A parametric LTR solution for discrete-time systems

Niemann, Hans Henrik; Jannerup, Ole Erik

Published in:
Proceedings of the 28th IEEE Conference on Decision and Control

Link to article, DOI:
10.1109/CDC.1989.70160

Publication date:
1989

Document Version
Publisher's PDF, also known as Version of record

Link back to DTU Orbit

Citation (APA):
A PARAMETRIC LTR-SOLUTION FOR DISCRETE-TIME SYSTEMS.

Hans Henrik Niemann and Ole Jannerup
Control Engineering Institute, Tech. University of Denmark
Technical University of Denmark
Building 424, DK-2800 Lyngby, Denmark.

ABSTRACT.
A parametric LTR-solution for discrete-time compensators incorporating filtering observers which achieve exact recovery will be presented for both minimum and non-minimum-phase systems.

First the recovery error, which define the difference between the target loop transfer and the full loop transfer function, is manipulated into a general form consisting of the target loop transfer matrix and the fundamental recovery matrix. Based on the recovery matrix a parametric LTR-solution will be developed. At least it will be shown that the LQC/LTR solution is included in this new parametric solution as a special case.

1 INTRODUCTION.
The LQG/LTR feedback design methodology for robust model-based compensation for both continuous-time and discrete-time systems has received much attention in the recent years, see e.g. [7-10]. For continuous-time minimum-phase systems the asymptotic recovery techniques works very effective. Unfortunately a similar asymptotic recovery technique is not feasible in discrete-time. When the LQG/LTR-solution is applied to the discrete-time case, the recovery error will in general remain finite when a prediction observer is used. [6]. It is still possible to make LTR, but other methods must be applied. Based on the equation for the recovery error, a parametric exact LTR-solution has been developed in [9] for minimum-phase as well as non-minimum phase systems.

If the processing time of computing the control signal is negligible in comparison to the sampling interval, a filtering observer can be used in the compensator instead of a prediction observer. It is then possible to achieve exact recovery in this case by using the LQG/LTR-solution when the system is minimum phase and has maximal number of zeros [7]. However, when these conditions isn't satisfied the LQG/LTR-solution will result in a finite recovery error, but it will still be possible to make exact recovery by applying the principle from the parametric LTR result in [9]. It will then be possible to make exact recovery for both non-minimum-phase systems as well as for systems with less than the maximal number of zeros.

2 RECOVERY EQUATION.
In the following square discrete-time systems $A,B,C$ are considered. The plant transfer matrix $G(s)$ is given by:

$$ G(z) = C(z)B(z), \text{dim } G(z) = n \times n \tag{1} $$

It will be assumed that the model is minimal and the number of transmission zeros be $p$. The plant is controlled by using a feedback compensator consisting of a filtering observer in serie with a state-feedback controller. The compensator transfer matrix $H(z)$ can be rewritten as [7]:

$$ H(z) = F(z) - (1 - F(z)A - K(z))^{-1}F(z) \tag{2} $$

where $F$ is the full-order observer gain, $F(z)$ is the filtering observer gain (the relation between $F$ and $F(z)$ is $F = AF_z$), and $K$ is the full-state feedback gain.

Now let the two step LTR-design method [1,3,4,7] be used for the design of $H(z)$ which consist of a target design of one of the compensator gains $(K$ or $F$) followed by a recovery design of the other gain.

In order to formulate the loop-shape robustness constraint the uncertainties (disturbance, noise and modelling errors) are reflected to the plant input [6]. The target loop transfer function is then the full-state loop transfer $K(z)$ and the full loop transfer in $H(z)$. Let $H_2(z)$ define the recovery error as [6,9]:

$$ H_2(z) = K(z) - H(z) \tag{3} $$

In order to have exact recovery it is required that $H_2(z) = 0$ for all $z$. Now let eq. (3) be rewritten in an equivalent form:

$$ H(z) = H_1(z) + H_2(z) \tag{4} $$

where $H_1(z)$ is the recovery matrix defined by:

$$ H_1(z) = F(z)A - F(z)C(z)K(z) \tag{5} $$

and $H_2(z)$ is the equation for the recovery error.

It is simple to see by using the form of $H_1(z)$ in eq. (4) that:

$$ H_2(z) = 0 \iff H(z) = 0 \tag{6} $$

The recovery matrix $H_1$ will only be used for making exact recovery in this paper, but $H_2$ have also other attractive properties. By using eq. (3) and (4) it is easy to derive that [10]:

$$ S(z) = E(z) + H_2(z) \tag{7} $$

where $E(z)$ and $S(z)$ are the input sensitivity functions for the full loop and the target full loop transfer function. For more details, please see [10].

3 THE PARAMETRIC LTR-SOLUTION.
Now let the equation for the recovery matrix, eq. (5), be rewritten in the residual form [6]:

$$ H_1(z) = F(z)A - F(z)C(z)K(z) \tag{8} $$

where $V$ is:

$$ V = [v_1, \ldots, v_n] \tag{9} $$

and $V$ are the right and left eigenvectors associated with the eigenvalue $\lambda$ of $A - F(z)C(z)K(z)$. It is then easy to show that:

$$ H_1(z) = 0 \iff \xi_1 = 0 \tag{10} $$

$$ \xi_1 = 0 \iff \xi_1 = 0 $$

$$ \xi_1 = 0 \iff \xi_1 = 0 $$

If $A - FC(z)$ is non-defective, from eigenspectrum assignment it is known that the left eigenvector $\xi_1$ with the eigenvalue $\lambda$ of $A - FC(z)$ are given by [11]:

$$ [w_1^T \xi_1^T] \begin{bmatrix} \lambda - \xi_1 \end{bmatrix} = 0, i = 1, \ldots, n \tag{11} $$

$$ w_1^T \xi_1^T = - \xi_1^T $$
The last condition in eq. (9) implies that:

\[
[w_{k}^{T} \ z_{k}^{T}] \begin{bmatrix} I_{p} & A & C \end{bmatrix} \begin{bmatrix} \bar{z}_{k} \end{bmatrix} = 0
\]  

(11)

Maximally \(p\) eigenvectors \(w_{k}^{T}\) can satisfy this condition, if \(z_{k}\) is selected as a transmission zero of \(S(A,B,C)\). Let these \(p\) eigenvectors/vectors be selected from eq. (11) so the \(p\) conditions in eq. (9) are satisfied.

The second condition in eq. (9), \(z_{k} = 0\), can be satisfied by placing maximally \(m\) eigenvalues \(r_{i}\) at the origin.

Again from eigenstructure assignment (11) it is easily found that with \(z_{k} = 0\):

\[
w_{k}^{T} = z_{k}^{T}CA^{-1} \quad \text{implying}\]

(12)

The choice of \(z_{k}\) depends on the type of the system which is treated: A square uniform rank system or a square non-uniform rank system.

The non-uniform rank case

The square uniform rank case is defined as:

\[
CA_{n} = 0, \quad 1 = \ldots = (n-2), \quad \text{det}(CA_{n-1}) \neq 0
\]  

(13)

The \(n\) eigenvalues \(w_{k}^{T}\) corresponding to the \(n\) zero eigenvalues are given by: [31]

\[
w_{k}^{T} = \sigma_{p}^{T}CA_{n-1}^{-1}, \quad i = 1, \ldots, p
\]

(14)

where \(E\) is a \(m \times m\) non-singular matrix of the appropriate coefficients of the linear combination of the rows of \(CA^{-1}\), see Shaked (13) for calculating \(E\).

Again \(F\) can be parametrized by:

\[
F = \begin{bmatrix} w_{1}^{T} & \ldots & w_{p}^{T} \end{bmatrix} E_{n}^{-1} n_{1}^{T} n_{2}^{T} \ldots n_{p}^{T}
\]

(15)

\(n_{i}^{T} = n_{i}^{T}CA_{1}^{-1}, \quad i = 1, \ldots, p\)

and \(N_{0}, N_{1}, \ldots, N_{m}\) are free design parameters.

The uniform rank system

The square uniform rank case is defined as, see (13):

\[
CA_{n} = 0, \quad 1 = \ldots = (n-2), \quad \text{det}(CA_{n-1}) \neq 0
\]

(16)

The solution for the uniform rank case is given by eq. (15) and (16) provides the same closed-loop eigenvalues as eq. (4).

If \(p < n\) the remaining \(n-p-m\) conditions in eq. (9) must be satisfied by selecting \(F\) suitable. The first condition in eq. (9) implies:

\[
k_{r} = \begin{bmatrix} z_{0}^{T} & \ldots & z_{n-1}^{T} \end{bmatrix} = [0 \ 0 \ldots 0]
\]  

(17)

with \(k_{r} = (n \times p)\) but otherwise it is arbitrary.

\[K = \begin{bmatrix} \sigma_{1}^{T} & \ldots & \sigma_{p}^{T} \end{bmatrix} = \begin{bmatrix} A & \text{DTA} \end{bmatrix}
\]

(18)

The uniform rank case is, of course, a special case.

5 REFERENCES


APPENDIX

Proof of eq. (4).

\[
E_{y}(s) = E_{1}(s)(s-I(s^{-1}z_{(s-F)c}(A-BK)+F_{c}(s-I(s^{-1}z_{(s-F)c}(A-BK)+F_{c}))))^{-1}
\]

with \(E_{y}(s)\) being the closed-loop transfer function.