Backfire Antennas with Dipole Elements

ERIK DRAGØ NIELSEN and KNUD PONTOPPIDAN

Abstract—A method is set up for a theoretical investigation of arbitrary backfire antennas based upon dipole structures. The mutual impedance between the dipole elements of the antenna is taken into account, and the field radiated due to a surface wave reflector of finite extent is determined by calculating the surface current distribution on the reflector plate. Numerical results obtained for Yagi backfire antennas and short-backfire antennas using this theory are compared with experimental results.

I. INTRODUCTION

THE PRINCIPLES of backfire antennas were first discussed by Ehrenspeck in 1901 [1]. The backfire antenna is a modified version of the ordinary endfire slow wave antenna, which may be, e.g., a Yagi antenna, a helical antenna, or a dielectric rod antenna. According to the Hansen-Woodyard condition, the maximum directivity for such endfire antennas is directly proportional to the length of the surface wave structure traversed by the wave, provided that the phase velocity of the surface wave is adjusted for optimum directivity for any specific length of the antenna. The backfire antenna, as proposed by Ehrenspeck, consists of an endfire surface wave structure terminated with a large conducting plate—usually called the surface wave reflector—placed perpendicular to the axis of the endfire structure. The large conducting plate acts as a mirror which turns the surface wave backwards, thus forcing the wave to traverse the physical length of the antenna structure twice. This means that the backfire antenna acts as an endfire antenna of double physical length, provided that the phase velocity of the surface wave is adjusted to optimum directivity for such an endfire antenna. The advantage of backfire antennas compared to endfire antennas of the same type, therefore, is expected to be a 3-dB increase in directivity for antennas of equal length or a reduction of the antenna length to half that of the usual endfire antenna length for a given directivity. However, experimental results obtained by Ehrenspeck and described in [3] show that the increase in directivity may be even larger than 3 dB, depending upon the size and shape of the surface wave reflector.

In 1964 Ehrenspeck obtained a patent on the backfire antenna principle [2]; since then he has performed a large number of experimental investigations on backfire antennas based upon the Yagi endfire array [3]-[6].

A Yagi backfire antenna for use as a telemetry antenna has been constructed by Longva [7], and measurements on a scale model of this antenna have been carried out.

Backfire antennas based upon slow wave structures other than the Yagi array have been investigated experimentally by Loh and Jacobsen [8], [9], and Spitz [10] has measured the performance of some backfire antenna-like structures which he named "resonant endfire antennas".

A partly theoretical investigation of the Yagi backfire antenna has been performed by Davies [11]. In his calculations, however, he uses measured values for the mutual impedances between the dipole elements of the antenna. His method is therefore not directly applicable for designing backfire antennas by optimization of the characteristics of the antenna, a problem which involves arbitrary changes in the length and position of the single dipole elements. A qualitative approach to the design of backfire antennas has been suggested by Zucker [12]; however, to the knowledge of the authors, no detailed theoretical treatment of the backfire antenna has been given yet. It is the purpose of this paper to present a direct theoretical analysis usable for designing backfire antennas numerically on an electronic computer.

II. THEORETICAL SOLUTION

The types of backfire antennas investigated are shown in Fig. 1. The antenna structure is based upon a Yagi antenna consisting of a feed dipole and a row of parasitic (short-circuited) elements, one of which acts as reflector and the remaining ones as director elements in the array. This Yagi antenna is supplied with a number of additional reflector elements—usually called outside reflectors—placed parallel to and symmetrically about the original reflector element in a plane through this reflector element and perpendicular to the array axis. The large plane surface wave reflector which changes the antenna from a Yagi to a Yagi backfire is placed perpendicular to and with its center on the axis of the Yagi array at the radiating end of the Yagi antenna at a certain distance from the last director element.

As shown in Fig. 2, the feed dipole of the backfire antenna is placed in the origin of a Cartesian coordinate system. All dipole elements of the antenna are parallel to the z axis and have their feeding points in the xy plane. The N - 1 director elements are placed along the positive x axis, and the M reflector elements are placed on a line parallel to the y axis through the center reflector which is placed on the negative x axis. The surface wave reflector is parallel to the yz plane at a distance \( x_p \) from the last
director element. The dipole elements are numbered from 1 to \( N + M \) as shown in Fig. 2. For description of the radiation properties, a polar coordinate system is introduced with its origin coinciding with the origin of the Cartesian coordinate system as shown in the figure.

**A. Method of Approach**

The calculations will proceed as follows. For determination of the induced dipole currents, the surface wave reflector is considered infinite in extent, and is therefore supposed to act as a perfect mirror. Using the image theory, an antenna system composed of real dipoles as well as image dipoles is considered. The mutual impedance between all the dipoles, real as well as imaginary, is found using the EMF method. By setting up a system of linear equations involving, as known quantities, the mutual impedances and the feeding voltages, the unknown currents induced in the dipoles are found. The surface current distribution on the infinite surface wave reflector is determined from the electromagnetic field due to the dipole currents set up at the surface of the reflector plate. Eventually, the total radiated field is determined by superposition of the field due to the real dipoles and the field due to that part of the calculated surface current distribution which corresponds to the finite area of the real surface wave reflector.

**B. Determination of Dipole Currents by Image Theory**

By assuming the plane surface wave reflector infinite in extent, the usual image theory is used for determination of the currents induced in the dipoles, real as well as image dipoles. Fig. 3 shows the Yagi array of the backfire antenna and its image. In the following, all quantities referring to the image part of the antenna configuration will be marked with a prime.

1) **Mutual Coupling Between Dipoles**: The self-impedance of the dipoles and the mutual impedance between them are found using the induced EMF method under assumption of sinusoidal current distribution on the dipoles. This approximation leads to good results for dipoles of length not much larger than half a wavelength which are always used for Yagi arrays of the types investigated here. The self-impedance of element \( i \) is called \( z_i \), and \( z_{ij} \) is the mutual impedance between the \( i \)th and the \( j \)th element.

Using the induced EMF method we have, for the mutual impedance \( z_{ij} \) between two dipoles of length \( l_i \) and \( l_j \),

\[
z_{ij} = -\int_0^{l_i} \frac{I_i(z) E_{ij}(z)}{I_j} \, dz
\]

and for the self-impedance of a dipole of length \( l_i \),

\[
z_{ii} = -\int_0^{l_i} \frac{I_i(z) E_i(z)}{I_i^2} \, dz.
\]

\( I_i \) is the terminal current of antenna \( i \), \( I_i(z) \) is the current distributed along this antenna, \( E_{ij}(z) \) is the electrical field strength at and parallel to antenna \( i \) due to the current in antenna \( j \), and \( E_i(z) \) is the electrical field strength at and parallel to the \( i \)th antenna due to its own current.

Using these formulas, the mutual impedances between all antenna elements, real dipoles as well as their images, have been calculated, assuming that all elements not under consideration have been removed. By comparison with measured results for the mutual impedances obtained by Davies [10], this approximation has been found to give good results for Yagi backfire antennas of lengths and element spacings as used in connection with this investigation.

2) **Induced Dipole Currents**: When the self-impedance and mutual impedance are known, a linear system of complex equations is set up for determination of the complex
the approximations involved, the surface currents do not wave reflect, or. The currents in the real dipoles are anticipated by the current.s which mould exist if the image theory. The surface currents on the finite plate are found as the mutual impedance between this element and its image. The unknown induced dipole currents are

\[ \{I_m\} = (I_1, I_2, \ldots, I_{N+M}) \]

and since all dipole elements except the feed dipole are short-circuited, the excitation voltages are

\[ \{V_m\} = (1, 0, 0, \ldots, 0, \ldots, 0). \]

Recognizing that

\[ z_{c'c'} = z_{rc} \]
\[ z_{c'c} = z_{rc'} \]
\[ I_{c'} = -I_c \]
\[ V_{c'} = -V_c \]

the matrix equation (3) shown above may be reduced to

\[ \{Z_{mm'}\} - \{Z_{mm'}\} = \{V_m\}. \]

The elements of the mutual impedance matrix are thus expressed by a difference found as the mutual impedance between elements \( i \) and \( j \) of the real antenna minus the mutual impedance between element \( i \) of the antenna and element \( j' \) of the image antenna, the latter being the image of element \( j \) of the real antenna. The diagonal elements of the matrix are found as the difference between the self-impedance of the dipole element in question and the mutual impedance between this element and its image.

By solving the complex matrix equation, the unknown complex currents \( \{I_m\} \) induced in the dipole elements of the antenna are determined. If the antenna is an ordinary endfire Yagi antenna, the impedances \( \{Z_{mm'}\} \), contributed by the image dipoles, vanish in this equation.

C. Radiated Field

The total field is found by superposition of the fields radiated from the physically existing sources, the dipole currents and the surface current distribution on the surface wave reflector. The currents in the real dipoles are anticipated to be the same as those found above using the simple image theory. The surface currents on the finite plate are approximated by the currents which would exist if the plate were a part of an infinite conducting plane. Due to the approximations involved, the surface currents do not vanish at the edges of the plate; this effect as well as edge diffraction phenomena in general are not taken into account in connection with this investigation. Once the currents are found, the field is determined in all directions of space.

1) Field Due to Dipole Elements: The far field in the direction \((\theta, \phi)\) from a linear antenna situated in the origin of the polar coordinate system and parallel to the \( z \) axis is given by

\[ E = E_\theta \hat{\theta} \]
\[ \hat{H} = H_\theta \hat{\phi} = \frac{1}{r} E_\theta \hat{\phi} \]

where

\[ E_\theta = -\frac{i\xi I \exp(ikR) \cos(kl \cos \theta) - \cos(kl)}{2\pi R \sin \theta} \]

\( \xi \) is the characteristic admittance of free space, \( k \) the propagation constant, \( R \) the distance to the field point, and \( 2l \) the length of the dipole element. The current flowing in the dipole is assumed to be given by

\[ I(z) = I \sin(l - z). \]

Consequently, the far field from dipole element number \( n \) which is situated at a point with radius vector \( \vec{r}_n \) in the \( xy \) plane can be written

\[ E_\theta^n = -\frac{i\xi I_n \exp[ik(R - \hat{R} \cdot \vec{r}_n)]}{2\pi R} \]

\[ \frac{\cos(kl_n \cos \theta) - \cos(kl_n)}{\sin \theta} \]

where \( \hat{R} \) indicates a unit vector in the direction to the field point. The total field from the \( N + M \) dipole elements is therefore given by

\[ E_\theta = -\frac{i\xi I_n \exp(ikR) \sum_{n=1}^{N+M} I_n \exp(-ik\hat{R} \cdot \vec{r}_n)}{2\pi R \sin \theta} \]

\[ \cdot \left[ \cos(kl_n \cos \theta) - \cos(kl_n) \right]. \]

2) Field Due to Surface Wave Reflector: The surface current density \( \vec{J} \) on a body of infinite conductivity is

\[ \vec{J} = \hat{n} \times \vec{H} \]

where \( \hat{n} \) is a unit vector pointing outwards from the body normal to the surface, and \( \vec{H} \) is the magnetic field vector at the surface. As mentioned above, the current density is assumed to be equal to the one which would exist if the plate were infinitely large. Therefore, the magnetic field vector in (9) can be calculated by using the image theory and adding the magnetic field from both the real and the image dipole elements.

The magnetic field at a point \( P \) from a linear antenna of length \( 2l \) is given by \([13]\]

\[ \vec{H} = H_\theta \hat{\phi} \]

where

\[ H_\theta = -(iI/4\pi) \{ \exp(ikr_1) + \exp(ikr_2) - 2 \exp(ikr_2) \cos(kl) \}. \]
The distances \( p, r_0, r_1 \), and \( r_2 \) are shown in Fig. 4. The expression for \( H_x \) is valid in both the near field and in the far field of the antenna. By use of (10) and (9) it is now possible to determine the surface current density for all points on the surface wave reflector. It should be noted that since the magnetic field has a component in the \( \phi \) direction only, induced current on the plate will be in the \( z \) direction only.

In order to determine the radiation from the current distribution, the surface wave reflector is subdivided into a number of small squares, as shown in Fig. 5 for the case of a circular surface wave reflector (only one quarter of the reflector is shown). Over the area of each square the current density is assumed to be constant and equal to the value calculated in the center of the square. Each square of side length \( h \) is approximated by a Hertzian dipole of length \( h \) pointing in the \( z \) direction and carrying the current \( J h \). The radiated field from each square in the direction \((\theta, \phi)\) is

\[
\hat{E}_{\theta} = E_{\theta} \hat{\theta}
\]  

(12)

where

\[
E_{\theta} = -i(k'h^2/4\pi R) J \exp(ik'R) \sin \theta
\]

(13)

where \( R \) is the distance to the field point. The total field from the surface wave reflector is then found by superposition.

The radiation pattern obtained in this way will vary according to the choice of the side length \( h \) of the square. Since the number of computations involved is approximately inversely proportional to \( h^2 \), it is important to use a value of \( h \) as large as possible. The computations performed in connection with this investigation have shown that \( h \) between 0.25\( \lambda \) and 0.125\( \lambda \) (\( \lambda = \) wavelength) will be sufficiently accurate for most purposes. As an example, Fig. 6 shows the directivity of a Yagi backfire antenna with seven dipole elements and a square 2\( \lambda \times 2\lambda \) surface wave reflector as a function of \( h/\lambda \). In the interval \( 0 < h/\lambda < 0.5 \) the variation is seen to be less than 0.2 dB.

### III. Numerical Results

In connection with the theory described in Section II, a computer program for determination of the properties of Yagi backfire antennas of the type shown in Fig. 1 has been developed. The program has been made in such a way that a numerical optimization of one or more of the properties of the antenna, e.g., gain, sidelobe level, backlobe level, beamwidth, and input impedance may be performed by properly changing the parameters involved. These parameters are the number of parasitic dipole reflector and director elements, the length, radius, and position of the dipole elements, and the size, shape and position of the surface wave reflector. The contour of the surface wave reflector may take any shape wanted, e.g., square, circular, elliptic, etc. This contour is determined in the computer program by a single two-dimensional functional expression.

The computer program for calculation of the mutual impedances is based upon a paper by Baker and LaGrone [14]. The self-impedance of the dipoles was determined using the induced EMF method as described by Jordan [15]. The complex matrix equation for determination of the induced currents was solved numerically using a computer program which employs the method of Gaussian elimination to triangulate the complex coefficient matrix.

So that the computer program could be used for other types of antennas besides the Yagi backfire antenna, the input parameters were chosen more general than required for this special type of antenna. Thus it is possible to compute the radiation properties of any type of antenna based on a configuration of parallel dipole elements—active as well as parasitic—of arbitrary dimensions and positions with feeding points in the same plane, and with or without a plane perfectly conducting reflector parallel to the dipoles. Consequently, the computer program was tested for Yagi backfire antennas and short-backfire antennas as well as for ordinary Yagi antennas by comparison with results obtained by others.

For a backfire antenna with an infinitely large surface wave reflector, it is evident that the field must be iden-
tically zero behind the reflector plate; that is, the fields from the dipoles and the surface current must cancel each other in this region. In connection with a test of the computer program, calculations were made for a Yagi backfire antenna with a square 10\lambda \times 10\lambda surface wave reflector. It was found that the field intensity in all directions behind this large plate was less than 0.01 percent of the intensity in the main lobe direction.

A. Yagi Backfire Antenna

1) Current Distribution on Surface Wave Reflector: The surface current on the reflector plate has been computed for a Yagi backfire antenna equal in dimensions to an antenna investigated experimentally by Ehrenspeck [6, sec. 4]. The surface current, which, with the approximations used in this theory, is independent of the size of the surface wave reflector, is shown in Fig. 7. Because of symmetries, only one quarter of the current distribution in the reflector plane is shown. In the lower left corner of the figure the Yagi backfire antenna is drawn. The surface wave reflector of this antenna is a square with side length 2\lambda.

The amplitude of the surface current is seen to decrease very rapidly with the distance from the antenna axis; at the boundary of the reflector plate it is about 10 percent of the value at the center. The curves of constant phase are approximately circles with their centers on the antenna axis. Since the maximum gain of the antenna may be obtained only if all the currents on the reflector add in phase, it was expected that a circular surface wave reflector will be more efficient than a square reflector. To verify this, computations were made for a circular reflector with diameter 2.2\lambda replacing the 2\lambda \times 2\lambda square reflector, but only a slight increase in the gain from 11.76 dB to 11.94 dB was obtained. The choice of this particular diameter of the surface wave reflector was based on the phase curves in Fig. 7. It was found that with this diameter of the reflector, the surface currents at the center and at the edges were +90° and −90°, respectively, out of phase with the field from the dipoles; therefore these currents gave no contribution to the field in forward direction, while all other regions of the plate contributed to the field in this direction.

2) Comparison with Experimental Results: In Fig. 8, a comparison is shown between the E-plane radiation pattern of a Yagi backfire antenna of length 1.5\lambda as determined experimentally by Strom and Ehrenspeck [6] and the computed E-plane radiation pattern of the same antenna found using the theory described in this paper. This antenna consists of nine parallel dipoles, three of which act as parasitic reflectors similar to what is shown in Fig. 1. The spacing is 0.2\lambda between directors and 0.3\lambda between reflectors, and the plane of reflectors is at a distance 0.2\lambda behind the feed dipole. The distance from the last director to the square 2\lambda \times 2\lambda surface wave reflector is 0.3\lambda. The lengths of the dipoles are as follows: feed: 0.450\lambda, directors: 0.381\lambda, center reflector: 0.514\lambda, and outside reflectors: 0.412\lambda. The radius of all dipole elements is 0.00635\lambda. From the curves in Fig. 8, it is seen that very good agreement has been found for the shape of the main lobe, the beamwidth, and the level and position of the first sidelobe. In the angular range from \theta = 60° to \theta = 180° a direct comparison between the two curves is not relevant because of the inaccuracy of the experimental results, which shows up mainly as asymmetry in the radiation pattern and as a radiation level approximately 20 dB below mainlobe in the direction \theta = 90°, where a null should appear in the E-plane pattern. This inaccuracy is probably due to reflections from the surroundings at the measuring site. However, the two sidelobes appearing at \theta = 120° and \theta = 150° in the computed curve seem to be present in the experimental curve, too.

In order to confirm, experimentally, the accuracy of the computed results for the low-level sidelobes of Yagi backfire antennas also, a test antenna has been constructed. This antenna, which is shown in Fig. 1, is designed for a frequency of 1.5 GHz and has the same dimensions (in wavelengths) as the antenna of Strom and Ehrenspeck [6].
mentioned above, except for the dipole radius which is 0.005λ. The feed arrangement of this antenna has been especially developed so as to avoid any leaking radiation from the system, which might disturb the direct radiation from the antenna.

In Fig. 9, a comparison is shown between computed and measured radiation patterns of this antenna. The measurements were carried out in a large, shielded, radio anechoic chamber, which has a reflectivity level of about -31 dB in the frequency range used. It will be noticed that an almost perfect symmetry is found in the experimental results because of the good measuring conditions in the radio anechoic chamber, and only small discrepancies from the computed patterns occur. This is the case even in directions of very low radiation intensity. The computed directivity is found to be 13.8 dB above isotropic and from the experimental results in the two principal planes, a
directivity of 12.4 dB was obtained by numerical integration, using the method mentioned by Silver [17].

The results for the Yagi backfire antenna confirm that the theory and the numerical methods employed in the present investigation may be useful for direct numerical design of backfire antennas with prescribed properties. Such design could be performed from a set of curves showing the radiation properties as functions of the parameters of the antenna [18].

B. Short-Backfire Antenna

1) Comparison with Experimental Results: It was found in the course of the investigations that the computer program developed for design of the Yagi backfire antenna was also usable for designing so-called short-backfire antennas [16]. The short-backfire antenna investigated numerically using the present computer program is of the type shown in Fig. 10. During the numerical computations the small reflector plate is considered as being composed of closely spaced, parallel, short-circuited dipole reflector elements with the center reflector of length 0.4λ and the outside reflectors of decreasing length in such a manner that a small reflector with a circular shape of diameter 0.4λ is obtained. In this way, the small reflector plate has been replaced in the computations by a unidirectionally conducting plate of the same size. However, since the current induced in the small reflector plate is parallel to the current in the feed dipole (cf. Section II-C2), the unidirectionally conducting reflector plate is a good approximation to the solid perfectly conducting reflector plate of the physical short-backfire antenna.
than those mentioned previously, and a comparison between measured principal planes. When comparing the directivity of the backfire antenna with the directivity of the half-wave dipole, the experimental results by numerical integration in the antenna with a number of parallel short-circuited dipole elements. The computed directivity was 12.4 dB above an isotropic source. A directivity of 11.9 dB was found from the experimental results by numerical integration in the principal planes. When comparing the directivity of the short-backfire antenna with the directivity of the half-wave dipole itself, it is seen that introduction of the system of reflector plates causes an increase in directivity of about 10 dB, even though the large plate is not equipped with the rim of 0.25λ width used by Ehrenspeck [16]. For this case, plus certain other minor modifications, Dod [19] reports a measured gain of 14.9 dB and Ehrenspeck [16] of 15.2 dB above an isotropic source.

**Conclusion**

A method of calculation has been set up which is applicable for investigating and designing arbitrary backfire antennas with dipole elements and with a plane surface wave reflector. Only dipoles of length less than one wavelength may be treated using this theory. The numerical results have been checked with experimental results and a good agreement has been found for Yagi backfire antennas as well as for the short-backfire antenna.

In connection with the investigation a computer program has been developed, which may be utilized in the design of backfire antennas with dipole elements. The numerical investigations have shown that the approximations employed in the method used have no serious effects upon the results obtained as long as the dipoles are small in diameter and about half a wavelength long, and the length of the Yagi surface wave structure is not much longer than 1.5 wavelengths.

Acknowledgment

The authors wish to express their appreciation to J. Appel-Hansen, who is in charge of the Radio Anechoic Chamber, Technical University of Denmark, where the reported measurements were carried out.

References