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A METHOD FOR UNIFIED OPTIMIZATION OF SYSTEMS AND CONTROLLERS

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Abstract

A unified method for solving control system optimization problems is suggested. All system matrices are allowed to be functions of the design variables. The method makes use of an implementation of a sequential quadratic programming algorithm (NLPQL) for solution of general constrained non-linear programming problems.

1. Introduction

The challenging problems, where the design variables are not limited to feedback gains and observer gains, but also may be plant parameters, are emphasized in this paper. Such unified optimization problems are important issues, especially in connection with active control of large space structures ([4] [7]). However, it turns out that the methods which are useful in the unified optimization problems also offer the possibility of optimization of less complex, but from a practical point of view very interesting cases, such as LQR output feedback (constant gain feedback from less than the full state vector), LQR design with eigenvalue assignment criteria ([5]). Likewise for Gℓ(p) (the equality and inequality side constraints). The side constraints may be used to assure the assumptions a1-a5 fulfilled during optimization. ρi,ι and ρι,υ are lower and upper bounds on the ith design variable, respectively.

2. Unified Optimization: Problem Formulation

Consider the nth order LTI closed loop system:

\[ x = Ax, \quad x(0) = x_0, \quad E(x'x^T) = S \] (1)

and the objective function

\[ J = E \left( x^TQx + dt \right) \] (2)

where \( A, Q \) and \( S \) are matrix functions of the elements \( \rho_i, \) in the vector of design variables, \( \rho, E \) denotes expected value.

The following assumptions are made:

a1: \( A \) is a stability matrix
a2: \( Q = Q^T \geq 0 \)

a3: \( (\lambda, \Omega) \) is observable for any \( \Omega \) such that \( \Omega = \Omega^T \)
a4: Eigenvalues of \( \lambda \) are distinct.
a5: The elements of \( \lambda \) and \( Q \) are all continuously differentiable functions of the design variables in the domain of \( \rho \).

Then with a1-a3 it is well known ([11]) that

\[ J = E \left( x^TPx + dt \right) = tr \left( P(x_0x_0^T) \right) \] (3)

where \( P \) is the unique (from a1) solution of the matrix Lyapunov equation

\[ \lambda^TP + P\lambda = -Q \] (4)

Let \( \psi \) and \( \phi \) be the bi-orthonormal left and right eigenvectors (normalized so that \( \psi^T\phi = 1 \)) corresponding to the j'th eigenvalue of \( \lambda \), \( \lambda_j \). Then from a4 and a5 it can be shown ([4]) that

\[ \frac{\partial \lambda_j}{\partial \rho_i} = \psi_1^T \left( \frac{\partial \lambda}{\partial \rho_i} \right) \psi_i \] (5)

Furthermore, with a1 and a5 we can compute

\[ \frac{\partial J}{\partial \rho_i} = \left( tr \left( \frac{\partial P}{\partial \rho_i} S + \frac{\partial S}{\partial \rho_i} P \right) \right) \] (6)

where \( \rho_i \) is the unique (a1) solution of the matrix Lyapunov equation ([5])

\[ \lambda_i^TP + P\lambda_i = -Q, \quad \lambda_i^TP + P\lambda_i = 0 \] (7)

With (1)-(7) general nonlinear constrained optimization problems of the form

\[ \text{Minimize} \quad F(p) \] (8)

Subject to \( G_j(p) = 0, j = 1, \ldots, M \)

\( G_j(p) \geq 0, j = M+1, \ldots, M \)

\( \rho_ι,ι \leq \rho_ι,υ \leq \rho_ι,υ \) i = 1, \ldots, N

can now be formulated and numerically solved. \( F(p) \) may be any combination of objective functions like (2), (3) and eigenvalue assignment criteria ([5]). Likewise for \( G_j(p) \) (the equality and inequality side constraints). The side constraints may be used to assure the assumptions a1-a5 fulfilled during optimization. \( \rho_i,ι \) and \( \rho_ι,υ \) are lower and upper bounds on the ith design variable, respectively.

It should be noted, that the assumed stochastic nature of the initial condition distribution in (1) is essential in (3) in order to avoid specific initial condition dependence of the optimal solution. Another closely related approach leading to a worst case problem specification is presented in [3].

Since a specific control law is not assumed in (1), this method may be used very generally. Special examples are the optimal tuning of classical \( P, PD, PI, PID \) controllers and of LQR output feedback controllers ([5]).

3. NLPQL

The program NLPQL is a FORTRAN implementation of a sequential quadratic programming method for solving general nonlinear programming problems, like (8). In each iteration step, a linearly constrained quadratic subproblem is formulated by approximating the Lagrange function quadratically and by linearizing the constraints. Subsequently, a one-dimensional line search is performed with respect to an augmented Lagrange merit function to obtain the new iterate. The merit function penalizes constraint violations. A further treatment of the algorithm and the flexibility it offers, can be found in [2].

In our implementation, MATLAB functions are
used to compute the function values and gradients (from (3)-(7)) which are necessary inputs to NLPOI. The combination of NLPOI and MATLAB makes the necessary problem specific programming efforts relatively small ([5]).

It is worth noticing, that the equations (4) and (7) only differ on the right hand side, so it is possible to reuse the factorisations, which are used to solve (4), in the N solutions of (7).

4. Example

The system under consideration is a solid clamped - free Euler-Bernoulli beam of length L=1, density \( \rho = 1 \) and Young modulus \( E = 1 \) with a circular cross section. The design variables are the beam radius \( r(x) \) (discretized into 40 elements of equal length), 2 positions of the collocated (point) force actuator/velocity sensor pairs and 4 feedback gains. Hence, the total number of design variables is 46. The objective is to minimize the criterion

\[
J = \int_0^1 (x^TQx + u^T R u) \, dt
\]

when the system is modelled as a \( n \)'th order LTI. The model is obtained through modal expansion and truncation ([6]):

\[
x = Ax + Bu, \quad y = BT x, \quad u = -Gy
\]

(10)

\[
E(x_0 x_0^T) = \frac{1}{r} \Rightarrow E(x_0^T Q x_0) = 1
\]

(11)

with

\[
A^T [-\text{diag}(\omega_i) - \text{diag}(2\xi_i \omega_i)] B = \begin{bmatrix} 0 \\ \phi_1(x) \phi_2(x) \\ \vdots \\ \phi_1(x) \phi_2(x) \end{bmatrix}
\]

\[
B^T \begin{bmatrix} \phi_1(x) \\ \vdots \\ \phi_1(x) \end{bmatrix} = \begin{bmatrix} q_1 \\ \vdots \\ q_n \end{bmatrix}
\]

\[
Q = \begin{bmatrix} \text{diag}(\omega_i^2) & 0 \\ 0 & 1 \end{bmatrix}
\]

\[ R = 0.01 \cdot I \]

where \( \omega_i = 0.04 \), \( \omega_i \) is the \( i \)'th modal eigenfrequency and \( \phi_i \) is the corresponding normalized eigenfunction (modeshape). \( \eta \) (eigenvalue) side constraints assure stability of the closed loop system and one side constraint limits the total volume to a maximum of 4. The radius is limited to a maximum of 2 and a minimum of 0.2. This yields (referencing to (1)-(3)):

\[
\lambda = A - BCG
\]

\[
S = Q^{-1}
\]

All matrices are continuously differentiable (no multiple eigenvalues) functions of \( \rho \). The solutions obtained depend strongly on the initial choice of the two positions (cf. [8]), but a (local) optimum (for \( n = 6 \)) is found to be the beam designed as shown in fig. 1 with actuator/sensor positions \( x_1 = 0.77 \), \( x_2 = 1.00 \) and the gain matrix

\[
G = \begin{bmatrix} 5.03 \\ -0.33 \end{bmatrix}
\]

with \( J = 0.0702 \) and volume=4.

This could be compared to the situation with a uniform beam of volume 4 with same actuator/sensor locations. Here the optimal gain matrix yields \( J = 0.1541 \).

References