Designing the dispersion for optimum supercontinuum bandwidth using picosecond pulses

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The minimizer will be calculated on the Microsoft Word. It is The Constraint can be rewritten as expressing mathematical formulae by plain

tial derivatives

Lagrange multiplier method is used for solving the problem. This method searches optimiz-

tions of these requirements.

The requirement on real time response means that the selected algorithm needs to be efficient. In this paper, we will consider the problem described in [1] and propose an algorithm to meet the challenges of these requirements.

2. PULSE-WIDTH

A very good introduction on PMD in optical fibers can be found in [1] [11] [5] [9]. We list only the results and formulae to form the optimization problem discussed in [1]. To present the proposed optimization formula we present the geometric figures in word format and to save the space, sometimes, we provide references but not details.

To solve the problem described above is equivalent to finding the best algorithm to select the polarization states of the input and the output to minimize the output pulse-width for the fiber transmission system described by transfer matrix.

The problem is transformed to the optimization of the pulse width defined as

\[ E(t) = \sum_n a_n \delta(t - nT) \]

where \( a_n \) is the complex amplitude of the nth pulse and \( T \) is the pulse period. The pulse width is defined as the full width at half maximum intensity (FWHM) of the pulse.

\[ \text{FWHM} = 2 \sqrt{\ln 2} \frac{T}{\pi} \frac{1}{|a_n|^2} \]

This proposed algorithm is implemented in MATLAB for a simple problem proposed in [1]. Starting from any random initial guess, the algorithm finds a local optimizer.

3. THE NEW OPTIMIZATION ALGORITHM

The problem described above is a typical nonlinear constrained optimization problem. In [1], Lagrange multiplier method was used for computer simulation to find the solution. Since Lagrange multiplier method is used for solving constrained optimization problems, it is not efficient for this very special problem. In addition, the set of complex nonlinear eigenvalue equations used in [1] are very difficult to be solved from the viewpoint of numerical analysis. Also, the convergence property of the Lagrange multiplier method for this problem was not addressed in [1]. We will use optimization techniques on Riemannian manifold (cf. [11]) to solve the problem. This method searches optimizers along geodesics of the smooth surface presented by the constraint set, and therefore all intermediate solutions are guaranteed.

Since \( a \) and \( \beta \) are complex, we denote by \( a_0, a_1, b_0, b_1 \) the real and imaginary parts of \( a \) and \( \beta \) respectively. Direct calculation will give the partial derivatives of \( \delta(t_n) \) with \( \theta, \phi, a_0, b_0, a_1, b_1 \) and \( \delta(t_n) \) (we omit here the proofs to save the space, and omit the formulae of the derivatives of the differentials of the differential forms expressing mathematical formulae by plain microsoft word). It is also straightforward to verify that these derivatives are real valued. The constraint can be rewritten as a product manifold of two spheres \( S^2 \times S^2 \times S^2 \), where \( S^2 \) is the 2-dimensional sphere.

The search for the minimizer will be carried out along the geodesic on the surface of \( S^2 \) corresponding to the partial derivatives. For this purpose, we first find, based on the partial derivatives, the gradient of \( q_i(\theta, \phi) \), which is a general derivative vector on to the tangent plane of \( S^2 \) at the current position \( \theta, \phi \).

\[ v_i = \frac{\partial q_i}{\partial \theta}, \quad w_i = \frac{\partial q_i}{\partial \phi} \]

Denote by \( v_i \) (diagonal matrix with its diagonal elements \( v_i \)) the tangent element at the diagonal as the elements of \( S^2 \), and let \( \text{d}v_i = 1 \) at 2. The matrix that projects the derivative vector on to the tangent plane is then given by (\( \text{d}v_i \)). The projection of \( q_i(\theta, \phi) \) on the space \( \text{d}v_i \) is therefore given by \( \text{d}v_i(q_i(\theta, \phi)) \) \( = q_i(\theta, \phi) \cdot \text{d}v_i \).

Using the geodesic formula for sphere \( S^2 \), it is easy to obtain the expression of \( \text{d}v_i(q_i(\theta, \phi)) \).

Claim: The geodesic of \( M \) emanating from \( \theta, \phi \) is given by

\[ \gamma(t) = \left( \gamma(t), \omega(t), \dot{\gamma}(t) \right) = \left( \sin(\theta(t)), \cos(\theta(t)), \dot{\gamma}(t) \right) \]

This result has been proven in [11] in the pulse-width compression problem. Algorithm:

Data: Parameters \( A(0), B \) in (0,1), \( B \in (0,1) \), \( t_0 = 0 \).

Step 1: Set \( k = 0 \) and select \( w_0 \).

Step 2: Compute \( q_k(w_0) \).

Step 3: Compute \( \text{grad}(q_k(w_0)) \).

If \( \text{grad}(q_k(w_0)) \cdot \text{proj} \neq 0 \), then go back to Step 1.

4. CONCLUSIONS

A new algorithm is proposed to optimize a pulse-width compression problem. This algorithm is better for real time implementation. Since many PMD compensation problems are constrained optimization problems with variables as Jones vector components, this technology can easily be extended to all these cases.

REFERENCE


L = 10 cm. Bottom row: same spectrum at L = 10 cm, 30 cm, 1 m, and 2 m. Left figures are for dispersion profile D1 and right figures for D2, respectively. For case D1 additional spectral line is shown for L = 1.7 m.

We use the standard split-step Fourier method with 2^11 points in a time window of T = 236 ps in our longest simulation out to L = 1.7 m. The photon number is conserved to within 5% of its initial value. Due to our large spectral window (405 nm - 1613 nm) we see in Fig. 1 (left) the emergence of FWM stokes and anti-stokes waves at the wavelengths \( \lambda_\text{s} = 1100 \text{ nm} \) and \( \lambda_\text{as} = 850 \text{ nm} \) for which the phase matching condition \( \Delta \phi = 180 \text{ ps/km} \) is satisfied. The spectral window presented in [6] was narrower and thus \( \lambda_\text{s} \) and \( \lambda_\text{as} \) were not observed. We find the maximum FWM parametric gain to be twice the maximum SRS gain, which explains why the FWM stokes and anti-stokes components appear before the SRS components.

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From Fig. 1 we see that the FWM stokes and anti-stokes components are generated they broaden much more in the same way as the central part of the spectrum around the pump. The merging of the spectral parts around \( \lambda_\text{s} \) and \( \lambda_\text{as} \) would create an ultra broad spectrum as observed in tapered fibers with femtosecond pulses [2, 4]. However, in this particular case the FWM stokes and anti-stokes lines are too far away for a merging of the generated SC to be observed.

In Fig. 2 we show the spectrum for dispersion profiles D3-D4, which have two Stokes and two anti-Stokes lines. The corresponding dispersion coefficients \( \beta_2, \beta_4, \beta_6 \) are \( \beta_2 = 0.285 \text{ ps}^2/\text{km}, \beta_4 = 0.05 \text{ ps}^4/\text{km}, \beta_6 = -0.29 \text{ ps}^6/\text{km} \) for D3, and \( \beta_2 = -1.3 \text{ ps}^2/\text{km}, \beta_4 = -2.6 \text{ ps}^4/\text{km}, \beta_6 = -5.8 \text{ ps}^6/\text{km} \) for D4. For the dispersion profile D4, cascaded direct degenerate FWM peaks appear. This leads to an even broader SC of approx. 600 nm within 20 dB.

For the dispersion profile D5 we have \( \beta_2 = -1.1 \text{ ps}^2/\text{km}, \beta_4 = 2.7 \text{ ps}^4/\text{km}, \beta_6 = -3.9 \text{ ps}^6/\text{km} \). In Fig. 4 the spectrum for the dispersion profile D5 is shown and for this case the direct degenerate FWM process generates three stokes and anti-Stokes lines. As seen this improves a lot the spectrum for shorter fiber lengths. For longer lengths, as \( \Delta \phi = 180 \text{ ps/km} \), the wavelengths of the stokes and anti-Stokes lines shift due to the input power depactivation and dips in the final SC spectrum are formed. To overcome this feature we increase slightly the input power to \( P = 600 \text{ W} \), and as is seen from Fig. 4 the resulted spectrum is 800 nm wide within 20 dB.

In Fig. 5 we show the spectrum for the dispersion profile D6, which has all three stokes and anti-Stokes lines. The input power is 500 W and the spectrum for the dispersion profile D6 is shown and it is seen that for this case the direct degenerate FWM process generates three stokes and anti-Stokes lines. As seen this improves a lot the spectrum for shorter fiber lengths. For longer lengths, as \( \Delta \phi = 180 \text{ ps/km} \), the wavelengths of the stokes and anti-Stokes lines shift due to the input power depactivation and dips in the final SC spectrum are formed. To overcome this feature we increase slightly the input power to \( P = 600 \text{ W} \), and as is seen from Fig. 4 the resulted spectrum is 800 nm wide within 20 dB.

The absence of stokes and anti-Stokes lines from the direct degenerate FWM process in the experiments in [6], is explained with the violation of the phase matching condition \( \Delta \phi \) due to irregularities along the fiber. We estimated the robustness of the proposed method for SCG, towards random dispersion fluctuations along the fiber. We consider the highly nonlinear PCF fiber reported in [7], and estimate the variation of the dispersion coefficients \( \beta_2 \) and \( \beta_6 \) to be: \( < \beta_2 > - < \beta_6 > = 9.5 \% \). We assume that the variation \( < \beta_2 > \) for all dispersion coefficients \( \beta_2, \beta_6 \) is 10%. The random process is Gaussian white noise. From Fig. 5 it is seen that for dispersion profile D1, the random dispersion variations significantly reduce the effectiveness of the direct degenerate FWM and it finally does not contribute to the generated SC.

This is explained by the narrow gain-band for direct degenerate FWM for the dispersion profile D1, strongly reducing the average gain. However for dispersion profile D2, the generation of the stokes and anti-stokes lines is more robust, due to the broader gain band-width. By a more thorough theoretical study it can be shown that for dispersion profiles D3-D5 the bandwidth of the direct degenerate FWM gain profile is even broader. This allows us to conclude that for the modified dispersion profiles D2-D5, the improvement of SCG due to generation and merging of stokes and anti-Stokes lines is robust to fiber imperfections.

We have shown that the direct degenerate FWM can significantly improve the SCG due to broadening and final merging of stokes and anti-stokes lines. The bandwidth of the achieved SC can be optimized by a proper design of the dispersion. The robustness of the process towards variations in the dispersion coefficients along the PCF was also investigated. It is shown that for a fiber with a modified dispersion profile the generation and final merging of the stokes and anti-Stokes lines with the main SC part is robust.

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References