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Analysis of Bit-Stuffing Codes and Lower Bounds on Capacity for 2-D Constrained Arrays using Quasi-Stationary Measures

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Abstract — A method for designing quasi-stationary probability measures for two-dimensional (2-D) constraints is presented. This method is derived from a modified bit-stuff coding scheme and it gives the capacity of the coding scheme. This provides a constructive lower bound on the capacity of the 2-D constraint. The main examples are checkerboard codes with binary elements. The capacity for one instance of the modified bit-stuffing for the 2-D runlength-limited RLL(2, ∞) constraint is calculated to be 0.4414 bits/symbol.

I. INTRODUCTION

We present a method for designing two-dimensional (2-D) constrained codes based on bit-stuffing. We consider 2-D arrays with elements taken from a finite alphabet. The constraint is specified by the set of admissible configurations on an N by M rectangle. For 2-D RLL(1, ∞), bit-stuffing provides efficient coding [1]. In [2] bit-stuffing for 2-D RLL(d, ∞), d ≥ 2 were considered. We shall take a slightly different approach to bit-stuffing in order to facilitate analysis e.g. providing a constructive lower bound on the capacity of the constraint. The method is generally applicable to checkerboard constraints, where a 1 must be surrounded by a certain pattern of 0s, meaning that a 0 is always admissible.

II. QUASISTATIONARY MEASURES

A quasi-stationary measure may be introduced by concatenating arrays. Given a constraint, let W denote a stochastic variable defined on an n by m array, which may take on any configuration admissible by the constraint. Let X and Z denote variables representing the first and last M – 1 columns (with n elements). Let Y denote a variable representing the middle m – 2M + 2 columns. We assume that the measures on the boundaries, X and Z are identical for the measures, W to be considered. Thus, starting with X0, Y0, Z0, arrays Y, Z may repeatedly be added to form X0(Y, Z)∞ such that Z−1, Y, Z has the same measure as W. The entropy (per symbol) is given by the conditional entropy of Y, Z given Z−1, which is

\[ H_W(m) - H_X(M - 1) \]

\[ m - M + 1 \]

where \( H_W(m) \) is the entropy of W (per row) and \( H_X(M - 1) \) is the entropy of X (per row). A simple way to specify W in (1) is to assign probabilities to the bit-stuffing scheme below. The boundaries X and Z are specified by identical bit-stuffing schemes. The middle columns Y are specified by bit-stuffing conditional on the boundaries X and Z.

III. NUMERICAL RESULTS

Two examples with binary elements and constraint size N = M = 3 are considered. For the RLL(2, ∞) constraint, analysis of the modified bit-stuffing was carried out calculating capacities, C for m = 12. The transition probabilities for a new row of W were determined by the products of (conditional) probabilities addressing and bit-stuffing the elements of the new line of X and Z before Y and using the same conditional probabilities for the corresponding elements of X and Z. Thus the prerequisites for (1) is satisfied. Let \( p_1 \) denote the probabilities of writing a 1 when this is admissible. Simple bit-stuffing writing an unbiased sequence with \( p_1 = 1/2 \) gave \( C = 0.388 \). Using a single biased sequence gave \( C = 0.437 \) for optimal choice of \( p_1 \). Finally the values of \( p_1 \) may be chosen independently for each column of X and Y. The \( p_1 \) values of Z are given by X.) This gave a best value of \( C = 0.4414 \), also providing a lower bound for the constraint. This is a fair improvement on the lower bound of 0.4267 on the capacity of (diagonal) bit-stuffing in [2].

Capacities were also calculated for applying the modified bit-stuffing scheme to the constraint given by a min. (1-norm) distance of 3 between 1s. The results obtained for m = 15 were \( C = 0.276 \) when writing an unbiased sequence, \( C = 0.344 \) for a single biased sequence and \( C = 0.3477 \) choosing different biased sequences for each column of X and Y. For this constraint the boundaries, Z−1 and Z, must be at least an additional row ahead in order to bit-stuff the elements of X and Z independently of past elements of Y. A more elaborate scheme for specifying W in (1) was also devised. The probabilities \( p_1 \) were made dependent on the other elements on the \((N - 1 =) \) 2 previous rows. The next row of X (and Z) is specified by probabilities conditioned on 3 rows of X and Z and 2 rows of Y. These conditional probabilities were obtained from the maxentropic solution [3] for W (with two rows forming the states). This gave a capacity of \( C = 0.3497 \), which also provides a new lower bound for the constraint.

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