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Nielsen, Palle Tolstrup

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On the Stability of a Double Integration Delta Modulator

P. TOLSTRUP NIELSEN

Abstract—A delta modulator with a second-order network in the feedback path is considered. For zero input signal the possible modes of oscillation are determined as a function of the zero of the linear network. The method could be employed with other types of feedback networks. A comparison is made with experimental results.

I. INTRODUCTION

It is well known that the performance of a delta modulation system in many cases can be improved by choosing a more complicated network than a single integrator in the feedback path of the modulator. With the configuration in Fig. 1 an often suggested transfer function for the linear network is

\[ H(s) = \frac{1 + sr_0}{(1 + sr_1)(1 + sr_2)}. \]  

The choice of the parameters \( r_1 \) and \( r_2 \) chiefly depends on the spectrum of the transmitted waveform \( s(t) \), whereas the zero is needed because of the demand for stability. From a quantizing noise viewpoint it is desirable to keep \( r_0 \) as small as possible. In fact it can be shown that with all other parameters fixed, the ratio of rms signal to rms noise is approximately inversely proportional to \( r_0 \). However, if \( r_0 \) is decreased below a certain limit, the system becomes unstable, and the follow-up characteristics of the modulator are destroyed. Hence there is an optimum value of \( r_0 \) which yields the best overall system performance. The purpose of this paper is to show how the degree of instability depends on the choice of \( r_0 \) when all other parameters remain fixed.

II. POSSIBLE MODES OF OSCILLATION

Consider the delta modulator in Fig. 1. For the purpose of this evaluation it is assumed that the input signal \( s(t) \) is zero and that the dc balance of the modulator is perfect, i.e., the number of ones equals the number of zeros in the output bit stream. Under these conditions the signal \( x(t) \) will be a symmetrical square wave, with a half-period equal to \( nT \), where \( T \) is the reciprocal of the clock frequency and \( n \) is some integer, \( n \geq 1 \). We denote the oscillation frequency by

\[ f_s = \frac{1}{2nT} = \frac{f_c}{2n} \]  

where \( f_c \) is the clock frequency. The case \( n = 1 \) corresponds to the highest possible degree of stability. If \( n \) is high, the oscillation will be of relatively low frequency and will therefore appear at the output of the demodulator with high amplitude. Our object is to determine the possible values of \( n \) for a given value of \( r_0 \).

All the waveforms of the system are shown in Fig. 2 for the case of a stationary oscillation. It is readily seen that \( y(t) \) is a true replica of \( x(t) \), except for a delay \( \Delta t \) due to the linear network \( H(s) \) and the phase shift of 180° due to the change of sign at the input of the comparator.

If the oscillation is to continue, the following inequality must be satisfied:

\[ (n - 1)T < \Delta t < nT. \]  

Our next objective is to calculate the zero-crossing delay \( \Delta t \) for the transfer function (1). It is shown in the Appendix that \( \Delta t \) can be found by solving the equation

\[ 1 + \frac{2\zeta_0}{1 + \exp(-nT/r_1)} \exp\left(-\frac{\Delta t}{\tau_1}\right) + 1 + \exp\left(-nT/r_2\right) \cdot \exp\left(-\frac{\Delta t}{\tau_2}\right) = 0 \]  

where \( \zeta_1 \) and \( \zeta_2 \) are given by

\[ \zeta_1 = \frac{r_0 - r_1}{r_1 - r_2} \]  

\[ \zeta_2 = \frac{r_0 - r_2}{r_2 - r_1}. \]
If we define
\[ \alpha \triangleq \frac{nT - t_d}{T} \]  
the inequality (3) can also be expressed as
\[ 0 \leq \alpha < 1. \]  
(8)
The quantity \( \alpha \) was calculated by numerical methods from (4) for several values of \( n \) and \( \tau_q \). In this calculation the other parameters were chosen as follows:

\[ \tau_1 = 1200 \mu s, \quad \tau_2 = 165 \mu s, \quad T = 18 \mu s. \]  
(9)
These are typical values for a delta modulation system designed for telephone quality speech transmission.

The result is shown in Fig. 3, where \( \alpha \) is plotted versus \( \tau_q \) with \( n \) as a parameter. Due to the choice of finite values for \( \tau_1 \) and \( \tau_2 \), a lower limit for \( f_0 \) is seen to exist even for \( \tau_q = 0 \). \( n = 2 \) is a possible mode for high values of \( \tau_q \). Consequently, no improvement of system performance could be expected when choosing \( \tau_q > 1.5T \).

### III. Experimental Results

Tests were carried out in the laboratory in an attempt to establish the modes of oscillation predicted in Fig. 3. It turned out that for \( \tau_q = T \), the only possible output pattern was

10101000000...

whereas for \( \tau_q = 0.25T \) three patterns could be stable:

10101000000000...

11001100000000...

111001100000000000...

The number of observed modes is considerably less than the theoretically expected number which, from Fig. 3, in the two cases should be 4 and 8. Obviously oscillations corresponding to high values of \( \alpha \) are hard to establish because of noise and jitter present in the system.

However, if a random signal is applied to the input of the delta modulator, short bursts of these high-order oscillations may occur from time to time, and unless the frequency of oscillation is well out of the voice band, these bursts will have enough inband energy to cause a noticeable decrease in the output SNR.

Experimental work by de Jager [1], Tomozawa and Kaneko [2], and other authors indicates that the optimum choice of \( \tau_q \) (maximizing the output SNR) is very close to \( T \). From Fig. 3 this choice is seen to eliminate oscillations at frequencies lower than about 7 kHz which is in good agreement with the previous discussion.

### IV. Conclusion

For a specific second-order delta modulator the possible modes of oscillation for zero input signal were determined as a function of \( \tau_q \), the zero of the feedback transfer function. The approach is general in nature and could be employed with other types of transfer functions. Since the analysis presented is valid only under the idle condition, it cannot lead to a rigorous optimization of \( \tau_q \). Nevertheless it should be clear that some relation exists between the degree of instability in the idle case and the follow-up characteristics when a signal is applied. The observation by other authors that a value of \( \tau_q \) close to \( T \) seems to be an optimal choice, is in good agreement with the results presented here.

### Appendix

Let \( u(t) \) be the function defined in Fig. 4. Then the Laplace transform of \( u(t) \) is

\[ U(s) = \frac{1}{s} \left( 1 - \exp(-as) \right), \]

This signal is applied to the input of \( H(s) \). The transform of the output is

\[ V(s) = \frac{H(s)}{s} \frac{1 - \exp(-as)}{1 + \exp(-as)}. \]

From the theory of Laplace transforms we know that if

\[ G(s) = \mathcal{L}[g(t)] \]

where \( \mathcal{L}[\cdot] \) denotes Laplace transform, then

\[ G(s) = \frac{1}{s} \frac{1 - \exp(-as)}{1 + \exp(-as)} = \mathcal{L}[-g(t)] + 2 \sum_{\epsilon \leq \epsilon' < \epsilon} \exp(j\pi\epsilon) g(t - \epsilon) \]

where \( q = 0, 1, 2, \ldots \) and \( a \) is a real constant, \( a > 0 \).

To invoke (13) we set

\[ G(s) = \frac{H(s)}{s} = \frac{1 + \epsilon\tau_q}{s(1 + \epsilon\tau_q)(1 + \epsilon\tau_2)} \]

and

\[ g(t) = 1 + \epsilon \exp\left(\frac{t}{\tau_1}\right) + \epsilon \exp\left(\frac{t}{\tau_2}\right) \]
with
\[ c_1 = \frac{\tau_2 - \tau_1}{\tau_1 - \tau_3} \] (16)
and
\[ c_2 = \frac{\tau_3 - \tau_2}{\tau_2 - \tau_1}. \] (17)
From (11) and (13) we obtain
\[ v(t) = -\left[ 1 + c_1 \exp \left( -\frac{t}{\tau_1} \right) + c_2 \exp \left( -\frac{t}{\tau_2} \right) \right] 
+ 2 \sum_{m \in \mathbb{Z}} \exp \left( j \omega m \right) \left[ 1 + c_1 \exp \left( -\frac{t - a \omega}{\tau_1} \right) 
+ c_2 \exp \left( -\frac{t - a \omega}{\tau_2} \right) \right]. \] (18)
Only the stationary solution is of interest, so we look for an expression for \( r(t) \) in the interval
\[ a m < t \leq a(m + 1) \]
where \( m \) is a positive integer. We define
\[ t' = t - a m \]
and obtain for the stationary solution in the interval (19)
\[ r(t') = \lim_{m \to \infty} r(t') \]
\[ = \lim_{m \to \infty} \left[ 2 \sum_{m \in \mathbb{Z}} \exp \left( j \omega m \right) \left[ 1 + c_1 \exp \left( -\frac{t' - a \omega}{\tau_1} \right) 
+ c_2 \exp \left( -\frac{t' - a \omega}{\tau_2} \right) \right] 
- \left[ 1 + c_1 \exp \left( -\frac{t' + a \omega}{\tau_1} \right) + c_2 \exp \left( -\frac{t' + a \omega}{\tau_2} \right) \right] \right]. \] (21)
But
\[ \sum_{m \in \mathbb{Z}} \exp \left( j \omega m \right) \exp \left( \frac{a \omega}{\tau} \right) = \frac{1 - (-1)^m}{1 + \exp \left( \frac{a \tau}{\tau} \right)}. \] (22)
Substituting (22) in (21) we get in the limit
\[ r(t') = (-1)^m \left[ 1 + \frac{2c_1}{1 + \exp \left( -\frac{\tau_1}{\tau_2} \right)} \right] \exp \left( \frac{-t'}{\tau_1} \right) 
+ \frac{2c_2}{1 + \exp \left( -\frac{\tau_1}{\tau_2} \right)} \exp \left( -\frac{t'}{\tau_2} \right). \] (23)
Using the same terminology we have
\[ x(t') = \lim_{m \to \infty} u(t') = (-1)^m. \] (24)
By now it should be clear that the value \( t_0' \), obtained by solving
\[ r(t_0') = 0 \] (25)
with \( a = n T \), is identical to the delay \( t_0 \) of Fig. 2. It is easy to verify that \( r(t') \) is increasing through the zero determined by (25) when \( m \) is even and decreasing when \( m \) is odd, provided only that
\[ \tau_1 < \max (\tau_1, \tau_2). \] (26)
This completes the proof of (4).

References

Correspondence

Intelligible Crosstalk in Multiple-Carrier FM Systems with Amplitude Limiting and AM–PM Conversion

Abstract—Intelligible crosstalk will occur in a multiple-carrier FM system when a gain slope versus frequency characteristic is followed by AM–PM conversion. This phenomenon can be particularly serious in a communication satellite repeater that utilizes a traveling-wave tube for output power amplification. A model that includes both amplitude saturation and AM–PM conversion is developed and the two-carrier intelligible crosstalk is calculated.

Correspondence approved by the Communication Theory Committee of the IEEE Communication Technology Group. Manuscript received December 7, 1970.

I. INTRODUCTION

The simultaneous transmission of frequency-division-multiplexed FM signals through a nonlinear system can result in intelligible crosstalk. In a voice-communication system this effect is particularly severe because it is manifested as intelligible whispering and can have a psychological impact considerably more severe than one might expect from the quantitative crosstalk level. In order for intelligible crosstalk to occur it is only necessary to have a frequency-dependent gain slope followed by AM–PM conversion [1]; however, amplitude limiting will affect the crosstalk magnitude.

The intelligible crosstalk level is determined in this analysis for a gain slope versus frequency characteristic followed by a