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# The Design of a 4'th order Bandpass Butterworth Filter with one Operational Amplifier

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**Abstract** – A numerical design method is presented for the design of all pole band pass active-RC filters applying just one operational amplifier. The operational amplifier model used is the integrator model:  $\omega_i/s$  where  $\omega_i$  is the unity gain frequency. The design method is used for the design of a fourth order band pass filter with Butterworth poles applying just one operational amplifier coupled as a unity gain amplifier. The unity gain amplifiers have the advantage of providing low power consumption, yielding a large dynamic range, sometimes simplifying the amplifier design and being usable over a larger frequency range than conventional constant gain amplifiers. The Schoeffler sensitivity index is used as a basis for a practical realization of the circuit.

**Keywords** – Numerical Design, Active-RC, Butterworth, Band-pass, Operational Amplifier, Unity Gain, Integrator Model, Sensitivity, Schoeffler Index, Stability.

## I. INTRODUCTION

Single-amplifier biquads are most often used for building higher order active filters by using the cascade technique. In order to reduce the amount and complexity of the active circuitry required it will be an advantage to design higher order active filters by using only one operational amplifier. In this way it is possible to have low power consumption, low noise and what is shown in this article to have reasonable low sensitivities. Unfortunately the analytical solution of the design equations for filters of higher order than two becomes very difficult if not unsolvable. In this paper we will use a numerical design technique for the solution of the design equations without the need of computing the transfer function coefficients analytically. This method has been published by the author in [6]. An analytical computation of the transfer function coefficient would have been necessary following the method described in [3] and [5]. The proposed method is general and may be used to design active filters applying real operational amplifiers. Real operational amplifiers may be described by the input and output impedances as well as the unity gain frequency. As the unity gain frequency is the key parameter of the integrator model of the operational amplifier we will in this paper neglect the influence of the input and output impedances.

Similar attempts to design higher order active RC-filters with one operational amplifier are reported in [1], [2] and [5]. In the approach described in [2] an increased amplifier gain has been used to reduce the parameter spreading

but with an increased sensitivity as a result. Kramer in [5] has reported interesting results by numerically solving the design equations for third order low-pass filters with the constraints of minimizing the spread of component values. In both cases ideal operational amplifiers are used.

## II. THE DESIGN METHOD

Consider a transfer function  $H(s)$  of a linear filter:

$$H(s) = \frac{B(s)}{A(s)} \quad (1)$$

$B(s)$  is the numerator polynomial and  $A(s)$  is the denominator polynomial. The transfer function  $H(s)$  which is called the target rational function is factored in the following form

$$H(s) = H_0 \frac{\prod_{k=1}^m (s - s_{zk})}{\prod_{j=1}^n (s - s_{pj})} \quad (2)$$

with  $m$  zeros  $s_{zk}$  and  $n$  poles  $s_{pj}$ . The real constant  $H_0$  is a scale factor.

If we assume that all the zeros and poles of  $H(s)$  are simple then we can formulate the following scalar error function  $\varepsilon(\mathbf{x})$ ,

$$\varepsilon(\mathbf{x}) = \sum_{j=1}^n \frac{1}{|H(\mathbf{x}, s_{pj})|^2} + \sum_{k=1}^m |H(\mathbf{x}, s_{zk})|^2 + \sum_{i=1}^{\ell} \{|H(\mathbf{x}, j\omega_i)| - |H(j\omega_i)|\}^2 \quad (3)$$

where  $H(s)$  is the target rational function and  $H(\mathbf{x}, s)$  is the realized rational function.  $\mathbf{x}$  is the vector of variable parameters e.g. resistors and capacitors of the circuit or unity gain frequencies of the operational amplifiers. The last term which utilizes some additional points of the magnitude of  $H(s)$  at real frequencies  $\omega_i$  is used if a specified output level is wanted. One point will normally be sufficient but more points may be incorporated if  $H(s)$  contains multiple poles and zeros. The evaluation of  $\varepsilon(\mathbf{x})$  could be done by any general purpose circuit simulation program. This program should have the possibility of evaluating the network function  $H(\mathbf{x}, s)$  in the whole  $s$ -plane.

If the function  $\varepsilon(\mathbf{x})$  from equation (3) has been reduced to zero a match of the target function  $H(s)$  and the realized function  $H(\mathbf{x}, s)$  has been obtained i.e. they have the same

poles and zeros. The evaluation of  $\epsilon(\mathbf{x})$  requires only knowledge of the explicit form of  $H(s)$  in terms of poles and zeros as well as the structure and initial component values of the circuit to be designed. We don't need the network function with the coefficients in analytical form. This analytical form is required when setting up the design equations for active filters by following the procedure described in [2], [3] and [5]. Stability is also guaranteed as long as the complexity of the network function fits the number of poles in the target rational function and all the real parts are negative. If the complexity of the network function exceeds the number of poles in the target function then stability of the final result should be checked by a network analysis program.

Any optimization strategy may be useful in reducing the function  $\epsilon(\mathbf{x})$  to zero, but it is recommended to use the damped least square algorithms due to Levenberg [9] and Marquardt [10]. This algorithm in a modified form has been described by Fletcher in [8]. Another very useful modification has been described by the author in [6]. This modification is a way of changing the damping constant from iteration to iteration allowing the algorithm to take diverging iterations when the normal equations of the linear least square problem tend to be ill-conditioned. This modification has meant a considerable improvement of the overall convergence rate especially for difficult optimization problems.

The whole analysis and optimization process is coded in FORTRAN. In our approach the network equations are set up from a net list and solved for each of the poles and zeros appearing in the problem at hand. Net list is used in the same way as in the SPICE program. This is a very flexible approach making it possible to change the structure and filter response without doing any additional coding. A typical run with 50 iterations takes less than 1sec on a personal computer.

III: A BUTTERWORTH BP-FILTER

To illustrate the utility of the proposed design method we have used it to design a 4<sup>th</sup> order band pass filter of the Butterworth type. The structure is an extended Sallen-Key filter applying only one operational amplifier and is shown in Fig.1. The operational amplifier is coupled as a unity gain amplifier since unity gain amplifiers have the advantage of providing low power consumption, yielding a large dynamic range, simplifying the amplifier design and being usable over a larger frequency range than more conventional constant gain amplifiers.

The conventional way of designing such a filter would be either a cascade approach of a second order high-pass filter and a second order low pass or by cascading two second order band pass sections. Such an approach would require two operational amplifiers with increased power consumption as a result. All thought the classical cascade approach would be preferable in the high-Q case due to lower sensitivities. Other possibilities with one operational

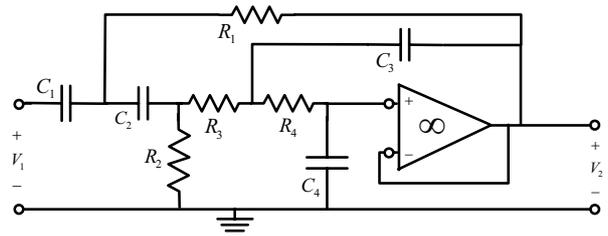


Fig. 1: Unity gain all-pole band-pass filter

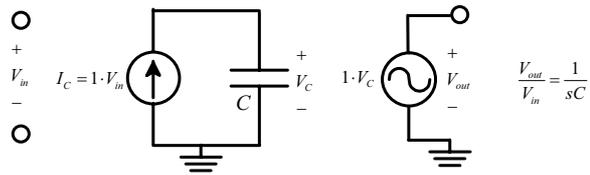


Fig. 2: Operational Amplifier Model

TABLE 1  
Normalized circuit components with ideal operational amplifier and with normalized operational amplifier model  $\omega_i = 62.5$  rad/sec

	Ideal Op.Amp.	Op.Amp. model	Rel.Dev. in %
C1 (F)	1.70000	1.70000	0
R1 ( $\Omega$ )	0.8677687	0.808192	-7.9
C2 (F)	0.991614	1.23019	24
R2 ( $\Omega$ )	1.28364	1.12370	-12
R3 ( $\Omega$ )	5.44854	5.96647	9.5
C3 (F)	1.70000	1.70000	0
R4 ( $\Omega$ )	1.97381	1.24352	-37
C4 (F)	0.02880	0.02880	0

TABLE 2,  
Normalized poles of the circuit response with operational amplifier model  $\omega_i = 62.5$  rad/sec

Poles (rad/sec)	Real part	Imaginary part
	-0.923877	0.382685
	-0.382684	0.923879
	-90.5906	0.0

amplifier also exist but this will be treated in another publication.

The operational amplifier model is shown in figure 2 with the unity gain frequency

$$\omega_i = \frac{1}{C} \tag{4}$$

In this case we will design a Butterworth band pass filter with a center frequency of 16 kHz and we will use an operational amplifier with a unity gain frequency of 1MHz. We have normalized the whole design such that

the center frequency is 1.0rad/sec. This gives a normalized unity gain frequency of  $\omega_r = 1\text{MHz}/16\text{kHz} = 62.5$  and a normalized model capacitor with the value of  $C = 1/\omega_r = 16\text{mF}$ . This corresponds to apply the low cost operational amplifier LM307.

We shall now show some results obtained by reducing the scalar error function  $\varepsilon(\mathbf{x})$  from equation (3) to zero. The optimization process has been started with an initial guess where all the passive component values are set to 1.0. In Table 1 we have shown the result with the constraint of minimizing the spread of component values both by applying an ideal operational amplifier and by applying an operational amplifier described by the integrator model with a unity gain frequency of  $\omega_r = 62.5\text{rad/sec}$ . The corresponding poles are shown in table 2. In the non-ideal case we observe a non dominant pole situated far outside the pass band of the Butterworth filter. This non dominant pole is caused by the unity gain frequency of the operational amplifier. The unity gain frequency also makes a shift of the dominant poles but this shift is compensated by a change of the resistors  $R_1, R_2, R_3$  and  $R_4$  as well as the capacitor  $C_2$ . This can be seen from the 3<sup>rd</sup> column of table 1 where we also have shown the relative deviation of the component of the non-ideal filter values compared with the ideal design. In fact the component values for the non ideal filter gives the Butterworth poles except for a non-dominant pole in the left half plane.

IV. SENSITIVITY ANALYSIS

Manufactured filters cannot guarantee to correspond exactly to the designed filter performances. The effect of component tolerances should be analyzed. The simplest way of predicting the effect of component tolerances is to use the concept of network sensitivity assuming that the component changes are small. The magnitude variability with respect to the passive components and the unity gain frequency  $\omega_t$  is given by,

$$\frac{\Delta|H(\mathbf{x}, s)|}{|H(\mathbf{x}, s)|} = \sum_{i=1}^4 S_{C_i}^{[H]} \cdot \frac{\Delta C_i}{C_i} + \sum_{i=1}^4 S_{R_i}^{[H]} \cdot \frac{\Delta R_i}{R_i} + S_{\omega_t}^{[H]} \cdot \frac{\Delta \omega_t}{\omega_t} \tag{5}$$

$S_{C_i}^{[H]} \ i=1,2,3,4$  are the relative sensitivities of the magnitude function with respect to the five capacitors and  $S_{R_i}^{[H]} \ i=1,2,3,4$  are the same sensitivities with respect to the resistors.  $S_{\omega_t}^{[H]}$  is the relative sensitivity with respect to the unity gain frequency.

The relative sensitivities  $S_x^{[H]}$  with respect to any network element  $x$  are easily obtained from the network analysis solution with an additional computational effort that is negligible as compared with the network analysis by itself. It is also noticeable that these sensitivities are ob-

tained with an error that is comparable with the error that is obtained by solving the network equations. This error of the sensitivities is also much lower than the error obtained by a numerical perturbation of the network elements.

Now we approximate the upper limit of the variability by the following sum

$$\left. \frac{\Delta|H|}{|H|} \right|_{\text{Schoeffler}} = \sqrt{\sum_{i=1}^4 \left| S_{C_i}^{[H]} \cdot \frac{\Delta C_i}{C_i} \right|^2 + \sum_{i=1}^4 \left| S_{R_i}^{[H]} \cdot \frac{\Delta R_i}{R_i} \right|^2 + \left| S_{\omega_t}^{[H]} \cdot \frac{\Delta \omega_t}{\omega_t} \right|^2} \tag{6}$$

We have chosen to subdivide the Schoeffler criterion in two parts a passive part with contribution only from the passive components

$$\left. \frac{\Delta|H|}{|H|} \right|_{\text{passive}} = \sqrt{\sum_{i=1}^4 \left| S_{C_i}^{[H]} \cdot \frac{\Delta C_i}{C_i} \right|^2 + \sum_{i=1}^4 \left| S_{R_i}^{[H]} \cdot \frac{\Delta R_i}{R_i} \right|^2} \tag{7}$$

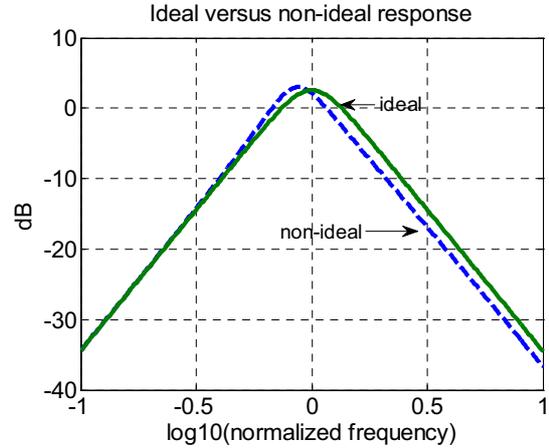


Fig. 3 Circuit responses with ideal operational amplifier and with operational amplifier model

and an active part with contribution only from the unity gain frequency of the operational amplifier

$$\left. \frac{\Delta|H|}{|H|} \right|_{\text{active}} = \left| S_{\omega_t}^{[H]} \cdot \frac{\Delta \omega_t}{\omega_t} \right| \tag{8}$$

We have chosen this way of subdividing the Schoeffler criterion as we may expect a much larger tolerance on the unity gain frequency of the operational amplifier compared with the tolerance of the passive components. But we can decrease the relative sensitivity  $S_{\omega_t}^{[H]}$  by choosing an operational amplifier with a higher unity gain frequency. A sensitivity analysis was performed assuming the relative changes of the resistors, capacitors and the unity gain frequency to be uncorrelated random variables with a zero mean Gaussian distribution and 1% standard deviation. The Schoeffler sensitivity index which is an estimate of the

upper limit of the gain variability in % is shown on figure 4. To obtain an analytical expression of the Schoeffler sensitivity index of the 4<sup>th</sup> order band-pass filter in terms of the component values is a complicated task. But in the limit when the Laplace variable  $s \rightarrow 0$  and  $s \rightarrow \infty$  we may in the all-pole filter case find the following transfer functions (The Butterworth filter is a special case of the all-pole filter)

$$\begin{aligned}
 H(s)_{s \rightarrow \infty}^{passive} &\approx \frac{1}{sC_3R_3sC_4R_4} \\
 H(s)_{s \rightarrow 0}^{passive} &\approx sC_1R_1sC_2R_2
 \end{aligned}
 \tag{9}$$

This corresponds to gain slopes of 40dB/decade. The passive sensitivity index of the gain function approaches

$$\begin{aligned}
 \left. \frac{\Delta|H|}{|H|} \right|_{\omega \rightarrow \infty}^{passive} &\approx \sqrt{\sum_{i=3}^4 \left| -\frac{\Delta C_i}{C_i} \right|^2 + \sum_{i=3}^4 \left| -\frac{\Delta R_i}{R_i} \right|^2} \\
 \left. \frac{\Delta|H|}{|H|} \right|_{\omega \rightarrow 0}^{passive} &\approx \sqrt{\sum_{i=1}^2 \left| \frac{\Delta C_i}{C_i} \right|^2 + \sum_{i=1}^2 \left| \frac{\Delta R_i}{R_i} \right|^2}
 \end{aligned}
 \tag{10}$$

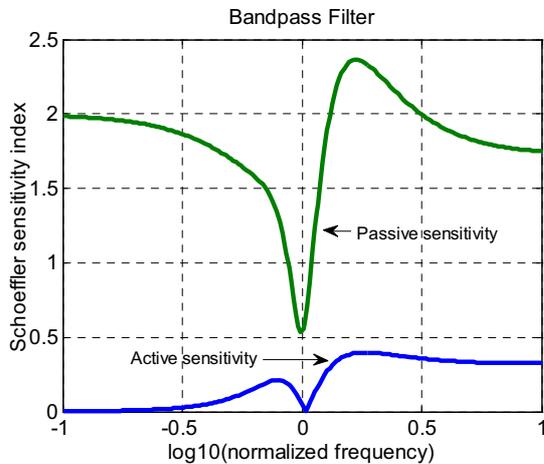


Fig. 4 Passive and active sensitivity index by applying the operational amplifier model  $\omega_c = 62.5$

This means that the passive Schoeffler sensitivity index approaches 2% with 1% standard deviation on each of the passive component. This is also the limiting value of the sensitivity index shown on fig. 4 when  $\omega \rightarrow \infty$  and  $\omega \rightarrow 0$ . As the sensitivity index from fig. 4 has a maximum value of 2.4 the circuit is easily built from a practical point of view.

The component spread is 59 which are satisfactory for integrated circuit design. But it is expected that the component spread as well as the sensitivity index increases when the proposed design method is applied to narrowband band-pass filters.

V. CONCLUSION

A numerical design method has been presented for the design of an all-pole Butterworth BP-filter of order 4 applying just one operational amplifier. The operational amplifier is described by the integrator model. The additional degrees of freedom have been used to reduce the component spread and we have found that the circuit is well suited for integrated circuit design. We have also found that the Schoeffler sensitivity has a maximum value that is 1.2 times the values at the slopes of the Butterworth characteristic. This is a highly desirable result for a practical realization of the filter. The disadvantage of this circuit is the component spread and sensitivities which increases when the circuit is designed in the narrow-band case.

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