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Letter to the Editor: Electric Vehicle Demand Model for Load Flow Studies
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Letter to the Editor:
Electric Vehicle Demand Model for Load Flow Studies

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Abstract This article introduces a specific and simple model for electric vehicles suitable for load flow studies. The electric vehicles demand system is modeled as a PQ bus with stochastic characteristics based on the concept of the queuing theory. All appropriate variables of stochastic PQ buses are given with closed formulas as a function of charging time. A specific manufacturer model of electric vehicles is used as study case.

Keywords electric vehicle demand, load flow analysis, steady state

1. Introduction

With the introduction of electric vehicles (EVs) into the transport system, the energy sector (mainly the electric power system) will suffer a dramatic change due to this important and expected issue. Therefore, there is a need to develop new models to facilitate the operation and optimal control of the new structure of the power systems.

In the state-of-the-art published material related to EV integration to the electric networks [1–6], there is no model available that incorporates these elements for load flow studies. This article covers this void by introducing a specific and simple model for EVs suitable for load flow studies.

The proposed model considers EV demand systems as PQ buses with stochastic characteristics. The development of the model is based on the queuing theory [7]. The presented formulas for stochastic real and reactive power demand (P, Q) are developed as a function of the charging time (time to be plugged into the power network). The Tesla Roadsters EV model [8] is adopted as a case study; its technical characteristics are considered, and related results are presented.

2. EV Model for Load Flow Studies

From the open literature, it is well known that all battery systems in EVs are chemical storage devices, and their charge/discharge modus operandi is the chemical process...
Therefore, they are exponential functions over time. In this study, the instantaneous charging status of the battery systems of Tesla Roadsters EV [8] is simulated by the following exponential formula:

\[ P_{EV}(t) = P_{EV,\text{max}} \cdot (1 - e^{-\alpha t / t_{\text{max}}}) + P_{EV,0}, \]  

(1)

where the current status of the battery system in kW is simulated by \( P_{EV,0} \), the maximum power capacity for the Tesla Roadster EV is \( P_{EV,\text{max}} = 189 \) kW, the maximum charging time is \( t_{\text{max}} = 3.5 \) hr [8].

The constant parameter is calculated at the value of \( \alpha = 6.9077 \), assuming that a fully discharged battery system of the Tesla Roadster EV (\( P_{EV,0} = 0 \) kW) absorbs 90% of maximum power capacity \( P_{EV,\text{max}} \) in approximately one-third of the maximum charging time \( t_{\text{max}}/3 = 70 \) min [8].

Figure 1 shows the charge process of the battery systems of a Tesla Roadster EV for three different initial active power status: \( P_{EV,0} = 0 \) kW, \( P_{EV,0} = 50 \) kW, and \( P_{EV,0} = 100 \) kW. From these curves, it is observed that the three battery systems need approximately 3.5, 0.6737, and 0.3225 hr, respectively, to be fully charged. Obviously, the time to be fully charged is related to its initial active power status. It is assumed that each plug-in EV needs \( t_c \) hours to be fully charged. Using Eq. (1), it is obtained that

\[ P_{EV,0} = P_{EV,\text{max}} \cdot e^{-\alpha t / t_{\text{max}}}. \]  

(2)

Equation (2) can be verified with the obtained values of the full charging time for the three different initial active power statuses in Figure 1. Then, the active power of each fully charged EV is given by

\[ P_{EV,\text{dem}} = P_{EV,\text{max}} - P_{EV,0}. \]  

(3)

From Eqs. (2) and (3), the active power demand of each Tesla Roadster EV is given by

\[ P_{EV,\text{dem}} = P_{EV,\text{max}} \cdot (1 - e^{-6.9077 t / t_{\text{max}}}). \]  

(4)
Furthermore, considering the unity constant power factor of the battery systems [3, 4, 8], the reactive power demand is given by

$$Q_{EV,dem} = 0.0.$$  \hspace{1cm} (5)

### 3. EV System Demand Model

In this article, the power demand model of the EV system at each load bus is developed based on the customer service model originated in the queuing theory [7]. In the $M/M/n_{max}$ queue, $M$ means the exponential distribution of incoming for charging EVs with a mean of inter-arrival times $1/\lambda$, mean of service times $1/\mu$, and $n_{max}$ maximum parallel charging EVs at each load bus (subsequently called the charging capacity of the load bus). Assuming that there is no time delay in the start of the charge for every EV in the network, it is considered that the mean rate of plugged-in EVs is approximately equal to the mean rate of plugged-out (charged) EVs: $\lambda \approx \mu$. So, the occupation rate of each load bus is [7]

$$\rho = \frac{\lambda}{\mu \cdot n_{max}} \approx \frac{1}{n_{max}}.$$  \hspace{1cm} (6)

Let $p(n)$ denote the probability that there is an $n$-EV system under the charging process at each load bus. In accordance to the $M/M/n_{max}$ queue theory and the above assumption, the probability $p(n)$ is given by [7]

$$p(n) \approx \frac{1/n!}{\sum_{i=0}^{n_{max}-1} \frac{1}{i!} + \frac{1}{(1 - 1/n_{max}) \cdot n_{max}!}},$$  \hspace{1cm} (7)

where $n = 0, 1, 2, \ldots, n_{max}$.

During the off-line load flow process, uniformly distributed random numbers are used in order to calculate the value of charging EVs ($n$) at each load bus following the process.

For each load bus, generate a random number $r[0, 1]$ and set as $n$ the lowest value in the set of values $[0, 1, 2, \ldots, n_{max}]$, which satisfies the inequality

$$r[0, 1] > p(n).$$  \hspace{1cm} (8)

The full charging time (the service time in the point-of-view of the queuing theory) of each EV is ($t_c$) follows the exponential possibility distribution with mean of $1/(\mu \cdot n_{max})$.

The density function of this distribution is given by

$$f(t_c) = \mu \cdot n_{max} \cdot e^{-\mu \cdot n_{max} \cdot t_c}.$$  \hspace{1cm} (9)

During the off-line load flow process, uniformly distributed random numbers $r(0, 1]$ are used for simulating the charging time of each $i$ EV ($t_{ci}$) at each load bus as follows [9]

$$t_{ci} = \begin{cases} 
  t_{max}, & \text{if } r(0, 1] < \mu \cdot n_{max} \cdot e^{-\mu \cdot n_{max} \cdot t_{max}} \\
  \frac{1}{\mu \cdot n_{max}} \cdot \ln \left( \frac{r(0, 1]}{\mu \cdot n_{max}} \right), & \text{if } r(0, 1] \geq \mu \cdot n_{max} \cdot e^{-\mu \cdot n_{max} \cdot t_{max}}.
\end{cases}$$  \hspace{1cm} (10)
Conclusively, a system of $n$ Tesla Roadster EVs demands active power from the network given by the following stochastic formula:

$$P_D = P_{EV,max} \cdot (n - e^{-6.9077t_{max}/t_{max}} - e^{-6.9077t_{max}/t_{max}} - \ldots - e^{-6.9077t_{max}/t_{max}}).$$  \hspace{1cm} (11)$$

For remanding purposes, a fully discharged battery system of a Tesla Roadster EV needs approximately $t_{max} = 3.5$ hr in order to achieve the 90% of maximum power capacity $P_{EV,max} = 189$ kW. The number of charging Tesla Roadster EV ($n$) at each load bus, and the full charging time of each of them ($t_{ci}$) are calculated using Eqs. (7), (8), and (10), respectively. The charging capacity of each load bus ($n_{max}$) and the mean rate of charging the EVs ($\mu$) at each load bus would be derived from measurements on a real system. Since there are none at the moment, in this article they are given as constant parameters.

4. Results

Figure 2 shows the real power demand (kW) of the Tesla Roadster EV [8] as a function of full charging time $t_c$, varying in the range of 0–3.5 hr. This study also represents the relation between the full charging time $t_c$ and the initial status of the battery systems in Figure 3. Figures 2 and 3 are confirmed by similar experimental plots that appeared in [4] and [6].
Considering the charging capacity of a load bus \( n_{\text{max}} = 10 \) and mean rate of a charging EVs \( (\mu = 8) \) at the same load bus, the stochastic real power demand of an \( n \)-EV system is given in Figure 4. The real power demand (Eq. (11)) is calculated, generating \( 10^4 \) samples of \( (n + 1) \)-tuples \((n, t_{c1}, t_{c2}, \ldots, t_{cn})\). From Figure 5, the basic load of this small test case study \( (n_{\text{max}} = 10, \mu = 8) \) is approximately calculated at 64 kW. Based on this study, and for further assessments, the basic load demand of larger \( n \)-EV systems and their impact on the energy market capacity will be evaluated.
5. Conclusions

In this article, a simple model for an EV demand system suitable for load flow studies is introduced. The developed model considered the EV demand systems as $PQ$ buses with stochastic characteristics. The proposed demand system is developed as inspired by the queuing theory. Closed formulas for the real and reactive power of the EV demand system are developed as a function of the charging time. The proposed model is confirmed using the Tesla Roadster EV as a case study.

References