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Demonstration of deterministic and high fidelity squeezing of quantum information

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By employing a recent proposal [R. Filip, P. Marek, and U.L. Andersen, Phys. Rev. A 71, 042308 (2005)] we experimentally demonstrate a universal, deterministic, and high-fidelity squeezing transformation of an optical field. It relies only on linear optics, homodyne detection, feedforward, and an ancillary squeezed vacuum state, thus direct interaction between a strong pump and the quantum state is circumvented. We demonstrate three different squeezing levels for a coherent state input. This scheme is highly suitable for the fault-tolerant squeezing transformation in a continuous variable quantum computer.

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The implementation of a direct nonlinear quantum operation is often hampered by decoherence due to inevitable practical imperfections in physical systems. Because of the necessity of invoking such unitary transformations in a fault-tolerant quantum information processor, the future of developing such units was not too bright. However, new optimism arose from the introduction of the so-called off-line schemes, where a nonlinear transformation is executed on a quantum state through simple linear interference with some off-line resource states that can be prepared at any time.

The first simple example of such an off-line scheme is teleportation, [6] which demonstrates the implementation of the most trivial unitary quantum operation—namely the identity operation: The off-line resource is a bipartite entangled state which is detected jointly with the fragile quantum information in a Bell measurement and the classical outcomes are fed forward to finalize the identity (or teleportation) operation. Remarkably it was found that by manipulating the off-line entangled state in the teleporter it is possible to implement any unitary transformation through teleportation. This was first realized for qubits [1] and subsequently used in the linear optical quantum computer [2], and later extended to continuous variables (CVs) which benefit from the easy Bell measurement [3].

Such a teleportation-based off-line scheme can, for example, be used for the implementation of a unitary and nonlinear squeezing operation. It was, however, realized in Ref. [7] that a much simpler off-line scheme relying only on a single vacuum squeezed ancilla suffices to implement the squeezing operation [see Fig. 1(a)]. In essence, this simple setup allows for the experimentally feasible fault-tolerant squeezing transformation of quantum information, and it can be seen as the CV analog to the one-qubit teleportation approach in Ref. [8].

In this Rapid Communication we construct a squeezing transformation using the off-line approach proposed in Ref. [7], and we demonstrate its function with coherent state inputs. Such a transformation is ideally described by a single mode Bogoliubov transformation, which maps the input Wigner function $W(\hat{x},\hat{p})=W(xe^{i\phi},pe^{-i\phi})$ [9] where $x$ and $p$ represent the amplitude and phase quadrature of the field and $r$ is the squeezing factor. Although this simple transformation is standard in any textbook on quantum optics, its experimental realization for arbitrary inputs (that is quantum information) has remained extremely challenging. Previously demonstrated squeezing transformations have either been suffering from large decoherence (as is the case for fiber or cavity implementations), thus corrupting the fragile quantum information of a quantum state, or been using an input dependent nondeterministic approach [10].

![FIG. 1. (a) Schematic of high-fidelity squeezing. (b) Experimental setup for high-fidelity squeezing. A variable beam splitter is realized by a half-wave plate (HWP) and two polarizing beam splitters (PBS). EOM: electro-optic modulator, LO: local oscillator, and OPO: optical parametric oscillator.](image-url)
contrast to previous implementations, the squeezing transfor-
mation demonstrated in this Rapid Communication is deter-
mindistic and it processes quantum information with very high
fidelity. It is therefore the first demonstration of a near
fault tolerant squeezing transformation that could be used in
CV quantum computation [5,11,12].

The scheme is illustrated in Fig. 1 and goes as follows.
The input state under interrogation is combined with a
squeezed vacuum at a beam splitter. A quadrature to be an-
tisqueezed is measured using homodyne detection, and after
appropriate rescaling of the outcomes the remaining field
is displaced accordingly. Mathematically, the transformation
can easily be derived in the Heisenberg picture. First, we
consider the input-output relations for the beam splitter:

\[
\hat{x}_i' = \sqrt{T} \hat{x}_i + \sqrt{1-T} \hat{x}_a, \quad \hat{\rho}_i' = \sqrt{T} \hat{\rho}_i + \sqrt{1-T} \hat{\rho}_a,
\]

\[
\hat{x}_a' = \sqrt{T} \hat{x}_a - \sqrt{1-T} \hat{x}_i, \quad \hat{\rho}_a' = \sqrt{T} \hat{\rho}_a - \sqrt{1-T} \hat{\rho}_i,
\]

where \( \hat{x} \) and \( \hat{\rho} \) represent the quadratures to be squeezed and
antisqueezed, the indices “i” and “a” refer to the input and
ancillary mode, respectively, and \( T \) is the transmittance of
the beam splitter. The quadratures of the ancilla are written as
\( \hat{x}_a = x_a^{(0)} e^{r_a} + \hat{\rho}_a = \rho_a^{(0)} e^{r_a} \) where \( r_a \) is the squeezing parameter
and \( x_a^{(0)} \) and \( \rho_a^{(0)} \) represent vacuum fluctuations. In the re-
lected part, the quadrature \( \hat{\rho}_a' \) is measured using homodyne
detection. The measurement outcomes are subsequently res-
caled by a factor denoted by \( g \) and finally used to displace
the remaining part of the system, which is equivalent to the
transformation \( \hat{x}_i \rightarrow \hat{x}_i'' = \hat{x}_i \) and \( \hat{\rho}_i \rightarrow \hat{\rho}_i'' = \hat{\rho}_i' + g \hat{\rho}_a' \). By choosing
\( g = -\sqrt{(1-T)/T} \), we arrive at the following input-output relations:

\[
\hat{x}_i'' = \sqrt{T} \hat{x}_i + \sqrt{1-T} x_i^{(0)} e^{-r_a},
\]

\[
\hat{\rho}_i'' = \frac{1}{\sqrt{T}} \hat{\rho}_i,
\]

In the limit of the infinitely squeezed ancilla, corresponding
to \( r_a \rightarrow \infty \), the transformation coincides with a perfect unitary
squeezing operation, with the actual squeezing parameter \( r \)
\( = \ln \sqrt{T} \) which is directly controlled by the transmittance
of the beam splitter. Furthermore, the quadrature being
squeezed can also be easily controlled through adjustment of
the relative phase between the signal and the squeezed an-
cilla and correspondingly the measured quadrature in the
feedforward loop [7]. Therefore full control of the squeezing
process is accessed through simple operations on linear pas-
sive devices. Let us note that by changing some of the set-
ings of the setup (such as the local oscillator phase, the
feedforward gain, and the ancilla state) the setup can func-
tion as a nonunitary noiseless amplifier [13], a nonunitary
quantum nondemolition measurement device [14], or as a
squeezed state purifier [15].

In a realistic situation, the ancilla state is not infinitely
squeezed and some extra quantum noise will inevitably be
added to the squeezed quadrature as indicated by the second
term in Eq. (3). Note that the noise suppression performance
never goes further than that of the ancilla. In contrast, the
imperfections of the ancilla state do not degrade the quality of
the transformation of the antisqueezed quadrature as well
as the mean values; The excess noise of the ancilla is not
coupled into the mode nor does it disturb the mean value
transformation.

The operation described above is universal and thus
squeezes all input states. In the following experimental in-
vestigation, however, we consider the squeezing of particular
states, namely coherent states. To ensure that the coherent
states are truly pure, we define them to be a sideband at a
radiofrequency relative to the carrier of a laser beam. This
beam as well as other auxiliary beams are delivered by a
Ti:sapphire laser operating at 860 nm. The experiment is di-
vided into three parts; preparation, processing, and verifica-
tion which will now be discussed.

Preparation. In the preparation stage, we generate the
input coherent state and the squeezed ancilla state. The coer-
herent state is prepared by traversing a part of the laser beam
through an electro-optic modulator operating at 1 MHz and
set to modulate the amplitude and phase simultaneously. As
a result, a true coherent state is generated at a 1 MHz sideband
and we assume the bandwidth to be 30 kHz. The power of
the optical carrier is about 3 \( \mu \)W whereas the power of
the sideband is about 15 dB above the corresponding shot noise
level. The ancillary squeezed state is produced in an optical
parametric oscillator (OPO). It is a 500 mm long bow-tie-
shaped cavity consisting of two plane mirrors and two mir-
rors with a 50 mm radius of curvature. The nonlinear crystal
is a 10 mm periodically poled KTiOPO4 (PPKTP) crystal
(see [16] for details). We pump the OPO with light at 430 nm
stemming from a second harmonic generator with the same
configuration as the OPO cavity but with a KNbO3 crystal
and pumped with the light from the Ti:sapphire laser. To
monitor and lock the squeezing phase we inject a weak co-
herent beam to the OPO. The output from the OPO and the
coherent state are then directed to the processing part. They
have 97 and 143 kHz modulation sidebands for phase lock-
ing.

Processing. At this stage the actual squeezing transforma-
tion is implemented. First the two states from the preparation
stage merge at a variable beam splitter composed of a half-
wave plate (HWP) sandwiched between two polarizing beam
splitters (PBS). The beam splitting ratio is thus easily con-
trolled via a wave plate rotation. One output of the beam
splitter is directed to a homodyne detector which measures
the \( p \) quadrature. The visibility between the output and a
local oscillator is 96% and the quantum efficiency of the
detectors is more than 99%. The measurement outcomes are
amplified electrically in a low-noise amplifier and subse-
quently used to drive a phase modulator which displaces an
auxiliary beam. Finally, the displacement of the signal is
achieved by combining it with the displaced auxiliary field
using a highly asymmetric beam splitter (99/1).

Verification. In the final stage of the experiment, the pro-
tocol is verified by measuring the input states as well as the
squeezed output states. The states are fully characterized by
balanced homodyne detection. The visibility between the
squeezed output beam and a local oscillator is 96% and the
total propagation efficiency is 96%. The electronic noise is
always 19 dB smaller than the optical noise. After detection
FIG. 2. Results of the homodyne measurements. (a)–(e) Raw quadrature data as a function of the phase of the local oscillator and (f) is the reconstructed Wigner function (using inverse Radon transformation [9]) for one realization of the experiment.

The photocurrents are used to reconstruct the quantum states. The 1 MHz component of the measured output signal is extracted by means of a lock-in detection scheme. The signal is mixed with a 1 MHz sine-wave signal from a function generator, low pass filtered (30 kHz), and finally digitized and fed into a computer with the sampling rate of 300 kHz.

First we present in Fig. 2 the raw data of the time resolved measurements of the input states and the output states. The time series for the input coherent states and blocking the displacement beam from the right

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ellipses are the measured standard deviations of \(x\) and \(p\). Obviously the mean values are transformed almost ideally.

In Fig. 4 the noise powers of the squeezed and antisqueezed quadratures are plotted as a function of the transmittance. The three curves represent theoretical predictions for the noise power of the antisqueezed quadrature (curve i), the squeezed quadrature with the ancilla 5.1 dB squeezed (curve ii), and infinitely squeezed (curve iii). Note again that the antisqueezed noise does not depend on the ancilla. Experimental data taken with and without the feedforward in place are also shown in Fig. 4: The noise powers of \(x\) (\(p\)) with feedforward are indicated by dots (closed diamonds), and without feedforward by circles (open diamonds). We see that the antisqueezed noise of the ancilla is cancelled and the transformation in \(p\) becomes almost ideal after the feedforward. The noise powers of the squeezed quadrature, however, deviate from the ideal operation due to the finite squeezing in the ancilla states. Furthermore, we observe a small degradation of the noise suppression due to some imperfections of the feedforward, such as phase fluctuation.

We now calculate the fidelities \([17]\) of these transformations. For the case of Gaussian states the fidelity between the ideal squeezed state, \(|\psi_{id}\rangle\), and the actual obtained mixed state, \(|\tilde{\rho}_{\text{out}}\rangle\), is given by (in the unit of \(\hbar =1/2\))

\[
F = \langle |\psi_{id}\rangle |\tilde{\rho}_{\text{out}}| |\psi_{id}\rangle = \frac{1}{2\sqrt{V_{\text{out}}^x + V_{\text{id}}^x}} \frac{1}{2\sqrt{V_{\text{out}}^p + V_{\text{id}}^p}} \exp \left[ -\frac{(\langle x_{\text{out}}\rangle - \langle x_{\text{id}}\rangle)^2}{2(V_{\text{out}}^x + V_{\text{id}}^x)} - \frac{(\langle p_{\text{out}}\rangle - \langle p_{\text{id}}\rangle)^2}{2(V_{\text{out}}^p + V_{\text{id}}^p)} \right],
\]

where the subscripts “id” and “out” denote the ideal squeezing and the experimental output, respectively, and \(V\) denotes the variance. Actually, due to small propagation and detection losses in the experiment, the fidelity ultimately depends on the input state. We therefore quantify the individual single shot fidelities for the inputs considered in the experiment, though the average fidelity will be found by integrating the fidelity in Eq. (5) over all possible input states. From the measured means and variances we compute the fidelities between the ideally squeezed states of the inferred inputs (accounting for losses) and the directly measured squeezed states, and we find 94\% ±1\% for \(T=0.75\) (1.2 dB squeezing), 89\% ±1\% for \(T=0.50\) (3.0 dB squeezing), and 78\% ±2\% for \(T=0.25\) (6.0 dB squeezing). We note that the fidelity between the measured input states and the inferred ones is found to be 97\% ±1\%. For comparison, the theoretically calculated fidelities with vacuum ancilla states (which correspond to the classical limits) are 93\%, 82\%, and 63\% for the transformations corresponding to 1.2, 3.0, and 6.0 dB squeezing, respectively.

In summary, we have succeeded in demonstrating deterministic and universal squeezing transformation using a feedforward technique. The squeezing operation associated with three different squeezing degrees corresponding to 1.2, 3.0, and 6.0 dB were demonstrated and quantum noise suppressions of 0.7, 1.6, and 2.5 dB below the shot noise were obtained, yielding the fidelities 94\% ±1\%, 89\% ±1\%, and 78\% ±2\%, respectively. Although the transformation only was tested for a single coherent state, it will work equally well for any other state due to its universality.

Finally, we should note that the high-fidelity squeezing in this work completes the set of demonstrated Gaussian operations \([11,18]\). An arbitrary multimode Gaussian transformation can be physically generated by the use of phase space displacement and rotation, beam splitting interaction, phase insensitive amplification \([19]\), and universal squeezing transformation. With the work presented in this Rapid Communication, we therefore pave the way for the experimental demonstration of new interesting CV Gaussian protocols such as the CV controlled-NOT gate \([7]\) and eventually quantum computation.

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