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COMPUTATION OF SCATTERING AND RADIATION FROM
OPEN-ENDED WAVEGUIDES AND SMALL HORES

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Introduction. In spherical near-field (SNF) measurements as well as
in paraboloidal reflector antenna systems, the conical horn and open
waveguide antenna is an important part of the system, whether being
used as a measuring probe or as a feed element. In both cases the
radiation and scattering properties of the horn are of interest.
With a knowledge of the radiation pattern, the spherical mode expan-
sion coefficients, necessary for the probe-corrected SNF-transfor-
mation, can be evaluated [1]. Further, aperture illumination and
efficiency of reflector antennas are derived from the radiation
pattern of the feed. As the theory behind the SNF-technique does
not take multiple scattering between test-antenna and probe into
account, it is important to obtain a knowledge of the scattering
properties, as reflections from the probe could introduce errors.
In reflector antennas, scattering from the feed might be used to
study the effect of aperture blockage on the radiation pattern.

The objective of this paper is to present a numerical approach to
the determination of the scattering and radiation characteristics
of antennas, with special emphasis on rotationally symmetric struc-
tures.

Theory. We consider an arbitrary, lossless and reciprocal antenna,
illuminated by an incident electromagnetic field. Defining the
scattered field as the difference between the total field with the
antenna present and the undisturbed incident field, it can be shown
that the scattered field from an antenna with an arbitrary load
impedance is given by

$$E^{sc} = \frac{\Gamma_L}{1 - S_{oo} \Gamma_L} \cdot E_o^{s}(\theta, \phi) + E_d^{s}(\theta, \phi)$$

(1)

Here $\Gamma_L$ and $S_{oo}$ are the load-reflection coefficient and antenna-re-
fection coefficient, respectively. $(\theta, \phi)$ are the usual spherical
coordinates. The first term on the right-hand-side is the re-radi-
ated field, i.e. the field received by the antenna, reflected from
the load and transmitted according to the radiation properties. The
pattern of $E_o^{s}$ is therefore the radiation pattern of the antenna.
The second term is the scattered field when the antenna is matched
$(\Gamma_L = 0)$, in which case there is no re-radiation. This field $E_d^{s}$ may
be considered to consist of two contributions, namely the field the
antenna will scatter in order to absorb power from the incident
field, and a field due to unloaded currents on the antenna structure,
i.e. currents which do not couple power to the load, but radiate.

If we know $E^{sc}(\Gamma_L, \theta, \phi)$ for three values of $\Gamma_L$, we are able to compute
$S_{oo}$, $E_o^{s}(\theta, \phi)$ and $E_d^{s}(\theta, \phi)$, which in turn allows the determination of
commonly encountered antenna characteristics. The basis for the
calculations will be purely numerical and restricted to rotationally
symmetric antennas, illuminated with an axially incident plane wave.
In order to solve eq. (1) for the three unknowns $S_{oo}$, $E_o^{s}(\theta, \phi)$ and

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We need three equations which can be obtained by varying \( r \). As the moment-method computer program [3] used to calculate \( E_{sc}^{s}(r,\theta,\phi) \) can only handle perfectly conducting bodies of revolution, this can be achieved by varying the position of a short. Fig. 1 shows the model for the numerical calculations.

\[ \Delta = \frac{\lambda_d}{8} \]

\( \lambda_d \) is the waveguide-wavelength

As a check on the accuracy of the calculations, the scattered field \( E_{sc}^{s}(r,\theta,\phi) \) is calculated for four different positions of the short \((z_1, \ldots, z_d, \text{cf. fig. 1})\), thus enabling us to solve \( 4 \times (3 \text{ nonlinear eqs. with 3 unknowns}) \) and get four independent solutions to the same variable. Of course, they should be identical so that differences are a measure of the accuracy of the calculations. From the solution it is now straightforward to calculate various antenna characteristics. The on-axis gain can be shown to be

\[ G = 4\pi \cdot \frac{R}{\lambda} \cdot \frac{|E_{0}^{s}(\theta=0)|}{|E_{1}|} \cdot \frac{1}{1 - |S_{\infty}|^2} \]

where \( R \) is the radius of the far-field sphere. The scattering cross-section for the matched antenna is found from

\[ \sigma_{sc, m}^{s} = 4\pi \cdot \frac{R^2}{|E_{d}^{s}|^2} \cdot \frac{|E_{1}|^2}{|E_{d}^{s}|^2} \]

Results. Numerical calculations have been performed for an open circular waveguide and a conical horn. Table I and II summarize some of the results. Where possible, comparison with available theoretical and/or experimental data is given. The numbers are the mean-values obtained from the four solutions to eq. (1), the standard-deviations being in the range 0.1 - 1%. \( A \) is the physical area of the aperture, i.e. the area within the horn outer rim.
Table I. Results for open waveguide

\[ L = 2.5\lambda, \ r = 0.35\lambda, \ F = 0, \ \alpha = 0. \]

<table>
<thead>
<tr>
<th>This work</th>
<th>(S_{\infty})</th>
<th>12 dB half-beamwidth</th>
<th>Peak cross-pol level (dB)</th>
<th>(\sigma_{sc,m}/A_p)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.134</td>
<td>81.4</td>
<td>74.2</td>
<td>-26.4</td>
</tr>
</tbody>
</table>

Table II. Results for conical horn

\[ L = 1.6\lambda, \ r = 0.375\lambda, \ F = 1\lambda, \ \alpha = 20 \text{ deg}. \]

<table>
<thead>
<tr>
<th>This work</th>
<th>(S_{\infty})</th>
<th>12 dB half-beamwidth</th>
<th>Peak cross-pol level (dB)</th>
<th>(\sigma_{sc,m}/A_p)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.026</td>
<td>37.8</td>
<td>47.6</td>
<td>-21.4</td>
</tr>
</tbody>
</table>

Fig. 2 shows further results for the horn in table II. The four solutions (four curves on each plot!) are seen to agree well, as the curves are nearly coincident for all \(\theta\)-values. Although the cross-polarization is difficult to calculate accurately in the main-beam, as it is the difference between almost equal numbers, it is seen to be very well behaved, indicating an accurate solution of eq. (1).

Conclusions. A numerical technique to predict scattering and radiation from antennas is presented. The method allows the determination of radiation patterns, antenna reflection coefficient and scattering with an arbitrary load impedance. The problems of modelling feed-arrangement and load-impedance are avoided in this method.

References

Figure 2. Results for the horn in Table II.

a. E-plane amplitude pattern (dB)
b. E-plane phase pattern (deg.)
c. E-plane scattering pattern ($\Gamma_L=0$) (dB)
d. Cross-polarization in $\phi=45$ deg. plane (dB)