Oscillators and operational amplifiers

Lindberg, Erik

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Oscillators and Operational Amplifiers

Erik Lindberg, IEEE Lifemember

Abstract — A generalized approach to the design of oscillators using operational amplifiers as active elements. A piecewise-linear model of the amplifier is used so that it makes sense to investigate the eigenvalues of the Jacobian of the differential equations. The characteristic equation of the general circuit is derived. The dynamic nonlinear transfer characteristic of the amplifier is investigated. Examples of negative resistance oscillators are discussed.

1 INTRODUCTION

When you want to design an oscillator the Barkhausen Criteria is normally used as a starting point i.e. a linear circuit with poles on the imaginary axis is designed [1, 2]. In order to start-up oscillations some parameters are changed so that the poles are in the right half of the complex frequency plane. The linear circuit becomes unstable and the signals will grow until infinity i.e. we must introduce nonlinearity in order to limit the signal amplitude.

- Oscillators are nonlinear circuits.

In the following an oscillator design procedure is described. It is based on the idea of "frozen eigenvalues" [3]. By means of piecewise linear modelling of the nonlinearities the circuit is linear in time slots so that it makes sense to study the eigenvalues (poles) of the Jacobian of the differential equations and see whether the signals are increasing or decreasing in amplitude in a certain time slot. If you observe that the poles are moving back and forth between the right half (RHP) and the left half (LHP) of the complex frequency plane the mechanism behind the behavior of an oscillator may be described as an act of balance between the energy you gain from the dc power supply when the poles are in RHP and the energy you lose when the poles are in LHP. The characteristic equation for a general linear circuit with a perfect amplifier (op amp) RC4136 is derived by means of PSpice analysis.

2 THE CHARACTERISTIC EQUATION

Figure 1 shows the general circuit which is an op amp with positive and negative feed-back. If we introduce memory elements - capacitors, coils, hysteresis - in the four admittances various types of oscillators may be obtained. The nonlinearity needed for oscillations may be introduced as nonlinear losses or hysteresis in connection with the memory elements. If we introduce linear memory elements with no hysteresis a nonlinear transfer characteristic of the amplifier must be introduced. Let us assume that the amplifier is a perfect amplifier with infinite input impedance, zero output impedance and piecewise linear gain A. The gain is very large for small signals and zero for large signals. Now a network-function may be calculated e.g. the transfer function $V(3)/V_{IN}$. The relation between the output voltage $V(3)$ and the input voltage $V_{IN}$ becomes:

$$V(3) \times \left( \frac{YB}{YA+YB} - \frac{YC}{YC+D} - \frac{1}{A} \right) = V_{IN} \times \left( \frac{YD}{YC+YD} \right)$$

(1)

If we observe that $V(3)$ is different from zero when $V_{IN}$ is zero then the coefficient of $V(3)$ must be zero i.e.

$$\left( \frac{YB}{YA+YB} - \frac{YC}{YC+YD} - \frac{1}{A} \right) = 0$$

(2)

Equation (2) is called the characteristic equation. The characteristic polynomial of the linearized differential equations describing the circuit
may be derived from this equation. For infinite gain
\( A = \infty \) the equation becomes:
\[
(YA 	imes YC) - (YB 	imes YD) = 0 \quad (3)
\]
For zero gain \( A = 0 \) the equation becomes:
\[
(YA + YB) \times (YD + YC) = 0 \quad (4)
\]
Table 1 is derived from the characteristic equation. It shows that it is possible to place the frequency
determining circuit in any of the four admittances.

<table>
<thead>
<tr>
<th>Case</th>
<th>Gain:</th>
<th>( A = 0 )</th>
<th>( A = \infty )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>( YA )</td>
<td>+RB</td>
<td>-RB \times RD/RC</td>
</tr>
<tr>
<td>B</td>
<td>( YB )</td>
<td>+RA</td>
<td>-RA \times RC/RD</td>
</tr>
<tr>
<td>C</td>
<td>( YC )</td>
<td>+RD</td>
<td>-RD \times RB/RA</td>
</tr>
<tr>
<td>D</td>
<td>( YD )</td>
<td>+RC</td>
<td>-RC \times RA/GB</td>
</tr>
</tbody>
</table>

Table 1: The "load" of an admittance "YX" for
gain zero and gain \( \infty \) when the other admittances
are conductors (\( GX = 1/RX \)).

simple capacitor \( C \) or a simple coil \( L \) and the other
three admittances are chosen as conductors (resis-
tors) multi-vibrator oscillators might be designed.
If one of the admittances of Fig. 1 is chosen as a
simple capacitor \( C \) or a simple coil \( L \) and the other
three admittances are chosen as conductors (resis-
tors) multi-vibrator oscillators might be designed.

<table>
<thead>
<tr>
<th>Case</th>
<th>( D )</th>
<th>( A )</th>
</tr>
</thead>
<tbody>
<tr>
<td>RA</td>
<td>180Ω</td>
<td>RLC</td>
</tr>
<tr>
<td>RB</td>
<td>180Ω</td>
<td>180kΩ</td>
</tr>
<tr>
<td>RC</td>
<td>180kΩ</td>
<td>180Ω</td>
</tr>
<tr>
<td>RD</td>
<td>RLC</td>
<td>180Ω</td>
</tr>
</tbody>
</table>

Table 2: Resistor values for two cases \( D \) and \( A \).
\( RLC \) is a two-terminal with a capacitor \( C = 1nF \)
in parallel with a coil \( L = 256\text{mH} \) with series resis-
tance \( RL = 14.8Ω \).

3 NEGATIVE RESISTANCE OSCILLA-
TORS

In [4] a 10kHz negative resistance oscillator is de-
signed where \( YD \) is chosen as a coil \( L = 256\text{mH} \)
(with a series loss resistor \( R_D = 14.8Ω \)) in parallel
with a capacitor \( C_D = 1nF \). The op amp \( RC4136 \) is
used. The components \( YA, YB \) and \( YC \) are chosen
according to case \( D \) of Table 2.

Figure 2 case \( D \) shows the dynamic transfer func-
tion of the op amp. The gain \( A \) is varying almost
piecewise linear between zero and \( +2.3305 \) corre-
sponding to the poles:
\[-2.801k\pm j \times 62.44k \quad (Q = +11, f = 9.94kHz) \]
and
\[+36.37k\pm j \times 50.78k \quad (Q = -0.86, f = 8.08kHz) \]
respectively in the complex frequency plane.

The \( Q \) of a pole specified above is defined as
\[
Q = \sqrt{(\alpha^2 + \omega^2)/(-2 \times \alpha)} \quad (5)
\]
It is a measure for the distance of the pole from the
imaginary axis. It is seen that \( Q \) becomes \( \infty \) for poles
on the imaginary axis. \( Q \) is negative for poles in the
right-half-plane RHP and positive for poles in the
left-half-plane LHP. The real part of the pole may be calculated from
\[
\alpha = \omega/\sqrt{4 \times Q^2 - 1} \quad (6)
\]
or \( 2\alpha = \omega/Q \) for large \( Q \).

Now the sign of the gain of the op amp is changed

corresponding to inverting the input terminals or
to interchange \( YA, YB \) and \( YD, YC \). This corre-
sponds to case \( A \) of Table 2.

Figure 2 case \( A \) shows the dynamic transfer
function of the op amp. The gain \( A \) is varying
almost piecewise linear between zero, -12.7 and
-565 corresponding to the poles:
\[-2.801k\pm j \times 62.44k \quad (Q = +11, f = 9.94kHz) \]
\[+1.993k\pm j \times 62.46k \quad (Q = -16, f = 9.94kHz) \]
and
\[+2.729k\pm j \times 62.43k \quad (Q = -11, f = 9.94kHz) \]

From Fig. 2 it is seen that both in case \( D \) and in
case \( A \) the poles follows the whole trajectory i.e. a
complex pole pair goes to the real axis and split up
into two real roots. One real root goes against zero
and the other real root goes to \( \infty \). The two real
roots meet again and a complex pole pair leaves
the real axis and crosses the imaginary axis [4].
In case D the fast transition from positive gain to zero gain via infinite gain happens at large values of the op amp input voltage \(V(1,2)\). In case A the fast transition from negative gain to zero gain via infinite gain happens at small values of the op amp input voltage \(V(1,2)\).

Please note that it is not a jump when the real pole goes from \(+\infty\) to \(-\infty\). The Figures 3 and 4 show the gain as function of time for positive gain and negative gain respectively.

An old rule says that if your oscillator do not work then change the input terminals of the op amp. Here is given an example where the oscillator works then change the input terminals of the op amp and negative gain respectively.

\[
\begin{align*}
\text{Figure 3: Gain } A &= dV(3)/dV(1,2) \text{ and op amp inverting input } V(2) \text{ as functions of time, “positive gain”, case } D. \\
\text{Figure 4: Gain } A &= dV(3)/dV(1,2) \text{ and op amp non-inverting input } V(1) \text{ as functions of time, “negative gain”, case } A. 
\end{align*}
\]

From equation (2) we get:
\[
YC + \left( \frac{1}{RD} \right) \times \left( \frac{\text{Num}}{\text{Denom}} \right) = 0 \quad (8)
\]

where \(\text{Num} = RB - RA \times (A - 1)\) and \(\text{Denom} = RA + RB \times (A + 1)\)

The characteristic polynomial becomes:
\[
s^2 + 2\alpha s + \omega^2_0 = 0 , \quad (9)
\]

where
\[
2\alpha = \left( \frac{1}{L_c} \right) \times \left( \frac{R_c \times \text{Num} + RD \times \text{Denom}}{(RB - RA \times (A - 1))} \right) \quad (10)
\]

and
\[
\omega^2_0 = \frac{1}{L_c \times C_c} \quad (11)
\]

\(\alpha = 0\) for
\[
A = \frac{(RA + RB) \times (R_c + RD)}{(R_c \times RA - RD \times RB)} \quad (12)
\]

Table 3 shows corresponding values of \(A\) and \(\alpha\).

<table>
<thead>
<tr>
<th>(A)</th>
<th>(\alpha)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(\frac{(R_c + RD)}{(2 \times L_c)})</td>
</tr>
<tr>
<td>(\infty)</td>
<td>(\frac{(R_c \times RA - RB \times RD)}{(2 \times L_c \times RA)})</td>
</tr>
</tbody>
</table>

Table 3: Corresponding values of \(A\) and \(\alpha\)

The poles or the natural frequencies of the circuit - the eigenvalues of the Jacobian of the differential equations - are the roots of the characteristic polynomial.

\[
p_{1,2} = -\alpha \pm j \sqrt{\omega^2_0 - \alpha^2} = -\alpha \pm j \omega \quad (13)
\]

In order to place the poles close to the imaginary axis \(\alpha\) must be much smaller than \(\omega_0\). For \(Q = -10\) and \(\omega = 1/\sqrt{256m \times 17} = 62.5k\), from equation (6) \(\alpha\) becomes 3.128913589k. From Table 3 \((A = 0)\) the resistor \(RD\) is calculated as \(RD = 2\alpha \times L_c - R_c = 1.587203758k\Omega\) and from Table 1: choose \(RA = RB = 190\Omega\). From equation (12) the gain is calculated as \(A = -2.037649363\) which is negative. This indicates that we have to switch the input nodes of the op amp corresponding to interchange \(YA, YB\) and \(YD, YC\) i.e. \(YB\) becomes a RLC series circuit and we have a case B oscillator instead of a case C oscillator.

The complex pole pair on the imaginary axis is found as: For \(A = +2.037649364\), pole: \(+0.70\mu \pm j \times 62.5k\), \(Q = -0.5e + 12\) and for \(A = +2.037649363\), pole: \(-64n \pm j \times 62.5k\),

\[
ZC = \frac{1}{YC} = sL_e + R_e + \frac{1}{sC_c} \quad (7)
\]

with \(L_e = 256mH\), \(R_e = 14.8\Omega\) and \(C_c = 1nF\).
Figure 5: Op amp dynamic transfer-characteristic \( V(3) \) as function of input voltage \( V(2,1) \).

Figure 6: Gain \( A = dV(3)/dV(1,2) \) as function of time.

Figure 7: Gain \( A = dV(3)/dV(1,2) \) as function of time.

Table 4: The poles as function of the gain.

<table>
<thead>
<tr>
<th>Poles</th>
<th>Real</th>
<th>Imaginary</th>
<th>Gain A</th>
</tr>
</thead>
<tbody>
<tr>
<td>(+1e+12)</td>
<td>+3.071k</td>
<td>62.42k</td>
<td>(-10.0)</td>
</tr>
<tr>
<td>(+38.83)</td>
<td>+2.767k</td>
<td>62.44k</td>
<td>(-11.0)</td>
</tr>
<tr>
<td>(+1e-12)</td>
<td>-3.129k</td>
<td>62.42k</td>
<td>(+10.0)</td>
</tr>
<tr>
<td>(-2136)</td>
<td>-3.870k</td>
<td>62.38k</td>
<td>(+8.1)</td>
</tr>
<tr>
<td>(-8.64)</td>
<td>-4.939k</td>
<td>62.30k</td>
<td>(-6.3)</td>
</tr>
<tr>
<td>(-12.26)</td>
<td>-4.280k</td>
<td>62.35k</td>
<td>(-7.3)</td>
</tr>
<tr>
<td>(-360)</td>
<td>-3.106k</td>
<td>62.42k</td>
<td>(-10.0)</td>
</tr>
<tr>
<td>(-1917)</td>
<td>-3.078k</td>
<td>62.42k</td>
<td>(-10.0)</td>
</tr>
<tr>
<td>(-1e+12)</td>
<td>+3.071k</td>
<td>62.42k</td>
<td>(-10.0)</td>
</tr>
</tbody>
</table>

Table 4 shows the poles for the selected values of the gain. When the gain \( A \) is in the interval from \(-2.20865\) to \(-1.81090\) the poles are real. For \( A = -2 \) there are no poles. The dc-gain is found to \(+38.83\) which corresponds to a complex pole-pair in the RHP.

5 CONCLUSIONS

Insight in the mechanisms behind the behavior of oscillators based on operational amplifiers is obtained by means of the "frozen eigenvalues" approach [3]. The dynamic transfer characteristic of a specific operational amplifier (RC4136) is studied in connection with negative resistance oscillators. A design procedure is presented.

References