows such a typical example of the $E_y$ field component at the input and output ports for a pulse propagating through the QD PhC waveguide. The pulse is seen to propagate fairly undistorted through the waveguide, with only a small temporal widening of the pulse. Thus, in this regime a time delay between the input and output pulses can be attributed and hence a group index can be extracted. Furthermore, a drop in the transmittance is also observed. The actual total delay and the reduction of the transmittance depend on the length of the PhC waveguide, giving larger delay and lower transmittance for longer waveguides. Moreover, compared to the RWG EIT system we observed an enhanced drop in the transmittance. A detailed analysis of the absorption properties is however beyond the scope of this letter, but a similar effect has recently been reported in Ref. 13. Short probe pulses on the other hand, will in general, experience strong dispersion and the probe pulse is broadened or even broken up. In this regime the dynamic becomes nonlinear and a clear identification of, e.g., slow-down factors becomes impossible [see Fig. 2(b)].

In Fig. 3 we plot the group index for pulses propagating through a passive PhC waveguide, RWG embedded with QDs, and the QD PhC waveguide system as a function of the applied pump power density of the control beam $I_p = \pi \rho_{bac} |E_c|^2$. The probe pulse duration is fixed to $\tau_p = 10 \, \text{ps}$. The passive PhC waveguide group index is of course independent of the pump power density. The group index for the RWG EIT system is determined from the analytic model and decreases with increasing pump power density and reaches a plateau equal to the back ground index $n_b$ at high densities. As the pump power density is lowered the group index increases dramatically, as also discussed in Refs. 2. The group index for the combined system, which explores both material and waveguide dispersion, increases as the pump power is decreased, thus showing the same dependency as the simple RWG EIT system. It is observed, however, that the group index is increased dramatically for the combined system compared to the individual subsystems. Notice that a small change in the pump power density gives a large change in group index for the combined system compared to the RWG EIT system. Thus, by combining both material and waveguide dispersion it is possible to enhance the group index as well as obtaining a variable and controllable group index in an ultrasmall waveguide structure.

It is a complex task to determine the group index analytically in such systems as in general the material dispersion will influence the waveguide dispersion properties. However, the group velocity in a PhC waveguide structure can be estimated as follows for an ideal infinitely extended system
with linear constitutive material relations. The harmonic magnetic eigenmodes are the solutions to the equation \(\omega^2/c^2 = \langle H, \nabla \times \varepsilon(r,\omega) \times \nabla \rangle\). Solving this equation must be performed in a self-consistent way if the dielectric function \(\varepsilon\) is frequency dependent. For a propagating mode with propagation constant \(\beta\) the group velocity is then evaluated as \(v_g = \frac{\omega}{\beta}\). The total derivative of the eigenvalue \(\omega\) with respect to \(\beta\) may be evaluated as

\[
\frac{2\omega}{c^2}v_g = \frac{d}{d\beta} \langle H, \nabla \times \varepsilon(r,\omega) \times \nabla \rangle + \frac{\partial \varepsilon}{\partial \omega} \frac{\partial \langle H, \nabla \times \varepsilon(r,\omega) \times \nabla \rangle}{\partial \beta}.
\]

For low-loss dielectrics, with negligible imaginary dielectric function \(\varepsilon'\), the group velocity then reads \(v_g = \frac{\omega E_d}{\beta H}\), where \(E_d\) is group velocity for the PhC waveguide without any material dispersion, and \(E_d = \langle D, E\rangle_{\text{QD}} / \langle D, E\rangle\) is the fraction of the electric energy inside the dielectric having a frequency dependent dispersion. Using \(\varepsilon = n^2\), the partial derivative of \(\ln(\varepsilon)\) may be expressed in terms of the dispersive group index, and the total group index can be written as

\[
n_g = n_g^{\text{PhC}} \left[ 1 + E_d \left( \frac{n_{\text{EIT}}}{n_b} - 1 \right) \right].
\]

This result shows that the group index has a contribution that scales linearly with the product of \(n_g^{\text{PhC}}\) and \(n_{\text{EIT}}\). The total group index in a system which combines both material and waveguide dispersion may then be evaluated from the passive PhC waveguide group index \(n_g^{\text{PhC}}\), filling factor \(E_d\), and from the QD ensemble group index \(n_g^{\text{EIT}}\), alone. In Fig. 3 we have also plotted the prediction of Eq. (1). The electric energy in the side the QD region is evaluated from the numerical simulations as follows: \(E_d = \int d\langle D, E\rangle_{\text{QD}} / \int d\langle D, E\rangle\). From our calculations we obtain \(E_d = 0.535\) and \(n_g^{\text{PhC}} = 38.5\). We observed good agreement between numerical simulations and theory.

In conclusion, we have investigated light slow-down in a PhC structure including QDs. The slow-down factor may be significantly enhanced compared to systems relying only on material or waveguide dispersion. This could be important in achieving efficient and variable control over pulse propagation in compact semiconductor waveguides.

This work has been partially financed by the European Commission through the IST Project No. 29283, QPhoton, the project QUEST financed by the Danish Research Councils, and the project NATEC financed by the VILLUM KANN RASMUSSEN FOUNDATION. The authors also acknowledge access to the Danish Center for Scientific Computing (DCSC) at the Technical University of Denmark under Grant No. DCSC CPU-0107-12.

11Our numerical implementation is equivalent to Eqs. (1)–(4b) of Ref. 10 with the obvious changes needed in describing a semiconductor system. Notice that the Eq. (4a) of Ref. 10 is the equivalent of Eq. (1) in Ref. 2.