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Analysis of Bit-Stuffing Codes and Lower Bounds on Capacity for 2-D Constrained Arrays using Quasi-Stationary Measures

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Abstract — A method for designing quasi-stationary probability measures for two-dimensional (2-D) constraints is presented. This method is derived from a modified bit-stuffing coding scheme and it gives the capacity of the coding scheme. This provides a constructive lower bound on the capacity of the 2-D constraint. The main examples are checkerboard codes with binary elements. The capacity for one instance of the modified bit-stuffing for the 2-D run-length-limited RLL(2,∞) constraint is calculated to be 0.4414 bits/symbol. For the constraint given by a minimum (1-norm) distance of 3 between 1s a code with capacity 0.3497 bits/symbol is given.

I. INTRODUCTION
We present a method for designing two-dimensional (2-D) constrained codes based on bit-stuffing. We consider 2-D arrays with elements taken from a finite alphabet. The constraint is specified by the set of admissible configurations on an N by M rectangle. For 2-D RLL(1,∞), bit-stuffing provides efficient coding [1]. In [2] bit-stuffing for 2-D RLL(d,∞), d ≥ 2 were considered. We shall take a slightly different approach to bit-stuffing in order to facilitate analysis e.g. providing a constructive lower bound on the capacity of the constraint. The method is generally applicable to checkerboard constraints, where a 1 must be surrounded be a certain pattern of 0s, meaning that a 0 is always admissible.

II. QUASI-STATIONARY MEASURES
A quasi stationary measure may be introduced by concatenating arrays. Given a constraint, let \( W \) denote a stochastic variable defined on an \( n \) by \( m \) array, which may take on any configuration admissible by the constraint. Let \( X \) and \( Z \) denote variables representing the first and last \( M - 1 \) columns (with \( n \) elements). Let \( Y \) denote a variable representing the middle \( m - 2M + 2 \) columns. We assume that the measures on the boundaries, \( X \) and \( Z \) are identical for the measures, \( W \) to be considered. Thus, starting with \( X_0 Y_0 Z_0 \), arrays \( Y, Z \) may repeatedly be added to form \( X_0 Y_0 Z_0(X, Y, Z)\) \( \mathbb{Z}^\infty \), such that \( Z_{i-1}, Y_i, Z_i \) has the same measure as \( W \). The entropy (per symbol) is given by the conditional entropy of \( Y, Z_i \) given \( Z_{i-1}, \) which is

\[
H_W(m) - H_X(M-1) - H_X(M-1) = \frac{H_W(m)}{m - M + 1}
\]

where \( H_W(m) \) is the entropy of \( W \) (per row) and \( H_X(M-1) \) is the entropy of \( X \) (per row). A simple way to specify \( W \) in (1) is to assign probabilities to the bit-stuffing scheme below. The boundaries \( X \) and \( Z \) are specified by identical but independent bit-stuffing schemes. The middle columns \( Y \) are specified by bit-stuffing conditional on the boundaries \( X \) and \( Z \).

III. NUMERICAL RESULTS
Two examples with binary elements and constraint size \( N = M = 3 \) are considered. For the RLL(2,∞) constraint, analysis of the modified bit-stuffing was carried out calculating capacities, \( C \) for \( m = 12 \). The transition probabilities for a new row of \( W \) were determined by the products of (conditional) probabilities adding and bit-stuffing the elements of the new line of \( X \) and \( Z \) before \( Y \) and using the same conditional probabilities for the corresponding elements of \( X \) and \( Z \). Thus the prerequisites for (1) is satisfied. Let \( p_1 \) denote the probabilities of writing a 1 when this is admissible. Simple bit-stuffing writing an unbiased sequence with \( p_1 = 1/2 \) gave \( C = 0.388 \). Using a single biased sequence gave \( C = 0.437 \) for optimal choice of \( p_1 \). Finally the values of \( p_1 \) may be chosen independently for each column of \( X \) and \( Y \). The \( p_1 \) values of \( Z \) are given by \( X \). This gave a best value of \( C = 0.4419 \), also providing a lower bound for the constraint. This is a fair improvement on the lower bound of 0.4267 on the capacity of (diagonal) bit-stuffing in [2].

Capacities were also calculated for applying the modified bit-stuffing scheme to the constraint given by a min. (1-norm) distance of 3 between 1s. The results obtained for \( m = 15 \) were \( C = 0.276 \) when writing an unbiased sequence, \( C = 0.344 \) for a single biased sequence and \( C = 0.3477 \) choosing different biased sequences for each column of \( X \) and \( Y \). For this constraint the boundaries, \( Z_{i-1} \) and \( Z_i \) must be at least an additional row ahead in order to bit-stuff the elements of \( X \) and \( Z \) independently of past elements of \( Y \). A more elaborate scheme for specifying \( W \) in (1) was also devised. The probabilities \( p_1 \) were made dependent on the other elements on the \( (N-1) = 2 \) previous rows. The next row of \( X \) (and \( Z \) is specified by probabilities conditioned on 3 rows of \( X \) and \( Z \) and 2 rows of \( Y \). These conditional probabilities were obtained from the maxentropic solution [3] for \( W \) (with two rows forming the states). This gave a capacity of \( C = 0.3497 \), which also provides a new lower bound for the constraint.

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