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Improved Analysis and Modelling of Leakage Inductance for Planar Transformers

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Abstract—Planar transformers have often been mistaken to essentially have lower leakage inductances. The “radial effect” is a nature characteristic for planar windings due to a higher aspect ratio of conductor width to conductor thickness, which gives a reduction in leakage inductance. Traditional formulas for leakage inductance in traditional transformers where the winding width is much smaller than the winding height are not suitable for planar transformers. This paper specifically tailors the traditional 1-D solution of leakage inductance by decomposing the leakage flux into longitudinal and transversal flux. In this manner the “eddy current effect” and the “radial effect” in leakage inductance can be analyzed individually. The proposed new formula including both ac (high frequency eddy current effect) and dc effects (radial effect) offers an accurate prediction of leakage inductance in planar transformers. Finite Element Analysis (FEA) and measurements are carried out to validate the proposed formula.

Index Terms—leakage inductance, transformer, switched mode power supply, magnetic field.

I. INTRODUCTION

Planar magnetics have gained the popularity in recent years because they have several advantages such as low profile, modularity, good thermal characteristics, ease of manufacture, and predictable parasitics etc. [1]-[9]. In traditional wire-wound components, it is very difficult to control the winding layout, which means significant variations in leakage inductance and winding capacitance appear in devices manufactured at the same time. On the other hand, with planar magnetics, the windings manufactured by automated PCB machines are more precise and consistent, yielding magnetic designs with highly controllable and predictable parasitic parameters [5].

Developments in wide bandgap devices (such as GaN and SiC semiconductors) and in advanced magnetic materials (such as Hitachi ML91S and TDK PC200) push the switching frequency into the MHz range, to minimize the size of the inductive components, however, the switching loss increases with frequency. PCB windings with enclosed planar magnetic cores have been widely used in such high-frequency and high-power-density converters [6]-[9]. Therefore, an accurate prediction of leakage inductance for planar transformers is needed in order to implement an optimized design in terms of high efficiency. Planar transformers have also been studied in resonant converters, such as LLC converters, where the leakage inductance is critical for a well-matched resonant frequency [10]-[14].

Characterization and modeling of the planar transformers have been widely published [15]-[28]. Most of the efforts are oriented to obtain winding loss based on modifications to Dowell’s formula [15]. Dowell’s formula may also be used to find the leakage inductance in conventional transformers [16]. A new set of formulas for calculating self and mutual impedances of planar coil on homogenous ferromagnetic substrates was proposed in [21]. These formulas are well suitable for planar air coils, thick film and thin film cases, but they are not directly applicable to planar PCB transformers where the core encloses the winding. But it is difficult to apply these results to planar PCB transformers. 1-D magnetic component model for planar structures based on transmission lines has been described in [24], [25], and an important characteristic of the “radial effect” for planar transformers has been reported as well. A general 2-D analytical method which is based on the partial-element-equivalent-circuit (PEEC) method was proposed in [30], [31] to compute the window field, and thus to calculate the leakage inductances of planar transformers. Publications [32] and [33] studied the leakage inductance influenced by the high frequency eddy current effect, and accurate expressions of frequency-dependent leakage inductance were proposed. However, these expressions can only be applied in traditional transformer structures where the “radial effect” of planar transformer was not taken into account.

It has often been misunderstanding assumed that planar transformers intrinsically have low leakage inductances compared to that of conventional counter parts. In fact, leakage inductance is proportional with the mean turn length (MTL) of the conductors. Planar transformers intrinsically have a longer mean turn length than that of the traditional vertical structures, resulting in a higher leakage inductance [1]. However, the important benefit of planar transformers in this regard is the relative ease with which primary and secondary windings can be heavily interleaved in a manufacturing or automation environment. Dowell and many of its modified forms for calculating the leakage inductance has been widely applied in traditional wire-wound transformers. However, significant errors have been observed when they are applied to planar transformers. The errors are mainly due to: 1) the traditional analysis only express the leakage inductance at low frequency (with frequency
inside the core. The magnetic core window height $h_w$ is assumed to be negligible. The leakage inductance is related to fully fill the core window width, so that end effects can be shown in Fig. 1. If the secondary is short-circuited, the main coupling to the secondary winding in a planar transformer as winding layers and insulator layers, causing imperfect ($\mu_r \rightarrow \infty$, $\sigma \rightarrow 0$), the magnetic field intensity goes to zero inside the core. The magnetic core window height $h_w$ is much smaller than the core window width $b_w$, and the conductors fully fill the core window width, so that end effects can be assumed to be negligible. The leakage inductance is related to the energy stored in the core window area. Therefore, the magnetic field strength accommodated in the conductors and dielectric insulators are very important to give an accurate account of leakage inductance.

### II. Improved Calculation of Leakage Inductance for Planar Transformer

Some flux will leak from the core and return to the air, winding layers and insulator layers, causing imperfect coupling to the secondary winding in a planar transformer as shown in Fig. 1. If the secondary is short-circuited, the main flux in the core which links both windings will be negligible because the primary and secondary ampere turns almost cancel. This means only the leakage energy is retained. Assuming that the permeability of core material is infinite ($\mu_r \rightarrow \infty$, $\sigma \rightarrow 0$), the magnetic field intensity goes to zero inside the core. The magnetic core window height $h_w$ is much smaller than the core window width $b_w$, and the conductors fully fill the core window width, so that end effects can be assumed to be negligible. The leakage inductance is related to the energy stored in the core window area. Therefore, the magnetic field strength accommodated in the conductors and dielectric insulators are very important to give an accurate account of leakage inductance.

#### A. Radial Effect

Planar transformer windings situated in a horizontal plane typically have a very high aspect ratio of width ($b_w$) to height ($h_w$) of the conductor cross-section, with the result that the current is concentrated in the inner path due to lower resistance in the shorter path. In any winding with spiral or circular coils conductors, the shorter path on the inside edge (at $r = r_1$ in Fig. 2) of the conducting section means that the resistance to current flow is lower and, therefore, the current density is higher than that on the outside. On the basis of this observation, it is reasonable to assume that there is an inverse relationship between the dc current density $J(r)$ and the radius $r$ [35]:

$$ h \int_{r_1}^{r_2} J(r) \cdot dr = h \int_{r_1}^{r_2} \frac{k}{r} \cdot dr = I \tag{1} $$

where $h$ is the conductor thickness, $r_1$, $r_2$ are the distances from the center to the inner edge and the outer edge of the core window respectively. The constant $k$ can be obtained from equation (1):

$$ k = \frac{I}{h \cdot \ln(\frac{r_2}{r_1})} \tag{2} $$

Therefore, the dc current density $J(r)$ can be expressed:

$$ J(r) = \frac{I}{r \cdot h \cdot \ln(\frac{r_2}{r_1})} \tag{3} $$

This relationship has been verified by Finite Element Analysis (FEA) simulation as shown in Fig. 6.
B. High Frequency Eddy Current Effect

We often observe from the measurement that the leakage inductance is reduced when the frequency increases. This is mainly due to the fact that the high frequency effect concentrates the current on the surface of the conductors, and thus equivalently reduces the thickness of the conductors in which the part of leakage energy is stored. The magnetic field is no longer linearly distributed within the conductors at high frequency because of the eddy current effect, this is illustrated in Fig. 3. The area of the MMF curve at high frequency is smaller than the one at low frequency, which means the stored leakage energy is actually lower at high frequency.

In a previous contribution [33], the authors gave a concrete analysis in connection to frequency-dependent leakage inductance particularly for traditional transformer structures. Maxwell’s equations can be rewritten in Cartesian coordinates as a second-order ordinary differential equation, namely the Helmholtz differential equation:

\[
\frac{d^2 H_r}{dy^2} = j \omega \mu_0 \sigma H_r \tag{4}
\]

The general solution of the Helmholtz equation is given by,

\[
H_r(y) = H_1 e^{\gamma y} + H_2 e^{-\gamma y} \tag{5}
\]

where \( H_1 \) and \( H_2 \) are determined by the boundary conditions and complex constants. The complex propagation constant is:

\[
\gamma = \sqrt{j \omega \mu_0 \sigma} = \frac{1 + j}{\delta_w} \tag{6}
\]

and \( \delta_w \) is the skin depth and \( \delta_w = \frac{1}{\sqrt{j \omega \mu_0 \sigma}} \).

However, this analysis only considers the magnetic field along the window height (y-component that is parallel to the surface of conductors). This part of flux is the main contribution to the high frequency eddy current (AC) effect. The DC current distribution \( J(r) \) in equation (3) due to the radial effect induces a magnetic field only having r-component, \( H_r(r) \), (changing along with r axis). The two magnetic fields are orthogonal and decoupled as shown in Fig. 4, so that they can be treated individually. A piecewise decomposition approach is needed to ensure a uniform current distribution in each piece, in order to apply the r-component magnetic field \( H_r(r) \) into the Helmholtz differential equation.

C. Piecewise Decomposition

The first conduction layer may be decomposed into many small segments with the width \( \Delta r (\Delta r \to 0) \), as shown in Fig. 2. Taking Ampere’s law at radius \( r \):

\[
J(r) \cdot \Delta r \cdot h = [H_o(r, 0) + H_o(r, h)] \cdot \Delta r + [H_o(r - \Delta r) + H_o(r + \Delta r)] \cdot h \tag{7}
\]

where the magnetic field intensity in the boundary between the first layer and the high permeability core material \( H_o(r, 0) = 0 \); and \( H_o(r - \Delta r) = -H_o(r + \Delta r) \) due to the condition of \( \Delta r \to 0 \), this leads to a simplification of equation (7):

\[
H_o(r, h) = J(r) \cdot h = \frac{l}{r \cdot \ln \left( \frac{r}{r_1} \right)} \tag{8}
\]

Invoking Ampere’s law for the closed loops \( L_n \) and \( L_{n+1} \) (see in Fig. 2) in a high permeability core, the upper and lower boundary conditions for the \( n \)th layer of the winding are obtained,

\[
H(r, y_n) = (n - 1) \cdot H_o(r, h) \tag{9}
\]

\[
H(r, y_{n+1}) = n \cdot H_o(r, h) \tag{10}
\]

and \( y_n \) and \( y_{n+1} \) are the distance from the upper surface of the first conductor to the upper surface and the lower surface of the \( n \)th layer respectively. Applying the boundary conditions (9) and (10) to the Helmholtz equation (5), the magnetic field inside the \( n \)th layer is obtained:

\[
H(r, y) = H_o(r, h) \cdot \frac{n \cdot \sinh(y) + (n - 1) \cdot \sinh(y - y)}{\sinh(y)} \tag{11}
\]

D. Leakage Energy in Each Element

With the consideration of AC and DC effects described above, the double integrals are used to calculate the leakage energy stored in each element. In the corresponding [33] for conventional transformer, this was not necessary since the radial effect was not presented. The differential volume of
each element is $2\pi r \cdot dr \cdot dy$, therefore the energy stored in each layer is:

$$E_i = \frac{1}{2} \cdot \mu_0 \cdot \int_{0}^{b} \int_{r_1}^{r_2} H(r, y)^2 \cdot 2\pi r \cdot dr \cdot dy \quad (12)$$

Substituting (11) into (12), the energy stored in the primary winding is:

$$E_p = \sum_{i=1}^{n_p} E_i = \frac{\mu_0 \cdot \pi \cdot l_p^2 \cdot n_p}{ln(n_p)} \left[ k_1(2n_p^2 + 1) + 4k_2(n_p^2 - 1) \right]$$

$$\cdot 12 \cdot \gamma \sinh^2(\gamma h_p) \quad (13)$$

where,

$$k_1 = \sinh(2\gamma h_p) - 2\gamma h_p$$

$$k_2 = \gamma h_p \cosh(\gamma \cdot h_p) - \sinh(\gamma \cdot h_p)$$

The same approach can be applied to secondary winding. The expression for the leakage energy stored in the secondary winding $E_s$ is similar to equation (13). The dielectric layers are placed between each conducting layer, and the magnetic field inside the $n^{th}$ dielectric layer remains constant and is equal to the magnetic field of the lower surface of the $n^{th}$ conductor layer. The energy stored in the dielectric layer is, therefore,

$$E_d = \frac{1}{2} \cdot \mu_0 \cdot h_1 \cdot \int_{r_1}^{r_2} H(r, h)^2 \cdot 2\pi r \cdot dr$$

$$= \frac{\mu_0 \cdot \pi \cdot h_1}{ln(h_2)} \left[ l_p^2 \sum_{i=1}^{n_p} i^2 + l_s^2 \sum_{i=1}^{n_s-1} i^2 \right]$$

$$= \frac{\mu_0 \cdot \pi \cdot h_1}{6ln(h_2)} \left[ l_p^2 \cdot n_p(2n_p + 1) \right. \left. + l_s^2 \cdot n_s(2n_s - 1) \right] \quad (14)$$

where $h_p$, $h_s$, and $h_1$ are the thickness of primary winding layers, secondary winding layers, and dielectric layer, respectively. The total leakage energy sum is the energy stored in each elementary layer which can be expressed by:

$$E_{total} = E_p + E_s + E_d \quad (15)$$

For a configuration with one turn in each layer, the turns ratio $n = \frac{n_s}{n_p}$ is defined, and the thickness of all conductors is kept the same ($h_p = h_s$), then the total leakage inductance is:

$$L_{lk} = \frac{\mu_0 \cdot \pi \cdot n_p}{3ln(h_2)} \left[ \frac{n_p^2(k_1 + 2k_2)(n + 1)}{\gamma \sinh^2(\gamma h_p)} \right.$$

$$\left. + \frac{(k_1 - 4k_2)(n + 1)}{2\gamma \sinh^2(\gamma h_p)} \right.$$

$$\left. + \left[ 2(1 + n) \cdot n_p^2 + \frac{1}{n} + 1 \right] h_1 \right] \quad (16)$$

Equation (16) can be also modeled for the case with multiple turns in each layer. It should be noted that more turns in each layer leads to a smaller effect from the dc current distribution.

III. FEA SIMULATIONS AND EXPERIMENTAL VERIFICATIONS

An example of the planar magnetic core ER 51/10/38 was selected to experimentally validate the proposed model for planar transformers. The parameters of the planar transformer are shown in Table I, the actual PCB windings are shown in Fig. 5. Both primary winding and secondary winding have 8 turns with one turn in each layer. The FEA simulation is carried out with ANSYS MAXWELL, and the transformer model is built in a 2D plane under the “eddy current” type.

Fig. 6 and Fig. 7 show the current density along with the $r$ axis (horizontal) and the $y$ axis (vertical), respectively. At low frequency, the current distribution $J_r$ along the $r$ axis well matches the equation (3) where the dc current concentrates on
the inner edge of conductors. At high frequency, the current concentrate on the surface of conductors due to the eddy currents.

Fig. 8 shows the magnetic field strength $H_0(r, h)$ of the first conductor layer along with the $r$ axis. The FEA simulation for the magnetic strength matches the equation (8) very well and validates its correctness. Fig. 9 shows the FEA simulation results of the current density $J$ in the conductors. Fig. 9(a) is simulated at 1MHz with the cylindrical axis in which both the radial effect and the high frequency eddy current effect are included. Fig. 9(b) is simulated at low frequency (dc current) with cylindrical axis which means only the radial effect is included. Fig. 9(c) is simulated at 1MHz with the Cartesian XY plane where the radial effect is not included, and only the high frequency ac current effect is included. It may be observed from Fig. 9 that the currents are concentrated on the inner edge of the conductor. The current density $J_r$ can be decomposed into two orthogonal components $J_r$ and $J_y$. The two components are decoupled in which $J_r$ is caused by the radial effect (DC current distribution) due to the unequal impedance path, and $J_y$ is caused by the skin and the proximity effects (AC current distribution) due to the transverse flux at high frequency.

IV. CONCLUSIONS

Planar transformer windings situated in a horizontal plane typically have an extremely high aspect ratio, thus concentrating the current in the inner edge of the conductors.
This phenomenon gives rise to a lower leakage inductance because the area of the stored leakage energy is effectively reduced. This so-called "radial effect" has been reported in planar transformer modelling before, but it is rare to link with frequency-dependent leakage inductance calculation. This paper gives an accurate prediction of leakage inductance for planar transformers with considerations of the high frequency eddy current (AC) effect and the radial (DC) effect. FEA simulations and experimental measurements show good agreement with these predictions. The proposed approach is an improved 1-D solution for leakage inductance, and thus only suitable for the cases that the leakage magnetic flux is substantially parallel to the surface of rectangular conductors, meaning that the external magnetic flux is perpendicular to the surface of conductors (2-D consideration) is negligible. The proposed 1-D solution is applicable to most of planar transformers with PCB windings enclosed by high magnetically permeability cores, and it is also applicable for interleaved windings where each layer is interleaved, but not applicable for complex interleaved cases such as primary and secondary windings on the same layer where 2-D consideration may be needed.

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