Experimental and Theoretical Investigation of Subnanosecond Pulse Propagation in Graded Index Fibers

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EXPERIMENTAL AND THEORETICAL INVESTIGATION OF SUBNANOSECOND
PULSE PROPAGATION IN GRADED INDEX FIBERS

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ABSTRACT

The propagation in a fibre which does not exhibit any mode coupling is investigated by varying the launching conditions. It is shown that for this fibre there exists a trade-off between dispersion and power coupling efficiency. The measurements are compared to theoretical calculations taking leaky modes and material dispersion into account and good agreement is obtained.

INTRODUCTION

It is the purpose of this paper to perform a detailed experimental and theoretical analysis of the influence of the launching conditions on the impulse response of a graded index fibre. A theoretical investigation for step index fibres has earlier been made [1].

THEORY

We are investigating fibres in which many modes can propagate and can therefore use the WKB method to calculate the propagation constant and group delay of each mode.

In a graded index fibre with refractive index $n(r)$ as a function of radius $r$ the propagation constant of each $LP_{\mu\nu}$ mode (linearly polarized) is determined by

$$\int_{r_1}^{r_2} u \, dr = (\mu+\frac{1}{2})\pi$$

where

$$u = \sqrt{n^2(r) \left( k^2 - \beta^2 - \nu^2/r^2 \right)}$$

(2)

here $\mu$ and $\nu$ are the number of zeros in radial and azimuthal direction, respectively. The free-space wave number is $k = 2\pi/\lambda$ and $r_1$ and $r_2$ are the zeros of the integrand. Guided modes are determined by $\beta \geq n(a)k$ and leaky modes by $\beta < n(a)k$, $a$ is the core radius.

We express the refractive index by

$$n^2(r) = n_1^2 \left( 1 - 2\Delta f(r/a) \right)$$

(3)

where $f(0) = 0$ and $f(1) = 1$

We can then calculate the group delay $\tau$ as

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\[ \tau = \frac{\phi}{\omega} = \frac{1}{c} \frac{\phi}{\kappa} = \frac{1}{c} \frac{\phi/\beta}{\phi/\beta} \]

by using equation (1) we get

\[ \tau = \frac{\int_{r_1}^{r_2} k \left( \frac{N_1}{n_1} n^2(r) - \lambda \frac{\Delta'}{2\lambda} \left( n^2(r) - n_1^2 \right) \right) \, u \, dr}{\int_{r_1}^{r_2} \beta / u \, dr} \]

(5)

where \( N_1 = n_1 - \lambda n_1' \) is the group index and a prime denotes differentiation with respect to the wavelength \( \lambda \).

If the index profile is a pure \( \alpha \)-profile i.e. \( f(r/a) = (r/a)^\alpha \), then \( \beta \) and \( \tau \) can be calculated analytically [2]. In all other cases they must be numerically calculated.

Equation (5) depends implicitly on the wavelength through \( \Delta' \). Analytical expressions for the refractive index as a function of wavelength have been determined for several glass compositions [3], [4].

The numerical calculations have been compared with the exact known values in the case of \( f(r/a) = (r/a)^2 \). It is found that \( \beta \) is determined with 8 correct digits and \( \tau \) with 6 correct digits. It is necessary to have such a high degree of accuracy since we are interested in the difference between the group delays.

**EXPERIMENTAL SET-UP**

A diagram of the set-up for the measurement of pulse response is shown in fig. 1.
The laser (RCA) is operating self-pulsing with light pulses of 250 psec (FWHM). The spectral width has been measured to 35 Å (FWHM). The fibre is a GeO₂ doped graded index fibre from Schott at the length of 1112 m. The numerical aperture is N.A. = 0.25 and the core radius a = 21.5 µm. The index profile has been measured by the near-field method and is shown in fig. 2. In this fibre 100 guided and 31 leaky LP modes can then propagate.

The detector is an APD from Telefunken. A good SNR is obtained by integrating the signal in the lock-in-amplifier. The fibre is either excited without optics inserted between fibre and laser or with a small spot of size \( \frac{1}{3} \mu m \times 3 \mu m \) obtained by the 3.5 x M.O. as shown in fig. 1. The spot is placed at different positions on the fibre end face as shown in fig. 3.

**RESULTS**

The measured pulse responses when using excitation without optics and spot excitation are shown in fig. 4 and fig. 5, respectively.

We note that the pulse response has three peaks and that the second and third peaks are largest when the fibre is excited at the centre of the core. This indicates that the first peak consists of the many higher order modes and the second and third peaks consist of the few lower order modes. It should be noted that it is impossible to obtain such pulse responses if the index profile is a pure \( \alpha \)-profile. The calculated responses are shown in fig. 6 and fig. 7.

In fig. 6 is shown the pulse response with and without leaky modes taken into account. The leaky modes are modes of high order and all of them arrive in the first peak. Fig. 6 can be compared to fig. 4 since the excitation is nearly equal.

Since it is very difficult to calculate the excitation coefficients for the spot excitation shown in fig. 3,
we cannot calculate pulse responses like those in fig. 5, but in fig. 7 are shown pulse responses with different excitation coefficients, which qualitatively agree with the different spot excitations. If we compare the positions of the peaks, which are independent of the exact excitation, we get that the distance between the first and second peak is: 1.05 ns (measured) and 1.00 ns (calculated), and the distance between the second and third peak is: 0.90 ns (measured) and 0.95 ns (calculated).

The main difference between the calculated curves and the measured curves is the fourth peak on the calculated curves. This peak consists only of the LP$_{00}$ mode which propagate between $r_1/a = 0.036$ and $r_2/a = 0.203$ (eq. (1)). The refractive index within these points is not well determined as one can see in fig. 2 and the WKB approximation may fail since we here have the fastest variation of the index within distances of order of a wavelength. It is found that the third peak consists of the LP$_{01}$, LP$_{02}$, LP$_{13}$ and LP$_{23}$ modes. Finally it is seen both from the experimental and the theoretical curves that the dispersion decreases when we excite the higher order modes, but in this case we also decrease the power coupled into the fibre. This result is opposite to the one we get for step-index fibres [1].

Fig. 4. Measured pulse response with excitation without optics.

Fig. 5. Measured pulse response with spot excitation as shown in fig. 3.

Fig. 6. Calculated pulse response with equal excitation of the modes. Full line without leaky modes; dotted line with leaky modes.
CONCLUSION

We have obtained good agreement between theoretical calculations and measurements of the impulse response of a graded index fibre. We have shown that the impulse response highly depends on the launching conditions and a steady-state mode excitation therefore is preferable if one wants a reproducible measuring method.

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