On the role of lateral waves in the radiation from the dielectric wedge

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Published in:
IEEE Transactions on Antennas and Propagation

Publication date:
1973

Document Version
Publisher’s PDF, also known as Version of record

Citation (APA):
an absolute error of 0.02 in Gaussian method). The computation of \( s^t, W^t, \) and \( H^t \) requires \( N = 4 \) with \( k \) equals 10, 24, and 50, respectively. One may conjecture that these moderate values of \( N \) and \( k \) still hold for other simple shapes scatterers.

It has been shown that there exist some differences between the exact results and those given by Lewis method when it is applied to an experimental situation. Nevertheless this method provides us with interesting informations about the scatterer.

**REFERENCES**


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**On the Role of Lateral Waves in the Radiation from the Dielectric Wedge**

**PETER BALLING**

*Abstract—The field on the dielectric wedge is approximated by a plane-wave expansion as in [1]. Contributions from this solution to both the surface field and the radiation field are examined. Finally, an experimental radiation field is compared with the plane-wave solution and with a geometric-optical diffraction field.*

**Introduction**

The dielectric wedge (Fig. 1) can be considered as a prototype for a variety of tapered antennas. The tapering gives an additional design variable which especially for dielectric antennas can improve pattern, impedance, and broad-band properties [2, pp. 104-105, 108-114].

A good approximation to the wedge field can be obtained by expanding the field from the source in an angular spectrum of plane waves which then are reflected at the interfaces [3, pp. 22-32]. This solution agrees well with experimental surface fields and is more accurate than the local-mode approach which often has been applied to problems concerning tapered waveguides and antennas [1].

**Surface Field**

We consider a dielectric wedge excited by a magnetic unit line current along the \( y \) axis (see Fig. 1). The approximate plane-wave solution was given by [1, eqs. (1)-(3)]. The plane-wave solution is evaluated by numerical integration along paths of steepest descent. This gives a representation in terms of saddle-point contributions and lateral waves. While the former in the high-frequency limit become equal to geometric-optical rays, the latter are diffraction waves excited by waves which hit the dielectric-air interfaces at the critical angle [4].

Fig. 2 shows the components of the once reflected wedge field, e.g., the field which hits the upper wedge interface after one reflection at the lower wedge interface (see Fig. 1). Close to the source both angles of incidence are less than the critical angle and only the saddle point contribution appears. As the field point moves towards the tip, first the reflection and then the final incidence becomes critical. Each time a new lateral wave appears. The emergence of a lateral wave is neutralized by a discontinuity in the saddle point contribution so that the total once reflected field remains continuous. The lateral wave LW1 is excited at the first incidence, propagates along the lower interface, radiates back into the dielectric, and finally hits the upper interface where it appears as a fast wave. The lateral wave LW2 is excited at the second incidence. Ray paths of the lateral waves are shown in Fig. 1.

Manuscript received August 14, 1972; revised October 2, 1972. This work was supported by a Grant from the Danish Government Fund for Scientific and Industrial Research. The author was with the Laboratory of Electromagnetic Theory, Technical University of Denmark, Lyngby, Denmark. He is now with the University of Zambia, Lusaka, Zambia.

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**Fig. 1.** Dielectric wedge excited by line current along the \( y \) axis. Figure shows paths of once reflected waves which illuminate point \( F \) on upper wedge interface. ———— Saddle-point contribution. —— Lateral waves. This lines show paths before excitation of lateral waves.

**Fig. 2.** (a) Once reflected surface field on wedge. Wedge angle \( \beta \) = 20°. Dielectric constant \( \varepsilon = 2.53 \). Normalized half-width at source \( kdr = 3.69 \). ———— Saddle-point contribution. —— Lateral waves. Total once reflected surface field. (b) Radiation patterns due to once reflected surface-field components (a) on upper wedge interface, \( kr \) is normalized distance to far-field point. ———— Ray diffracted at the tip. ———— Lateral waves diffracted at the tip. ———— Total geometric-optical diffraction field. ———— Plane-wave result.
The field radiated from the wedge can be obtained by a numerical surface integration of the plane-wave wedge field. The radiation due to the once reflected surface field on Fig. 2(a) is shown in Fig. 2(b). As only the contribution from the field along the upper interface is considered, the pattern is asymmetrical. The figure also shows the corresponding geometric-optical field as well as corrections due to waves diffracted at the tip. These corrections are obtained by means of Monchau's compensation theorem, the geometric-optical surface field, and the asymptotic forms of the two lateral waves LW1 and LW2. A similar approach has earlier been applied to surface-wave antennas [5] and the dielectric rod antenna [2].

The geometric-optical field in Fig. 2(b) decreases abruptly for $\phi > 49^\circ$ because the reflection at the lower interface then becomes partial. The first correction is due to diffraction at the tip of the ray which is reflected at the lower wedge interface immediately before it hits the tip. As the tip is situated in the region with total reflection, the once reflected ray is a slow wave and the diffraction pattern very broad. When the tip is situated in the region with partial reflection, this contribution gives a smooth transition across the light-shadow boundary. The lateral wave LW2 is excited at the final incidence and travels along the upper interface with a velocity close to that of light before the diffraction at the tip. The resultant pattern is relatively sharp and important towards endfire.

The lateral wave LW1 is excited at the incidence on the lower interface and yields a fast illumination of the upper interface. The corresponding diffraction pattern is very narrow and only significant close to $\phi = 49^\circ$ where the reflection of the geometric-optical ray is critical. This contribution gives a smooth transition of the field. The total geometric-optical diffraction field agrees remarkably well with the plane-wave result. This agreement disappears, however, when the wedge tip is close to the point of critical incidence of a ray. Then it is not possible to approximate the field in the vicinity of the tip with the geometric-optical field and the asymptotic forms of the lateral waves.

In a measured radiation field, it is not possible, as in the plane-wave solution, to distinguish contributions from direct waves, once reflected waves, twice reflected waves, etc. Fig. 3 compares an experimental radiation pattern with the total plane-wave radiation pattern and total geometric-optical diffraction field. In the experiment, a three wavelengths wide wedge was excited by a two wavelengths wide $H$-plane sectoral horn embedded in the dielectric. (A cut perpendicular to the wedge apex has been inserted in Fig. 3.) The horn was approximated by a uniformly illuminated $2\theta$-high aperture. Hence, the plane-wave solution was modified with the spectrum of this aperture [3, pp. 61-64]. Similarly, the geometric-optical diffraction field was modified by the far-field pattern of the uniformly illuminated aperture. The plane-wave result and the experimental result are matched at endfire and agree well with each other. The geometric-optical diffraction field exceeds the plane-wave field with $1.2 \text{ dB}$ at endfire but contains the main features of the radiation field. The discrepancies are due to waves which hit the tip region critically. These contributions to the surface field cannot be approximated by simple asymptotic expressions.

When the wedge angle $2\theta$ and the susceptibility $\chi = \varepsilon - 1$ as in this case are small, we can neglect most waves which have undergone one partial reflection and all waves which have undergone more than one partial reflection. Therefore, the $n$ times reflected contribution to the geometric-optical diffraction field in Fig. 3 does not contain $n + 1$ lateral waves (one excited at each incidence) but only two, namely those excited at the last two incidences.

In conclusion, the lateral waves can give important contributions not only to the surface field but also to the radiation field, in particular towards endfire. The plane-wave field agrees better with experiment than the geometric-optical diffraction field which, however, requires much less computer time.

**Acknowledgment**

The author wishes to express his gratitude for the support he received at the Laboratory of Electromagnetic Theory, the Technical University of Denmark, especially from Dr. techn. J. Bach Andersen who supervised the work.

**References**


**Group Velocity Diagrams for a Ferrite Medium**

S. R. SESHADRI

Abstract—A procedure for obtaining the group velocity diagrams for a magnetized ferrite medium is given. Some results of group velocity diagrams are presented for the purpose of illustrating their general features.

The group velocity in a magnetized ferrite medium, in addition to being a function of frequency, is dependent on the direction with respect to the magnetostatic field. In many problems of wave propagation in ferrites, we have found it helpful to use group velocity surfaces which are such that the length of the radius vector terminating on any point on this surface gives the magnitude of the group velocity in the direction of the radius vector. Since there is cylindrical symmetry about the magnetostatic field, only the group velocity diagram given by the cross section of the group velocity surface by a plane containing the direction of the magnetostatic field need be considered. In this communication, we outline the

Manuscript received August 31, 1972. This work was supported in part by the NSF under Grant GK-16203.

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