Induced Kerr effects and self-guided beams in quasi-phase-matched quadratic media [CBC4]

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The quasi-phase-matching (QPM) technique has been studied intensively in the context of second-harmonic generation [1]. It relies on a periodic modulation of the nonlinear susceptibility and/or refractive index, by which an additional (grating) wavevector is introduced, that can compensate for the mismatch between the wavevectors of the fundamental and second-harmonic waves. With the QPM technique, phase-matching becomes possible at ambient temperatures, and does not introduce difficulties been overcome and stable techniques been developed [3]. The important question whether QPM can be used to support solitary waves in materials, has therefore not been addressed so far? To do this we derive effective equations that include both quadratic \((\chi^{(2)})\) and induced cubic nonlinearities. We use these to prove the existence of a novel class of solitary waves, QPM solitons.

We show that quasi-phase-matching of quadratic media induces Kerr effects, such as self- and cross-phase modulation, and leads to the existence of a novel class of solitary waves, QPM solitons.

We consider the interaction of a CW beam (carrier frequency \(\omega\) and wavenumber \(k_\omega\)), and its second harmonic, as they propagate in a QPM \(\chi^{(2)}\) slab waveguide, where only the \(\chi^{(2)}\) susceptibility is modulated. The propagation along \(z\) of the slowly varying envelope of the fundamental, \(\mathcal{W}(x, z)\), and the second harmonic, \(\mathcal{V}(x, z)\), is governed by the normalized equations (see [1,2])

\[
\begin{align*}
    i\partial_t W + 2\partial_x^2 W + d(z)W^2e^{-i\beta z} & = 0, \\
    i\partial_t V + \partial_x^2 V + d(z)W^2e^{i\beta z} & = 0,
\end{align*}
\]

where \(\beta=\Delta k|k_\omega|^2\) is proportional to the mismatch \(\Delta k=2k_\omega-k_{2\omega}\). Walk-off is neglected; it is usually not present in QPM media, since perpendicular or parallel polarization states can be used. The transverse coordinate \(x\) is measured in units of the normalization parameter \(x_0\), and \(z\) in units of the diffraction length \(l_d=x_0^2|k_\omega|\).

The periodic modulation of the \(\chi^{(2)}\) susceptibility is described by the grating function \(d(z)\) with amplitude \(1\) and domain length \(\pi/\kappa\) (see Fig. 1). Expanding \(d(z)\) in a Fourier series, \(d(z)=\sum_n d_ne^{inKz}\), and making the transformation \((\mathcal{W}, \mathcal{V})=(w, ve^{i\beta z})\), we obtain

\[
\begin{align*}
    i\partial_t w + 2\partial_x^2 w + d_1 w^* v + w^*\sum_{n\neq 1} d_ne^{i(n-1)Kz} & = 0, \\
    i\partial_t v + \partial_x^2 v - \beta v + \xi w^2 + w^2\sum_{n\neq 1} d_ne^{i(n+1)Kz} & = 0,
\end{align*}
\]

where \(\beta=\beta_\kappa\). Assuming the most efficient QPM of the first order, \(\kappa=\beta_\kappa\), the effective mismatch \(\beta\) is of order one or less (ideally 0), even though \(\beta\) might be large itself.

Equations (2) include coefficients that are periodically varying with the period \(2\pi/\kappa\). Thus the dynamics can be adequately described by averaged equations, if \(\kappa\) is sufficiently large. We consider therefore \(\kappa\gg 1\), and expand the fields in Fourier series, \(w=\sum_n w_ne^{inKz}\), \(v=\sum_n v_ne^{inKz}\), where \(w_n(x, z)\) and \(v_n(x, z)\) are assumed to vary slowly compared with \(e^{iKz}\). Assuming that the higher harmonics are of order \(\kappa^{-1}\) or smaller, compared to the averages \(w_0\) and \(v_0\), we then take into account only the lowest order perturbation in the equations for the harmonics, and derive the relations

\[
\begin{align*}
    w_n = (n\kappa)^{-1}d_{n+1}w_{n+1}v_0, \quad v_n = (n\kappa)^{-1}d_{n-1}w_n^2. \quad (3)
\end{align*}
\]

Inserting the harmonics (3) into the corresponding equations for \(w_0\) and \(v_0\), we obtain the average equations

\[
\begin{align*}
    i\partial_t w_0 + 2\partial_x^2 w_0 + d_1 w_0^* v_0 + (\gamma|w_0|^2 + \rho|v_0|^2)w_0 & = 0, \\
    i\partial_t v_0 + \partial_x^2 v_0 - \beta v_0 + \xi w_0^2 + 2\eta|w_0|^2 v_0 & = 0,
\end{align*}
\]

where \(\gamma, \rho, \) and \(\eta\) are all of order \(\kappa^{-1}\), and given by

\[
\begin{align*}
    \gamma = \kappa^{-1}\sum_{n\neq 0} \gamma_n/n, \quad \rho = \kappa^{-1}\sum_{n\neq 0} \rho_n/n, \quad \eta = \kappa^{-1}\sum_{n\neq 0} \eta_n/n,
\end{align*}
\]

with \(\gamma_n=d_{n-1}d_{n+1}, \rho_n=d_{n+1}^2d_{n+1}, \) and \(\eta_n=d_{n+1}^2d_{n-1}\). From Eqs. (4) follows the important result that the QPM grating induces an effective cubic nonlinearity in the form of self- and cross-phase modulation (SPM and CPM). However, the CPM term does not appear for the second

\[
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\]
harmonic, making the localized solutions and the system dynamics be different from earlier analyzed cases of competing nonlinearities [4].

In many physical applications the QPM grating is well approximated by the square function depicted in Fig. 1, for which \( d(z) \) contains only odd harmonics, \( d_{2n+1} = 2/\pi(2n+1) \). Then Eqs. (4) reduce to

\[
\begin{align*}
\frac{d^2 w_0}{dz^2} &= \left( 2a z + \beta \right) w_0 - i x w_0 \frac{\partial}{\partial z} w_0 + \gamma \left( |w_0|^2 - |v_0|^2 \right) w_0 = 0, \\
\frac{d^2 v_0}{dz^2} &= -i x w_0 \frac{\partial}{\partial z} v_0 + \beta v_0 + i x w_0 ^2 - 2i |w_0|^2 w_0 = 0,
\end{align*}
\]

(5)

where the nonlinearity coefficients are \( \chi = 2/\pi \), and \( \beta = \kappa = 0 \). Note the \( \pi/2 \) phase-shift in front of the quadratic terms and the opposite signs of the SPM and CPM terms \( (p=\eta=-\gamma) \). We will look for stationary solutions to Eqs. (5) in the form

\[
w_0(z, x) = \tilde{w}_0(x) e^{i \lambda z}, \quad v_0(z, x) = i \tilde{v}_0(x) e^{2i \lambda z},
\]

(6)

where the real and exponentially localized profiles \( \tilde{w}_0(x) \) and \( \tilde{v}_0(x) \) decay monotonically to zero as \( |x| \) increases. The real eigenvalue \( \lambda \) must satisfy \( \lambda > \max \{ 0, -\beta/2 \} \), in order for \( \tilde{w}_0 \) and \( \tilde{v}_0 \) to be exponentially localized. In the original variables, the stationary solutions correspond to self-guided beams with rapidly oscillating intensities given by \( I_w = \tilde{w}_0^2 + \Delta I \) and \( I_v = \tilde{v}_0^2 - \Delta I \), where

\[
\Delta I = \frac{8 \tilde{w}_0^2 \tilde{v}_0}{\pi \kappa} \sum_{n=1}^{\infty} \frac{\cos(2n \kappa z)}{4n^2 - 1}.
\]

(7)

Since \( d(z) \) is real, Eqs. (1) conserve the total power

\[
P = \int_{-\infty}^{\infty} (I_w + I_v) dx = \int_{-\infty}^{\infty} (\tilde{w}_0^2 + \tilde{v}_0^2) dx.
\]

Using QPM solitons as initial condition we solve Eqs. (1) numerically for the square grating, and \( \beta = 0 \). After a steady state is reached we record the peak intensities in a whole number of periods \( \pi/\kappa \), and calculate the average, and max. and min. values in a period. The measured ratio of the average peak intensities of the excited solitons, shown in Fig. 3a, deviates clearly from the prediction of the zero-order approximation \( (\gamma = 0) \), but is in excellent agreement with the theory that takes into account the induced Kerr effect. Also the measured max. and min. values, shown in Fig. 3b, are in perfect agreement with our theory.

In conclusion, we have shown that self-guided beams in QPM \( \chi(2) \) slab waveguides are strongly influenced by induced Kerr effects, such as SPM and CPM. We have numerically found the new families of rapidly oscillating, spatial solitary waves in the physically relevant regime of the QPM domain length and the phase-mismatch.