Global Optimal Design of Composite Laminates Including Failure Criteria Using Decomposition Techniques

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Global Optimal Design of Composite Laminates Including Failure Criteria Using Decomposition Techniques

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Problem Setting

- Discrete material optimization (DMO) with 0-1 design variables.

- Material selection among a discrete set of candidates materials.

- $t :=$ Density of material block (design variables):

  $$t_{ij} = \text{Density of the material } j \text{ used in the element } i.$$ 

- $u_l :=$ Displacement due to the load condition $f_l$, $l = 1, \ldots, m$.

- $M :=$ Available amount of material.
Problem Setting

- Material selection among a discrete set of candidate materials.

- Materials are defined by the specific stress-strain relationship.

- Material candidates could for example, be the same material oriented in specific angles (orthotropic materials).

- In this case, the problem becomes an angle selection problem.
Problem Formulation

- Minimum compliance, multi-material, local Failure problem:

\[
\begin{align*}
\text{minimize} & \quad \max_{1 \leq l \leq m} \{ f_l^T u_l \} \\
\text{s.t.} & \quad K(t)u_l = f_l, \quad l = 1, \ldots, m \\
& \quad \rho^T t \leq M \\
& \quad t_{ij} \in \{0, 1\}, \quad \forall \ i, j. \\
& \quad \sum_j t_{ij} = 1, \quad i = 1, \ldots, n^c \\
& \quad F(x, u_l) \leq 0, \quad l = 1, \ldots, m. \\
\end{align*}
\]  

(Compliance) (Equilibrium) (Mass) (0-1 cond.) (Mat. Select.) (Local Failure.)

\[ K(t) = \sum_{i,j} t_{ij} K_{ij}, \quad K_{ij} \succeq 0, \quad \text{linear elasticity.} \]
Solve (P) to global optimality is a quite difficult task, even if no failure is considered.
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Global Optimization

- Solve (P) to global optimality is a quite difficult task, even if no failure is considered.

- The task becomes even more difficult if the considered failure criterion does not have useful mathematical properties.

- Only few existing works in this area (and none for multimaterial problems).
We propose to use the Generalized Benders’ Decomposition (GBD) algorithm to attack problem (P).
Mixed Integer formulation: Features

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- The GBD method in general does not converge to global optima. Nevertheless, it does for the problem (P) when no failure criterion is considered.
Mixed Integer formulation: Features

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- The main reason is the convexity of the compliance function as a function only in the design variable $t$.

- This issue must be taken into account when attacking the problem (P) (i.e., including local failure).
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GBD Principle

Figure: Generalized Benders’ Decomposition Method

GBD Principle

Problem Setting

GBD Results

Local Failure Criteria

Summary and Conclusions
GBD: Relaxed Master Problem (m=1)

\[ \text{minimize } y \quad \text{subject to } v^k T t - y \leq -2f^T u^k \quad k = 1, \ldots, N \]
\[ \sum_{i,j} \rho_{ij} t_{ij} \leq M, \]  
\[ \sum_{j} t_{ij} = 1 \quad \forall i \]  

- \( v^k T = -u^k T \nabla_t K(t)u^k = -(u^k T K_1 u^k \ u^k T K_2 u^k \ldots \ u^k T K_n u^k) \)
- \( u^k \) is the displacement solution to \( K(t^k)u^k = f \).
- The \( t^k \)'s is the solution of the \((k - 1)\)-th Relaxed Master Problem (RMP)
GBD Performance

- Heuristics to find candidate solutions improve the performance of the method.
- Solve Sub-MIP problem by GBD.
- Designs without failure criterion:
  - C. Hvejsel, today 18h00, Solution with less than 2 % global optimality gap, for a multilayered problem of 23,000 design variables.
Local Failure Criteria

- Local failure criteria + Global Optimization: Possible?
- General types of failure criteria for composite structures
  - Max stress, max strain, Tsai-Hill, Tsai-Wu, Puck, Cuntze, Hashin, etc
- It is not clear which is the most convenient failure criteria for composite structures.
- In general, all local failure criteria functions are not convex.
- GBD does not guarantee global solutions in this case.
Local Failure Criteria

- Max strain: $Au \leq b$

- Tsai- Wu/Tsai-Hill:

$$\frac{1}{2} u^T W_j(t) u + w_j(t) u \leq Y_j \quad \forall \; j = 1, \ldots, n$$

$$W_j(t) = \sum_i t_{ij} W_{ij}, \quad w_j(t) = \sum_i t_{ij} w_{ij}$$

- $W(t)$ positive semidefinite matrix.

- If the failure is non convex a convex reformulation (if possible) is necessary for using GBD or any Global Optimization technique.
Local Failure Criteria

- “Big-M” reformulation of bilinear equations (Stolpe and Svanberg 2001)

\[ z_{ij} = t_{ij} u \]
Local Failure Criteria

- "Big-M" reformulation of bilinear equations (Stolpe and Svanberg 2001)
  \[ z_{ij} = t_{ij} u \]

  \[ t_{ij} c_{ij}^{\min} \leq z_{ij} \leq t_{ij} c_{ij}^{\max} \]

  \[ (1 - t_{ij}) c_{ij}^{\min} \leq u - z_{ij} \leq (1 - t_{ij}) c_{ij}^{\max} \]

- \( c_{ij}^{\min}, c_{ij}^{\max} \) are convenient bounds.
GBD for Problem (P)

- The "Big-M" reformulation can be applied to create GBD cuts for the max strain, max stress, Tsai-Hill/Wu failure criteria.
- However, the constraints generated by this method are in general too weak (there is no impact in performance).
- Algorithm becomes slower instead of becoming faster.
- Only possible for a few local failure criteria (if a convex reformulation exists).
- Even a feasible design is very difficult to find.
Toy Example: Topology Design, 24 DV

- No failure criterion, minimum compliance, connectivity imposed, optimal solution found

- Optimality gap: 0.5 %, CPU-time: 38[s], 54 iterations.
- maximum strain value: 5.397
Local Failure Criteria

- **Strategy:** Use this value as reference for setting the limit for the strain.
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- **Strain limit imposed**

\[ \| \epsilon \| \leq 5.397 - 0.001 = 5.396. \]
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- Attack (P) for a max strain failure criterion with the proposed GBD algorithm.
Local Failure Criteria

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• Attack (P) for a max strain failure criterion with the proposed GBD algorithm.

• Result: Algorithm runs for 12[h], and not a single feasible solution is found.
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- **Attack (P)** for a max strain failure criterion with the proposed GBD algorithm.

- **Result**: Algorithm runs for 12[h], and not a single feasible solution is found.

- **Conclusion**: Failure feasibility constraints from the GBD method are too weak for expecting an acceptable performance.
Local Failure Criteria

- Alternative: Solve the problem (P) in two stages. First, without the local failure criterion.

- When the problem is solved to optimality, reset the Upper bound to $+\infty$. Restart the GBD algorithm considering the local failure criterion in the problem formulation.

- If the current design at iteration $k$, $t^k$, is infeasible for the local failure, include a single linear constraint preventing $t^k$ (and only $t^k$) to be feasible in the master problem.

\[ c^T t^k \leq b^k, \quad c \in \mathbb{R}^{n+1}, b^k \in \mathbb{R}. \]

- The method can be used for any kind of failure function $F(x, u_l)$ (no special properties are needed).
- Angle selection problem: +45, -45, 0, 90.
- 400 FE discretization.
- 100 design element discretization × 4 candidate angles: 400 DV.
No Failure Solution

- No failure criterion, minimum compliance, solution found.

- Optimality gap: 2.64 %, compliance: 15.4716.
- maximum strain value: 5.74397E-03. CPU-time: 12[h]
Local Failure Criteria

- **Strategy**: Use this value as reference for setting the limit for the strain.

- **Strain limit imposed**

  \[ ||\varepsilon|| \leq 5.74397 \cdot 10^{-3} - 1 \cdot 10^{-8} = 5.74396 \cdot 10^{-3}. \]

- **Attack (P)** with the proposed GBD algorithm.
Local Failure Criteria

- Failure criterion included, solution found.

- Optimality gap: 2.87 %, Objective Value: 15.4902.
- maximum strain value: 5.74375 \cdot 10^{-3}. CPU-Time: 43[h]
Comparison

(a) No local failure
(b) Max strain failure criterion
Summary and Conclusions

- Generalized Benders’ Decomposition can solve medium size structural design problems to optimality.

- The inclusion of local failure criteria makes the feasible set smaller.

- $\Rightarrow$ Faster convergence is expected. However, the opposite occurs.
Summary and Conclusions

- The minimum compliance problem problem (P) + local failure can be attacked by Generalized Benders’ Decomposition.
Summary and Conclusions

- The minimum compliance problem $\text{(P)} + \text{local failure}$ can be attacked by Generalized Benders’ Decomposition.

- The algorithm converges theoretically to a global minimum solution in a finite number of steps, or stops with an infeasibility flag (if the problem is infeasible).
Summary and Conclusions

- The minimum compliance problem problem (P) + local failure can be attacked by Generalized Benders’ Decomposition.
- The algorithm converges theoretically to a global minimum solution in a finite number of steps, or stops with an infeasibility flag (if the problem is infeasible).
- The method can be used for almost any failure criterion, independently of convexity assumptions.
Thanks for your attention!