On the scattering cross section of passive linear arrays

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REFERENCES

On the Scattering Cross Section of Passive Linear Arrays
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Abstract—A general formula is derived for the scattering cross section of a passive n-element linear array consisting of isotropic radiators. When all the reactances are tuned out and scattering in the mirror direction is investigated, it is found that $A_{nn}$, the relative scattering cross section is equal to the square of the maximum gain the array can produce. As a consequence, for forward scattering in the limiting case of zero spacing between the elements, $A_{nn} = n^2$.

The cross section of a scattering object having cylindrical symmetry is defined as follows

$$A_s(\theta_1, \theta_2) = 4\pi r^2 \frac{\text{power density at distant point } r \text{ in direction } \theta_2}{\text{power density of plane wave incident from direction } \theta_1}. \quad (1)$$

For an array one is more interested in the relative scattering cross section

$$A_{sr}(\theta_1, \theta_2) = \frac{A_s(\theta_1, \theta_2)}{A_s(\theta_1, \theta_1)} \quad (2)$$

where $A_{sr}$ is the scattering cross section of the individual radiator.

In this Communication we shall express $A_{sr}(\theta_1, \theta_2)$ in terms of the geometry of the array and of the parameters of the matching matrix. A general formulation for an arbitrary three-dimensional array is certainly possible, but we shall, for simplicity, restrict the investigation to linear arrays consisting of isotropic radiators. In that particular case there are several theorems available making...
where the n-dimensional vectors can always be expressed in the form

$$\mathbf{e} = C_1 \mathbf{S}_1 e^{j \phi}$$

where $\mathbf{S}_1$ is the incident power density and $C_1$ is a constant the value of which is of no interest in the present context. Taking the phase zero at the chosen zero point, the excitation of the $Z$th element is of the form

$$\mathbf{e}_Z = C_1 \mathbf{S}_1 e^{j \phi}$$

The current flowing in the elements will depend on the self- and mutual-impedances of the array and on the matching network but can always be expressed in the form

$$\mathbf{I} = \mathbf{Y} \mathbf{e}$$

where $\mathbf{Y}$ is an $n \times n$ admittance matrix and $\mathbf{I}$ is an n-dimensional vector

$$\mathbf{I} = \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_n \end{bmatrix}$$

The power density produced at the distance $r$ by a single radiator of current $i_i$ is

$$S(r) = \frac{C_2}{4\pi r^2} |f(\theta)_i|^2$$

Substituting for $f$ from (7) and using (5) we get

$$S(r) = \frac{C_2 C_1 \mathbf{S}_1}{4\pi r^2} |f(\theta)_i|^2$$

Hence the relative scattering cross section (2) is of the form

$$A_m = \frac{R_1^2 |f(\theta)_i|^2}{|f(\theta)i|^2}$$

where $R_1$ is the self-resistance of the individual elements and the self-reactance is assumed to be zero. Note that $A_m$ equals unity for $n = 1$.

Let us now investigate the special case when the matching network is chosen so as to tune out all the mutual reactances that is

$$\mathbf{Y} = \mathbf{R}^{-1}$$

where $\mathbf{R}$ is the real part of the impedance matrix of the array. Choosing further

$$\theta_1 = \pi - \theta_1$$

that is evaluating the scattering in the mirror direction, we get

$$A_m = \frac{|f(\theta)_i|^2}{|f(\theta)i|^2}$$

The expression in the bracket may be recognized to be the maximum gain an array can produce in the $\theta_1$ (or $\pi - \theta_1$) direction. Thus

$$A_m(\theta_1, \pi - \theta_1) = G_{\max}(\theta_1)$$

An interesting limiting case is $\theta_1 = 0$ when the distance between the elements tends to zero. Then, according to Uzkov [1] the maximum gain is equal to $\pi^2$ leading to

$$A_m(0, \pi) = \pi^2$$

Equations (16) and (17) prove not only that linear arrays may have “supergain” qualities as scatterers but also that they can be “supergain” as “receivers” from the $\theta_1$ direction and radiators in the $\theta_2$ direction at the same time. It is easy to account for the latter property. A plane wave incident from the $\theta_1$ direction will induce equal voltage amplitudes and just the right phase for radiating in the $\pi - \theta_1$ direction. According to the travelling wave theorem of Bloch et al. [2] this voltage distribution will give the maximum gain.

It is more difficult to devise a simple argument for explaining the “receiving” properties of the array but perhaps it is only due to our unfamiliarity with the subject. One usually looks at transmission and reception separately and perhaps when they occur simultaneously there is little benefit in looking at them separately. A separation though is possible. One can express the power going into the array (and necessarily radiated if all the matching elements are reactive) as follows

$$P = \frac{1}{2} \text{Re} \mathbf{e}^* \mathbf{I}$$

Choosing now $\mathbf{Y}$ in the form of (13) it may be seen that $P$ becomes proportional to $G_{\max}(\theta_1)$. The gain in the mirror direction is also $G_{\max}(\theta_1)$ so according to this alternative derivation of (16) both “reception” and reradiation are proportional to $G_{\max}$. Note, however, that this relationship is no longer valid when scattering in other than mirror direction is considered.

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**References**
