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Mass and position determination of attached particles on cantilever based mass sensors

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An analytical expression relating mass and position of a particle attached on a cantilever to the resulting change in cantilever resonant frequency is derived. Theoretically, the position and mass of the attached particle can be deduced by combining measured resonant frequencies of several bending modes. This finding is verified experimentally using a microscale cantilever with and without an attached gold bead. The resonant frequencies of several bending modes are measured as a function of the bead position. The bead mass and position calculated from the measured resonant frequencies are in good agreement with the expected mass and the position measured.


In the recent years a wealth of nanoelectromechanical systems has emerged, and one of the obvious applications of these is in mass sensing. Since the method was proposed in 1995, cantilever based mass sensors have developed into promising candidates for ultrasensitive mass sensing, and recently systems capable of detecting masses in the atto- and zeptogram (10^-18 –10^-21 g) ranges have been reported. The ultimate goal of single molecule detection capabilities of mass sensors will change with the actual position of the added mass since the vibration velocity of the cantilever surface varies with the position of the added mass. These relations allow the mass and position of the added mass to be determined even without a priori knowledge about the position. The method relies on measurements of the native as well as the mass loaded cantilever resonant frequencies for several vibration modes. The method is verified using experimental data and proves to resolve both position and mass with high accuracy.

Consider a cantilever with the mass \(m_0\) loaded by a point mass \(\Delta m\) positioned at \(z_{\Delta m}\) (Fig. 1). The displacement function of the vibrating beam is \(W_n(z,t)=a_n U_n(z)e^{-int}\), where \(n\) denotes the modal number, \(a_n\) is the resonant frequency, \(U_n(z)\) is the time independent mode shape, and \(a_n\) is the modal amplitude at mode \(n\). The mode shape for a clamped-free beam (cantilever) takes the form

\[
U_n(z) = A_n (\cos \kappa_n z - \cosh \kappa_n z) + B_n (\sin \kappa_n z - \sinh \kappa_n z),
\]

where the modal wavenumbers \(\kappa_n\) are solutions to \(\cos(\kappa_n L) \cosh(\kappa_n L) = 1\), and the mode coefficients fulfill \(A_n/B_n = (\cos \kappa_n L + \cosh \kappa_n L)/(\sin \kappa_n L - \sinh \kappa_n L)\). Numerical values of the first modal wavenumbers and corresponding mode coefficients are \(\kappa_n L = 1.875, 4.694, 7.855, 10.996, \ldots\) and \(A_n/B_n = 1.362, -0.982, -1.001, -1.000, \ldots\) Below, normalized mode shapes, \(\int_0^L U_n^2(z)dz = L\), are assumed.

If the mass load is much less than the cantilever mass, \(\Delta m \ll m_0\), the cantilever mode shape will not change significantly; thus the resonant frequency of such a system can be accurately estimated using an energy approach and the Rayleigh-Ritz theorem. According to the Rayleigh-Ritz theorem the time average kinetic energy \(E_{\text{kin}}\) equals the time average strain energy \(E_{\text{strain}}\) at resonance. Thus, for a cantilever with an attached point mass, \(E_{\text{strain}} = E_{\text{kin}} + E_{\text{kin},\Delta m}\), where \(E_{\text{kin},\Delta m}\) is the kinetic energy due to the point mass. The kinetic energy of the cantilever is

\[
E_{\text{kin}} = \int_0^L \frac{1}{2} \rho a_{\Delta m}^2 \omega_{\Delta m}^2 U_n^2(z)dz
\]

\[
eq \frac{1}{2} \rho \omega_{\Delta m}^2 \int_0^L \frac{U_n^2(z)dz}{L} = \frac{1}{2} m_0 \omega_{\Delta m}^2 \Delta m,
\]

where \(\rho\) is the average mass density of the cantilever. The kinetic energy due to the added point mass at \(z_{\Delta m}\) is

\[
f_{\Delta m} U_n^2(z_{\Delta m})dz_{\Delta m}
\]

\[
= \frac{1}{2} \rho \omega_{\Delta m}^2 \int_0^L \frac{U_n^2(z)dz}{L} = \frac{1}{2} m_0 \omega_{\Delta m}^2 \Delta m.
\]
A robust fitting procedure is needed. Solving Eq. (2), to determine in this manner, it is useful to solve Eq. (3), and native resonant frequencies have been used. The cantilever length, width, and thickness were determined from optical measurement of the mass loaded cantilever. The position of the gold bead, the change in resonant frequency was recorded for the first four bending modes while the cantilever was positioned with a radius of 0.9 μm. For each position of the gold bead, the change in resonant frequency was evaluated with a radius of 0.9 μm, corresponding to a mass of approximately 60 pg, was positioned and manipulated on the cantilever. The position of the gold bead is determined from optical images with an estimated accuracy of ±1 μm. For each position of the gold bead, the change in resonant frequency was recorded for the first four bending modes while the cantilever was resonating in a low ambient pressure of 0.5 mbar to ensure a high quality factor for the resonator and good resonant frequency resolution. The measured unloaded resonant frequencies for the first four bending modes of the cantilever were m0 = 7 ng. In the experiments a single gold bead with a radius of 0.9 μm, corresponding to a mass of approximately 60 pg, is positioned and manipulated on the cantilever. The position of the gold bead is determined from optical images with an estimated accuracy of ±1 μm. For each position of the gold bead, the change in resonant frequency was recorded for the first four bending modes while the cantilever was resonating in a low ambient pressure of 0.5 mbar to ensure a high quality factor for the resonator and good resonant frequency resolution. The measured unloaded resonant frequencies for the first four bending modes of the cantilever were m0 = 7 ng. In the experiments a single gold bead with a radius of 0.9 μm, corresponding to a mass of approximately 60 pg, is positioned and manipulated on the cantilever. The position of the gold bead is determined from optical images with an estimated accuracy of ±1 μm. For each position of the gold bead, the change in resonant frequency was recorded for the first four bending modes while the cantilever was resonating in a low ambient pressure of 0.5 mbar to ensure a high quality factor for the resonator and good resonant frequency resolution.

The mass of the cantilever must be known in order to estimate the mass of the attached particle. The cantilever mass can be calculated from parameters of the materials used in fabrication and the designed or measured geometrical dimensions with an uncertainty of a few percent or less. This error on the actual cantilever mass directly affects the accuracy of the calculated mass of the attached particle and could well be the dominant contribution to the total mass error, but the error is small enough to be acceptable in most cases. To verify the theoretical findings, data from previously reported experiments on a micrometer sized cantilever have been used. The cantilever length, width, and thickness were approximately 153, 11, and 1.05 μm, respectively; the cantilever was fabricated in SiO2 with a Au/Ti (100 nm/10 nm) coating on the topside; thus the estimated total mass of the cantilever is m0 = 7 μg. In the experiments a single gold bead with a radius of 0.9 μm, corresponding to a mass of approximately 60 pg, is positioned and manipulated on the cantilever. The position of the gold bead is determined from optical images with an estimated accuracy of ±1 μm. For each position of the gold bead, the change in resonant frequency was recorded for the first four bending modes while the cantilever was resonating in a low ambient pressure of 0.5 mbar to ensure a high quality factor for the resonator and good resonant frequency resolution.
The extracted gold-bead mass ratio $\Delta m/m_0$ and position $z_{\text{opt}}$ obtained using the fitting procedure and the measured resonant frequencies are shown in Fig. 4 as a function of the visually determined bead position $z_{\text{opt}}$. The extracted positions are seen to agree very well with the visually determined positions almost within the estimated errors on these, except for the single point close to the cantilever base where a significant disagreement is seen. At this point the fit returns a false position and mass, but due to the correlation and the particular distribution of measurement errors, there is a global minimum at the reported erroneous position and mass. Apparently, except for this single point, the accuracy of the extracted positions seems to exceed the accuracy of the visually determined positions.

The average extracted mass ratio is $\Delta m/m_0 = 0.0084 \pm 0.0001$ when the point closest to the cantilever base is omitted. Thus, the calculated mass of the gold bead is $m_{\text{ bead}} = \Delta n = 7 \, \text{ng} \times (0.0084 \pm 0.0001) = 59 \pm 1 \, \text{pg}$, which is in perfect agreement with the expected value.

The almost perfect agreement between the extracted mass and the expected value when the mass is positioned on the outer 4/5 of the cantilever clearly demonstrates the possibility to do mass distribution analysis without prior knowledge about or control over the position of the added particle.

The method could very well be applied to single-cell measurements or nanoparticle detection, where a very accurate particle mass measurement is needed and a time consuming position measurement using scanning electron microscopy imaging is inconvenient. It could equally well be used for determining the mass of cells or particles flowing inside a cantilever as demonstrated by Burg and Manalis.

The method could also prove very useful for enhancing the functionality of cantilever based mass sensors. This could be achieved by having several areas on the cantilevers coated for sensing of specific and different target molecules. By measuring several vibration modes during operation, the binding of desired targets in one area could be differentiated from binding in other areas. In principle, this would make it possible to design an artificial nose using only a single cantilever.

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