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Jahn-Teller and Non-Jahn-Teller Systems Involving CuF$_6^{4-}$ Units: Role of the Internal Electric Field in Ba$_2$ZnF$_6$:Cu$^{2+}$ and Other Insulating Systems

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Abstract

The applicability of the Jahn-Teller (JT) framework to sixfold coordinated d$^9$ ions whose local symmetry is not strictly octahedral is explored by means of first principle calculations. Our results contradict much of the existing literature where these systems are analyzed within the quasi-JT regime which assumes the usual JT description with a small splitting between $b_{1g}(\sim x^2-y^2)$ and $a_{1g}(\sim 3z^2-r^2)$ orbitals and also the existence of three nearly equivalent distortions. To clarify this issue we investigate the equilibrium geometry (equatorial, $R_{eq}$, and axial, $R_{ax}$, Cu$^{2+}$-F distances) and optical transitions for CuF$_6^{4-}$ units formed in Cu$^{2+}$-doped the tetragonal Ba$_2$ZnF$_6$ host lattice. While the experimental d-d transitions cannot be reproduced through the isolated CuF$_6^{4-}$ unit at the equilibrium geometry, a reasonable agreement is reached adding in the calculation the internal electric field, $E_R(r)$, created by the rest of lattice ions on the electrons confined in the complex. It is shown that this tetragonal field, $E_R(r)$, already produces a gap $\Delta_0 \sim$ 0.35 eV between $b_{1g}(\sim x^2-y^2)$ and $a_{1g}(\sim 3z^2-r^2)$ orbitals of Ba$_2$ZnF$_6$:Cu$^{2+}$ when $R_{ax} = R_{eq}$. Nevertheless, as this internal field leads to a $\Delta_0$ value higher than typical JT barriers it drastically modifies the characteristic pattern of a JT effect. In particular, it prevents the existence of three equivalent distortions as confirmed by experimental EPR data. Furthermore, $E_R(r)$ is shown to be the main physical reason behind an unusual compressed ground state with the hole in the $a_{1g}(\sim 3z^2-r^2)$ level while it is always placed in the $b_{1g}(\sim x^2-y^2)$ level for MX$_6$ complexes (M = Cu$^{2+}$, Ag$^{2+}$, Ni$^+$; X = F$^-$, Cl$^-$) in cubic lattices displaying a static JT effect. While the experimental results of CuF$_6^{4-}$ in Ba$_2$ZnF$_6$ cannot be understood within the JT framework it is pointed out that a quasi-JT situation can however happen for a d$^9$ ion in a cubic lattice under a strain of $\sim 10^{-3}$ in agreement with experimental data. The present results stress the key role played by the internal electric fields for a quantitative understanding of compounds with transition metal cations. Moreover, they also demonstrate that in the interpretation of experimental data the use of a simple model should be avoided unless all its assumptions are well justified.
INTRODUCTION

In 1937 Jahn and Teller\(^1\) explored the structural stability of molecules and solids showing that, when the electronic ground state is orbitally degenerate, the initial structure becomes unstable, with the only exception of linear molecules. To be more specific this idea just means that adiabatic minima displaying a lower symmetry than the initial one are, in general, found when orbital degeneracy comes out\(^2\).

The Jahn-Teller (JT) effect has been widely invoked in the last fifty years for explaining the properties of systems involving d\(^9\), d\(^7\) and d\(^4\) cations under octahedral coordination. Accordingly, experimental results on Cu\(^{2+}\) systems like superconducting copper oxides or copper oxyfluorides and on silver(II) fluorides have often been analyzed assuming the existence of a JT effect\(^3\)\(^-\)\(^11\). Nevertheless, this assumption implies that there exist a high symmetry reference conformation possessing an orbitally degenerate electronic state, but this crucial condition has not been proved in all cases\(^2\)\(^,\)\(^12\)\(^,\)\(^13\). So, while it is fulfilled for the polaron in AgCl\(^{13\)\(^-\)\(^15\) and d\(^9\) impurities in cubic lattices without any close defect\(^16\)\(^-\)\(^18\), doubts have been raised analyzing the experimental and theoretical results for CuF\(_6^{4-}\) complexes in the tetragonal K\(_2\)ZnF\(_4\) lattice\(^11\)\(^,\)\(^19\)\(^-\)\(^22\). Indeed, the ground state in K\(_2\)ZnF\(_4\):Cu\(^{2+}\) is certainly unusual when compared to cubic fluoride\(^16\)\(^,\)\(^23\)\(^-\)\(^27\) or chloride lattices\(^28\)\(^-\)\(^33\) doped with d\(^9\) ions displaying a static JT effect. In the latter systems the hole is always in the b\(_{1g}(\sim x^2-y^2)\) level while it surprisingly resides in the a\(_{1g}(\sim 3z^2-r^2)\) level for K\(_2\)ZnF\(_4\):Cu\(^{2+}\)\(^19\)\(^,\)\(^22\)\(^-\)\(^23\). Furthermore, the optical properties of CuF\(_6^{4-}\) complexes in K\(_2\)ZnF\(_4\) cannot be quantitatively explained considering only the isolated CuF\(_6^{4-}\) unit at the equilibrium geometry derived for K\(_2\)ZnF\(_4\):Cu\(^{2+}\)\(^22\)\(^,\)\(^34\). Nevertheless, in a previous study it was found that both the d-d transitions and the unusual ground state of K\(_2\)ZnF\(_4\):Cu\(^{2+}\) are reasonably explained merely adding in the calculation the internal electric field, E\(_R\)(r), that the rest of ions, belonging to the tetragonal lattice, create on the complex where active electrons do reside\(^22\)\(^,\)\(^34\). These results also imply that due to the action of E\(_R\)(r) there is a gap, \(\Delta_0\), between b\(_{1g}(\sim x^2-y^2)\) and a\(_{1g}(\sim 3z^2-r^2)\) levels of CuF\(_6^{4-}\) units in K\(_2\)ZnF\(_4\) when the complex is perfectly octahedral\(^34\).

Bearing these facts in mind it is crucial to clarify whether or not the experimental data of d\(^9\) complexes inserted in tetragonal lattices can be understood in the realm of the so called quasi-JT effect. In this framework the JT Hamiltonian is assumed to be the right one with the exception of considering a small gap between the initially degenerate states and keeping the existence of three nearly equivalent distortions\(^11\). Although the applicability of the quasi-JT effect has been widely assumed, it is however necessary to clarify when such a framework is no longer valid for a quantitative understanding of experimental data.

Seeking to find an answer to this fundamental question we explore in this work the so-called centre I in Ba\(_2\)ZnF\(_6\):Cu\(^{2+}\) where Cu\(^{2+}\) impurities replace Zn\(^{2+}\) ions and are also sixfold coordinated\(^35\)\(^-\)\(^36\). Experimental electron paramagnetic resonance (EPR) and electron
nuclear double resonance (ENDOR) results show that the local symmetry around the Cu\(^{2+}\) impurity is tetragonal with a principal axis that is \textit{always} coincident with the crystal c axis of Ba\(_2\)ZnF\(_6\) (Figure 1). Moreover, the experimental g-tensor values, \(g_{||} = 1.99\) and \(g_{\perp} = 2.36\), measured for the centre I in Ba\(_2\)ZnF\(_6\):Cu\(^{2+}\) at \(T = 20\) K, undoubtedly prove that the unpaired electron lies in the \(a_{1g}(\sim3z^2-r^2)\) level\(^{36}\), a situation which is thus contrary to that observed for d\(^9\) ions in cubic fluorides or chlorides under octahedral coordination\(^{23-33}\).

Interestingly, although Ba\(_2\)ZnF\(_6\) \(^{37}\) is also a layered perovskite (Figure 1) the octahedron surrounding the Zn\(^{2+}\) cation is much more distorted than that corresponding to K\(_2\)ZnF\(_4\) which is practically perfect\(^{38}\). Calling \(R_{ax}^0\) and \(R_{eq}^0\) the axial and equatorial Zn\(^{2+}\)-F\(^-\) distances in the host lattice, it turns out that \(R_{eq}^0 - R_{ax}^0\) is equal only to 0.3 pm for K\(_2\)ZnF\(_4\) \(^{38}\) while the tetragonal compression is nearly 30 times bigger (i.e. 9 pm) for Ba\(_2\)ZnF\(_6\) \(^{37}\). Despite this fact the mean value \(R_m^0 = (R_{ax}^0 + 2R_{eq}^0)/3\) is found to be identical for both compounds within 0.3\% (Table 1). Therefore, this \textit{initial} distortion in the Ba\(_2\)ZnF\(_6\) host lattice could help to increase the \textit{final} distortion of the CuF\(_6^{4-}\) unit in Ba\(_2\)ZnF\(_6\):Cu\(^{2+}\) with respect to what is found for K\(_2\)ZnF\(_4\):Cu\(^{2+}\). For this reason, it is necessary to investigate the experimental splitting, \(\Delta\), between the antibonding \(b_{1g}(\sim x^2-y^2)\) and \(a_{1g}(\sim3z^2-r^2)\) levels measured at the ground state equilibrium geometry of Ba\(_2\)ZnF\(_6\). In particular it is crucial to elucidate whether the value \(\Delta = 0.80\) eV\(^{35}\), measured for Ba\(_2\)ZnF\(_6\):Cu\(^{2+}\) (Table 1) can or not be accounted for just considering the tetragonal distortion undergone by the CuF\(_6^{4-}\) complex in the Ba\(_2\)ZnF\(_6\) host lattice. Furthermore, it is of fundamental importance to clarify the actual physical reason making the hole in Ba\(_2\)ZnF\(_6\):Cu\(^{2+}\) to reside in the antibonding \(a_{1g}(\sim3z^2-r^2)\) level and not in the \(b_{1g}(\sim x^2-y^2)\) one. Both questions imply in a first step to determine the role played by the internal field \(E_{\text{int}}(r)\)\(^{17,22}\) and thus to know the gap, \(\Delta_0\), when the complex is perfectly octahedral\(^{34}\). Once this matter is cleared up we explore, in a second step, whether all available data can or not be explained in terms of a quasi-JT effect such as it happens for d\(^9\) ions in a cubic lattice and subject to a strain of \(\sim10^{-3}\) \(^{25,28,31}\).

In summary, in the present study we want to understand why the behavior of Cu\(^{2+}\) in the layered perovskites is clearly different from that displayed by d\(^9\) ions in cubic lattices where a static JT effect takes place. For gaining a better insight into this problem the role played by the internal \(E_{\text{int}}(r)\) field for other transition metal impurities\(^{17,39-40}\) in insulating compounds is discussed in some detail. This analysis also allows one to assess the reliability of simple models employed in the interpretation of experimental data of insulating compounds containing transition metal cations.

The present work is organized as follows. A short account of employed computational methods is described in the next section while main results obtained in the present work are shown and discussed in the Results and Discussion section together with the condition for having a quasi-JT effect under an internal electric field or an external strain. Moreover, the last part of this section is devoted to provide an ensemble view on the role played by \(E_{\text{int}}(r)\) in insulating compounds containing transition metal cations. Finally,
the general conclusions extracted from the present work are displayed in the last section.

**METHODS**

Geometry optimizations on periodic supercells simulating diluted Cu$^{2+}$ impurities doped in Ba$_2$ZnF$_6$ layered perovskite (tetragonal I4/mmm space group, lattice parameters $a = 410.1$ pm and $c = 1626.3$ pm$^3$, Figure 1) have been performed by means of the CRYSTAL (version 14) code$^{37}$ under the framework of the Density Functional Theory (DFT). Calculations were performed using 2x2x1 periodic supercells containing 72 ions. In order to be sure that impurity-impurity interactions between supercells are negligible some calculations were repeated on 3x3x1 supercells and the results were practically unmodified. In the CRYSTAL code, the Bloch wavefunctions are represented by a linear combination of atomic orbitals which, in turn, are expressed as a combination of Gaussian basis functions. All ions have been described by means of basis-sets taken directly from the CRYSTAL’s webpage$^{42}$. In particular, we have used all-electron triple-$\zeta$ plus polarization (TZP) basis recently developed for Peitinger et al. for Cu, Zn and F and the pseudopotential basis HAYWSC-3111(2d)G_zagorac_2012 for Ba. Following previous works, we have used the B1WC hybrid exchange-correlation functional (including 16% of Hartree-Fock exchange)$^{43}$ that has shown to be able to reproduce with great accuracy the geometry and properties of a large number of both pure and doped crystals. The integration in reciprocal space was carried out by sampling the Brillouin zone with the 8x8x4 Monkhorst-Pack net which is enough to provide a full energy convergence.

As the ability of the CRYSTAL program to treat excited states is limited, we have carried out additional geometry optimizations in clusters containing 49 and 109 ions around the Cu$^{2+}$ impurity in both $A_{1g}$ ($b_{1g}^2a_{1g}^1$ configuration) and $B_{1g}$ ($a_{1g}^2b_{1g}^1$) states, and also in the reference state involving the average $b_{1g}^{1.5}a_{1g}^{1.5}$ configuration. Calculations have been carried out with the Amsterdam density functional (ADF) code$^{44}$, that allows performing DFT calculations on each specific electronic configuration. For this goal we have used the popular B3LYP hybrid functional$^{45}$ in the spin-unrestricted Kohn-Sham formalism of the DFT and high-quality all-electron basis sets of triple-$\zeta$ plus polarization (TZP) type formed of localized Slater-type functions as implemented in the 2016.101 version of the ADF code$^{44}$. All calculations include the electrostatic $E_R(r)$ internal field on each computed cluster, that was generated by means of about 200 point charges with values previously fitted to reproduce the electric field corresponding to the infinite system.

Moreover, we have calculated the energies of the $d$-$d$ electronic transitions in CuF$_6^{4-}$-7 ion clusters in vacuo and also taking into account the electrostatic $E_R(r)$ field created by the rest of the infinite crystal lattice ions on the cluster. The use of 7 atom clusters for describing the $d$-$d$ transitions of these centers is consistent with the highly localized character of the unpaired electrons residing essentially in the CuF$_6^{4-}$ complex region.
Supporting this view our periodic calculations yield a hole residing less than 2% outside that region.

**RESULTS AND DISCUSSION**

**Equilibrium Geometry and Ground State for CuF$_6^{4-}$ embedded in Ba$_2$ZnF$_6$**

As there is no experimental information on the actual equilibrium Cu$^{2+}$-F$^-$ distances for CuF$_6^{4-}$ in Ba$_2$ZnF$_6$ 35-36 its calculation is a prerequisite for a further insight into the associated d-d transitions, discussed in the next part of this section.

For achieving this goal we have tested in a first step the reliability of the employed methods through the calculation of the lattice parameters $a$ and $c$, as well as and the axial, $R_{ax}^0$, and equatorial, $R_{eq}^0$, Zn$^{2+}$-F$^-$ distances corresponding to the pure Ba$_2$ZnF$_6$ tetragonal lattice (Figure 1). The comparison between experimental$^{37}$ and calculated values is reported on Table 2. It can be noticed that the deviation of values derived through periodic or cluster calculations with respect to experimental ones is smaller than 2%.

In a second step, we have derived, by means of DFT calculations under periodic boundary conditions, the equilibrium geometry and the associated electronic structure corresponding to the centre I in Ba$_2$ZnF$_6$:Cu$^{2+}$ involving a CuF$_6^{4-}$ complex (Figure 1). The obtained equilibrium geometry is found to be tetragonal with a $C_4$ axis always parallel to crystal $c$ axis. The associated electronic structure involves a hole in the $a_{1g}(\sim 3z^2-r^2)$ level. Both conclusions are in agreement with experimental EPR and ENDOR data.$^{36}$

The values of axial, $R_{ax}$, and equatorial, $R_{eq}$, Cu$^{2+}$-F$^-$ distances, corresponding to the CuF$_6^{4-}$ complex embedded in Ba$_2$ZnF$_6$ obtained from present calculations are collected in Table 3. When compared to $R_{eq}^0$ and $R_{ax}^0$ of the perfect host lattice (Table 1) we see that the Zn$^{2+}\rightarrow$Cu$^{3+}$ substitution involves a relaxation of ligands, smaller than 4% in all cases. As to the position of nearest ions to the CuF$_6^{4-}$ unit in Ba$_2$ZnF$_6$ it is modified by less than 1% with respect to the perfect lattice. In Table 3 the values of $R_{ax}$ and $R_{eq}$ found for Ba$_2$ZnF$_6$:Cu$^{2+}$ are also compared to those previously derived$^{22,34}$ for the same unit in K$_2$ZnF$_4$. It can be noted that the tetragonal distortion undergone by the CuF$_6^{4-}$ complex in Ba$_2$ZnF$_6$ is certainly higher than that obtained for the same complex in K$_2$ZnF$_4$. Specifically, the value $R_{eq} - R_{ax} = 18.5$ pm found in periodic calculations for CuF$_6^{4-}$ in Ba$_2$ZnF$_6$ is only 70% higher than the figure calculated for the same unit in K$_2$ZnF$_4$ and thus the quantity $R_{eq} - R_{ax}$ does not follow the pattern of $R_{eq}^0 - R_{ax}^0$ corresponding to host lattices (Table 2).

Seeking to investigate whether the behavior of CuF$_6^{4-}$ in Ba$_2$ZnF$_6$ follows or not the pattern of a typical JT effect we have calculated the equilibrium geometry for the average $b_{2g}^{1.3}a_{1g}^{1.5}$ configuration as well as for the $B_{1g}$ state where the hole is located in the $b_{3g}(\sim x^2-y^2)$ orbital. As shown in Table 3 the equilibrium geometry found for the average $b_{1g}^{1.5}a_{2g}^{1.5}$ configuration is not octahedral but tetragonally compressed with
$R_{\text{eq}}(av) = 205.0$ pm and $R_{\text{ax}}(av) = 199.6$ pm. This situation is thus different from that found for systems with a static JT effect where $b_{1g}(\sim x^2-y^2)$ and $a_{1g}(\sim 3z^2-r^2)$ levels are strictly equivalent and thus the equilibrium geometry for that average configuration is found to be perfectly octahedral\cite{17,33}. Therefore, the fact that $R_{\text{ax}}(av) \neq R_{\text{eq}}(av)$ for Ba$_2$ZnF$_6$:Cu$^{2+}$ already suggests that $a_{1g}(\sim 3z^2-r^2)$ and $b_{1g}(\sim x^2-y^2)$ orbitals are not degenerate when the geometry is perfectly octahedral. A further insight into this matter is given in the next part of this section.

The equilibrium geometry obtained for the B$_{1g}$ state (Table 3) is not consistent with what is expected on the basis of a JT effect. Indeed if for the A$_{1g}$ state the equilibrium geometry corresponds to a compressed situation with $R_{\text{eq}} - R_{\text{ax}} = 15.6$ pm one would expect an elongated geometry with $R_{\text{ax}} - R_{\text{eq}} \approx 15$ pm for the B$_{1g}$ state\cite{2,12,17,33}. However, the present calculations give only a much reduced value $R_{\text{ax}} - R_{\text{eq}} = 6.4$ pm.

For gaining a better insight into this important matter we have also explored the energy dependence of both A$_{1g}$ and B$_{1g}$ states varying the local geometry of the CuF$_6^{4-}$ complex placed in Ba$_2$ZnF$_6$. Taking as a departure point the values $R_{\text{ax}}(av)$ and $R_{\text{eq}}(av)$ derived for the average $b_{1g}$ configuration we have considered the different $R_{\text{ax}}$ and $R_{\text{eq}}$ values given as a function of the $\eta$ parameter as follows

$$R_{\text{eq}} = R_{\text{eq}}(av) \pm \eta \quad R_{\text{ax}} = R_{\text{ax}}(av) + 2\eta$$

(1)

For the sake of clarity, the results obtained for Ba$_2$ZnF$_6$:Cu$^{2+}$ are compared in Figure 2 with those for CuF$_6^{4-}$ in the cubic perovskite KZnF$_3$ where a static JT effect takes place. It can be seen that in the latter case the energy curves $E(j, \eta)$ of $j = A_{1g}$ and $B_{1g}$ states coincide for $\eta = 0$. Moreover, if we designate by $E_{\text{min}}(j)$ the minimum energy value for such states the condition $E_{\text{min}}(j) < E(j, \eta = 0)$ is also fulfilled. Although this behavior may, at first look, resemble a JT effect\cite{33} it is however far from being followed by the curves $E(j, \eta)$ derived for Ba$_2$ZnF$_6$:Cu$^{2+}$ such as it is depicted in Figure 2.

**Optical d-d transitions for CuF$_6^{4-}$ in Ba$_2$ZnF$_6$: role of the internal electric field**

Once the tetragonal distortion for CuF$_6^{4-}$ in Ba$_2$ZnF$_6$ has been determined it is now crucial to explore whether the experimental $d$-$d$ transitions\cite{35} can be explained considering only the isolated CuF$_6^{4-}$ complex at the calculated equilibrium geometry.

The values of three $d$-$d$ transitions derived for the isolated CuF$_6^{4-}$ unit when $R_{\text{ax}} = 188.5$ pm and $R_{\text{eq}} = 207.0$ pm are gathered in Table 4 and compared to experimental findings\cite{35}. As shown in Table 4 the calculated energies of $b_{2g}(\sim xy) \rightarrow a_{1g}(\sim 3z^2-r^2)$ and $b_{1g}(\sim x^2-y^2) \rightarrow a_{1g}(\sim 3z^2-r^2)$ transitions are significantly smaller than the corresponding experimental values. In particular the experimental energy gap, $\Delta = 0.80$ eV, between $b_{1g}(\sim x^2-y^2)$ and $a_{1g}(\sim 3z^2-r^2)$ orbitals, at the ground state equilibrium geometry, is found to be $\sim 50\%$ higher than the calculated value.
It is worth noting now that for an isolated $\text{CuF}_6^{4-}$ complex a simple relation between the gap $\Delta$ and the distortion parameter $R_{\text{eq}} - R_{\text{ax}}$ has previously been established:

\[ \Delta = \beta (R_{\text{eq}} - R_{\text{ax}}) \]  

(2)

where $\beta = 27$ meV/pm. Accordingly, if $R_{\text{eq}} - R_{\text{ax}} = 18.5$ pm for $\text{CuF}_6^{4-}$ in $\text{Ba}_2\text{ZnF}_6$ we expect a value $\Delta = 0.50$ eV, a figure which is practically coincident with that calculated for the isolated $\text{CuF}_6^{4-}$ complex at the right equilibrium geometry (Table 4) albeit far from the experimental value $\Delta = 0.80$ eV measured by optical absorption for the centre I in $\text{Ba}_2\text{ZnF}_6: \text{Cu}^{2+}$ 36. This significant discrepancy already points out that the $d-d$ transitions measured for $\text{CuF}_6^{4-}$ in $\text{Ba}_2\text{ZnF}_6$ can hardly be understood just considering the isolated $\text{CuF}_6^{4-}$ complex. Indeed, when the electrostatic potential, $V_{\text{R}}(r)$, generating the tetragonal field $E_{\text{R}}(r)$, is included in the calculation, we see in Table 4 that the three experimental $d-d$ transitions are reasonably reproduced. In particular, we notice that the energy of both $b_{2g}(-xy) \rightarrow a_{1g}(3z^2-r^2)$ and $b_{1g}(-x^2-y^2) \rightarrow a_{1g}(3z^2-r^2)$ transitions are raised by about 0.35 eV due to the action of $V_{\text{R}}(r)$ while this electrostatic potential has a smaller effect upon the $e_g(-xz,yz) \rightarrow a_{1g}(3z^2-r^2)$ transition.

The origin of this relevant result is cleared up looking at the shape displayed by $(-e)V_{\text{R}}(r)$. For the sake of clarity in Figure 3 is depicted the shape of this potential for the cubic perovskite $\text{KZnF}_3$ and also for the tetragonal $\text{Ba}_2\text{ZnF}_6$ lattice. In Figure 3 is portrayed $V_{\text{R}}(r)$ when the electron coordinate, $r$, moves along the principal tetragonal axis, $c$, or along the $a$ axis of $\text{Ba}_2\text{ZnF}_6$ while shows that $V_{\text{R}}(r)$ is essentially constant when $r$ moves along a principal axis of the cubic $\text{KZnF}_3$ perovskite. Nevertheless, in $\text{Ba}_2\text{ZnF}_6$, we see that when $r$ is varied along <001> and <100> directions, although $E_{\text{R}} = 0$ at the copper site, this is no longer true when we go at the ligands position. Specifically, when the unpaired electron is along a <001> direction the $(-e)V_{\text{R}}(r)$ quantity increases its energy while the opposite happens when it lies along the $a$ axis. Therefore, as the electronic density in the antibonding $a_{1g}(3z^2-r^2)$ level is mainly lying along the $c$ axis while in the layer plane for the $b_{1g}(-x^2-y^2)$ orbital the internal field, $E_{\text{R}}(r)$, contributes to open a gap between the two levels placing $3z^2-r^2$ above.

These results mean that, in general, there are two contributions to the energy, $E$, of an optical transition corresponding to a transition metal complex embedded in an insulating lattice46. In accord with this idea $E$ can be written as

\[ E = E_{\text{int}}(R_{\text{eq}}, R_{\text{ax}}) + E_{\text{ext}} \]  

(3)

Here, the first quantity means the intrinsic contribution coming from the isolated complex which depends on metal-ligand distances, while the second one, the extrinsic contribution, arises from the effects due to the internal field depending on the position of lattice ions lying outside the complex. Therefore, even if the octahedron surrounding $\text{Cu}^{2+}$ is perfect ($R_{\text{eq}} = R_{\text{ax}}$), the gap $\Delta_0$ is not necessarily equal to zero due to the existence of the extrinsic contribution, $\Delta_{\text{ext}}$, which can be different from zero for a non-cubic host
lattice\textsuperscript{34}. From data gathered in Table 4, it can be inferred $\Delta_0 = \Delta_{\text{ext}} = 0.35$ eV for CuF\textsubscript{6}\textsuperscript{4-} in Ba\textsubscript{2}ZnF\textsubscript{6}, a quantity similar to that previously found for K\textsubscript{2}ZnF\textsubscript{4}:Cu\textsuperscript{2+}\textsuperscript{22,34}.

This result also allows one to understand the actual origin of the anomalous ground state of Ba\textsubscript{2}ZnF\textsubscript{6}:Cu\textsuperscript{2+} when compared to that found for d\textsuperscript{9} ions in cubic fluorides or chlorides under octahedral coordination\textsuperscript{23-33}. Indeed if when $R_{\text{eq}} = R_{\text{ax}}$ the $a_{1g}(\sim 3z^2-r^2)$ level lies \textasciitilde 0.35 eV above $b_{\text{lg}}(\sim x^2-y^2)$ due to the action of $E_{\text{in}}(r)$, this fact already forces the hole to be located in the $a_{1g}(\sim 3z^2-r^2)$ level. Once the ground state of CuF\textsubscript{6}\textsuperscript{4-} in Ba\textsubscript{2}ZnF\textsubscript{6} involves a positive hole in the axial $a_{1g}(\sim 3z^2-r^2)$ it induces a reduction of the axial metal-ligand distance, $R_{\text{ax}}$. Moreover, this reduction is also favoured by the substantial increase of (-e)$V_{\text{in}}(r)$ for Ba\textsubscript{2}ZnF\textsubscript{6} when $r$ is along the [001] direction (Figure 3). These facts are thus the actual physical reason behind the unusual compressed equilibrium geometry found in Ba\textsubscript{2}ZnF\textsubscript{6}:Cu\textsuperscript{2+}. The rapid increase of (-e)$V_{\text{in}}(r)$ along the [001] direction (Figure 3) also explains, albeit qualitatively, why the equilibrium geometry for the $b_{\text{lg}}^{1.5}a_{1g}^{1.5}$ configuration is found to be slightly compressed or why the equilibrium $|R_{\text{eq}} - R_{\text{ax}}|$ value for the $A_{1g}$ state is 2.5 times higher than that found for the $B_{1g}$ state (Table 3). All these results, ultimately due to the existence of $E_{\text{in}}(r)$, point out that the properties displayed by Ba\textsubscript{2}ZnF\textsubscript{6}:Cu\textsuperscript{2+} cannot be understood in a pure JT framework. A similar situation holds for CuF\textsubscript{6}\textsuperscript{4-} in the tetragonal K\textsubscript{2}ZnF\textsubscript{4} lattice\textsuperscript{22,34}.

It is worth noting now that other characteristics of the JT effect have been shown not to be fulfilled by experimental and theoretical results on Cu\textsuperscript{2+} in a tetragonal lattice. For instance, while in a system displaying an $E\otimes e$ JT effect the $e$ vibration mode is degenerate this degeneracy has been shown to be clearly broken for CuF\textsubscript{6}\textsuperscript{4-} in the tetragonal perovskite K\textsubscript{2}ZnF\textsubscript{4}\textsuperscript{22}.

**d\textsuperscript{9} ions in cubic and layered lattices: macroscopic differences**

Let us now focus on the macroscopic differences between the behaviour of Cu\textsuperscript{2+} in layered perovskites and in cubic crystals where in all cases the impurity is sixfold coordinated. When a MX\textsubscript{6} complex (M = Cu\textsuperscript{2+}, Ag\textsuperscript{2+}, Ni\textsuperscript{2+}; X = F\textsuperscript{-}, Cl\textsuperscript{-}) is embedded in cubic lattices, like KZnF\textsubscript{3}, CsCdF\textsubscript{3} or NaCl, a static JT effect is observed\textsuperscript{23-29,32} whose main fingerprints are the following: (a) At low temperatures three centers with tetragonal symmetry are simultaneously observed by EPR. Although such centers are physically equivalent they differ in the orientation of its principal axis which is just one of the three $C_4$ axes of the cubic crystal. (b) In all cases, at equilibrium, the unpaired electron is lying in the $b_{1g}(\sim x^2-y^2)$ orbital and the ligand octahedron is elongated. (c) When temperature is raised the EPR spectrum becomes isotropic. The temperature of this transition, $T_\text{t}$, has been found to lie in the region 30-200 K working in the X band. For instance, $T_\text{t} = 30$ K for Cu\textsuperscript{2+}-doped CsCdF\textsubscript{3} or KZnF\textsubscript{3}\textsuperscript{24-25} while $T_\text{t} = 160$ K for KCl:Ag\textsuperscript{2+}\textsuperscript{29}. (d) The gap, $\Delta$, observed at equilibrium is explained exclusively in terms of the local distortion described by the actual $R_{\text{eq}}$ and $R_{\text{ax}}$ values. A detailed study on this matter has been carried out for KCl:Ag\textsuperscript{2+}\textsuperscript{33}.
The preceding characteristics of a static JT effect for Cu$^{2+}$, Ag$^{2+}$ or Ni$^+$ in cubic fluorides or chlorides are not followed by data for CuF$_6^{4-}$ in layered perovskites like Ba$_2$ZnF$_6$ or K$_2$ZnF$_4$. Main arguments supporting this assertion are: (a) All CuF$_6^{4-}$ complexes formed in the tetragonal host lattices are found to have the same principal axis, the c axis of the host crystal, and thus there are not three magnetically non-equivalent centres at low temperatures$^{19, 23, 36, 47}$. (b) The hole in layered perovskites is surprisingly placed in the $a_{1g}(\sim 3z^2-r^2)$ level and not in $b_{1g}(\sim x^2-y^2)$. (c) The EPR spectrum of both K$_2$ZnF$_4$:Cu$^{2+}$ $^{47}$ and Ba$_2$ZnF$_6$:Cu$^{2+}$ $^{36}$ has been shown to display a tetragonal symmetry in the whole temperature range 4-300 K. (d) While for a MX$_6$ complex (M = Cu$^{2+}$, Ag$^{2+}$, Ni$^+$; X = F$^-$, Cl$^-$) in a cubic lattice the extrinsic contribution to the gap, $\Delta_{\text{ext}}$, is necessarily equal to zero this is no longer true when we deal with a host lattice with tetragonal symmetry.

**Absence of a quasi Jahn-Teller effect in Ba$_2$ZnF$_6$:Cu$^{2+}$ due to $E_{\text{int}}(r)$**

The preceding sections have shown that the properties of CuF$_6^{4-}$ units in Ba$_2$ZnF$_6$ are not understandable in terms of a static JT effect mainly due to the existence of an internal electric field $E_{\text{int}}(r)$ displaying tetragonal symmetry. For this reason, it is now necessary to explore under what conditions the addition of an internal field of lower symmetry does not significantly modify the pattern characteristic of a static JT system$^{7, 12}$. In other words, we need to understand why an extrinsic contribution $\Delta_{\text{ext}}$ of only 0.35 eV prevents the existence of a quasi-JT effect for Ba$_2$ZnF$_6$:Cu$^{2+}$.

For achieving this goal let us firstly consider the case of a MX$_6$ complex (M = Cu$^{2+}$, Ag$^{2+}$; X = F$^-$, Cl$^-$) placed in a cubic lattice and displaying a static JT effect. Under these circumstances, $\Delta_{\text{ext}} = 0$ and also $\Delta E_{\text{int}}(R_{\text{eq}}, R_{\text{ax}}) = 0$ when $R_{\text{eq}} = R_{\text{ax}}$ (Figure 2). These facts just reflect that when the ligand octahedron is perfect the a$_{1g}(\sim 3z^2-r^2)$ and b$_{1g}(\sim x^2-y^2)$ levels are degenerate. Nevertheless, if we place the positive hole in the b$_{1g}(\sim x^2-y^2)$ level it forces an equilibrium geometry such as $R_{\text{ax}} > R_{\text{eq}}$ $^{17, 48}$ and involves a relaxation energy termed $E_{\text{R}}(B_{1g})$ (Figure 2). Similarly, if the hole is now located in the a$_{1g}(\sim 3z^2-r^2)$ level then $R_{\text{ax}} < R_{\text{eq}}$ at equilibrium and there is an associated relaxation energy $E_{\text{R}}(A_{1g})$ which is not exactly equal to $E_{\text{R}}(B_{1g})$ (Figure 2). For all explored MX$_6$ complexes (M = Cu$^{2+}$, Ag$^{2+}$, Ni$^+$; X = F$^-$, Cl$^-$) placed in a cubic lattice, displaying a static JT effect, it turns out that the energy gain $E_{\text{R}}(B_{1g})$ is found to be somewhat higher than $E_{\text{R}}(A_{1g})$ $^{30, 33-34}$. Thus, these systems exhibit an electronic ground state with a hole placed in the b$_{1g}(\sim x^2-y^2)$ level$^{23-29, 31}$.

According to this reasoning the quantity $B = E_{\text{R}}(B_{1g}) - E_{\text{R}}(A_{1g})$ (Figure 2) has a fundamental role in the realm of the JT effect. Indeed, $B$ not only measures the gap between the ground and first excited state computed at the corresponding equilibrium geometry but it is also the energy barrier among physically equivalent distortions$^{2, 12}$. The actual value of this key quantity for different JT systems has been investigated in recent years$^{12, 18, 30, 33-34}$.
Let us now consider the effects of adding an internal electric field of tetragonal symmetry, $E_R(r)$, on the electronic ground state of a given JT complex. As we have seen the internal field, $E_R(r)$, produces a different energy shift, $\varepsilon_t(x^2-y^2)$ and $\varepsilon_t(3z^2-r^2)$, on the $b_{1g}(\sim x^2-y^2)$ and $a_{1g}(\sim 3z^2-r^2)$ levels even if $R_{eq} = R_{ax}$. In other words, the extrinsic contribution to the gap between these two levels, $\Delta_{ext}$, is just given by

$$\Delta_{ext} = \varepsilon_t(3z^2-r^2) - \varepsilon_t(x^2-y^2)$$

According to the present reasoning, if in a $d^9$ JT system the hole lies in the $b_{1g}(\sim x^2-y^2)$ level and thus the ligand octahedron is elongated, the addition of $E_R(r)$ can change the nature of the electronic ground state as well as the existence of three quasi equivalent distortions. The non-existence of this change is thus favored by the condition

$$\Delta_{ext} < B$$

Results on several elongated systems displaying a static JT effect show that the barrier is lying in the 6-150 meV range\(^{12, 18, 30, 33-34}\). More precisely, typical values for this magnitude are found for KZnF$_3$:Cu$^{2+}$ and NaCl:Ag$^{2+}$ being, respectively, $B = 8$ and 62 meV, while the largest one we are aware corresponds to $B = 127$ meV for KCl:Ag$^{2+}$\(^{12, 18, 30, 34}\).

The scale of these values is a factor 3-50 smaller than the gap created between $b_{1g}(\sim x^2-y^2)$ and $a_{1g}(\sim 3z^2-r^2)$ levels by the internal field in Ba$_2$ZnF$_6$:Cu$^{2+}$ giving rise to $\Delta_{ext} = 0.35$ eV. In particular, this extrinsic contribution is about 40 times higher than the value $B = 8$ meV derived for the JT system KZnF$_3$:Cu$^{2+}$ (Figure 2). Thus, according to this simple reasoning, the extrinsic contribution $\Delta_{ext} = 0.35$ eV found in Ba$_2$ZnF$_6$:Cu$^{2+}$ largely suffices for drastically changing the JT pattern such it has been shown in the previous parts of this section. For this reason, an explanation of properties observed in Ba$_2$ZnF$_6$:Cu$^{2+}$ on the basis of a quasi-JT effect has to be ruled out. In particular, the ground state of Ba$_2$ZnF$_6$:Cu$^{2+}$, involving a hole in the $a_{1g}(\sim 3z^2-r^2)$ level, is different from that observed for $d^9$ ions in perfect cubic fluoride or chloride lattices where the hole is located in the $b_{1g}(\sim x^2-y^2)$ orbital. As we have proved, this relevant difference is mainly due to the action of the anisotropic internal field $E_R(r)$ in the layered perovskite and not to the JT effect.

Although for all MX$_6$ complexes ($M = Cu^{2+}, Ag^{2+}, Ni^{+}$) in fluoride or chloride cubic lattices explored up to now the hole is found in the $b_{1g}(\sim x^2-y^2)$ level and the ligand octahedron elongated this is not necessarily true for every system displaying a static JT effect. Indeed it has recently been demonstrated that Ni$^{+}$ in the cubic CaO lattice exhibits a static JT effect with a hole in the $a_{1g}(\sim 3z^2-r^2)$ level and a compressed geometry\(^{49}\). The main factors favoring the compressed geometry in a static JT system are discussed in Refs.\(^{17, 33, 49}\).

**Applied strains on d$^9$ ions in a cubic lattice: limits for a quasi-Jahn-Teller effect**
In this section we explore the effects of an applied uniaxial strain on a system initially displaying a static JT effect. The main objective is to determine the conditions for observing experimentally a quasi-JT effect when the strain is varied.

For the sake of clarity let us consider a $d^9$ ion in a cubic lattice displaying an elongated geometry ($R_{ax} > R_{eq}$) and a hole in $b_{1g}(x^2-y^2)$, being $OZ$ the principal axis. If the system is subject to a strain, described by a strain tensor whose only non-zero component is $e_{zz}$, this tetragonal perturbation induces shifts, termed $\varepsilon_S(3z^2-r^2)$ and $\varepsilon_S(x^2-y^2)$, on the two considered $d$-levels. If $e_{zz} < 0$ then $\varepsilon_S(3z^2-r^2) > \varepsilon_S(x^2-y^2)$. Thus, similarly to Eq. (5), the condition favoring not to modify the ground state under the applied strain is just given by

$$\varepsilon_S(3z^2-r^2) - \varepsilon_S(x^2-y^2) < B \quad (6)$$

For a JT system the quantity $\varepsilon_S(3z^2-r^2) - \varepsilon_S(x^2-y^2)$ is expressed in terms of the $e_{zz}$ strain as follows$^{2,12,28}$

$$\varepsilon_S(3z^2-r^2) - \varepsilon_S(x^2-y^2) = (3/2)V_2e_{zz} \quad (7)$$

where in turn $V_2$ can be expressed in terms of the linear vibronic coupling constant, $V_{1e}$,

$$V_2 = - (2/\sqrt{3})V_{1e}R_{av}e_{zz} \quad (8)$$

Eqs. (6-8) just mean that if $V_{1e} \sim 1 \text{ eV/Å}$ and $R_{av} \sim 2.5 \text{ Å}$ then a quasi-JT effect should be observed for $e_{zz} < 10^{-2}$ in the case of a barrier $B = 60 \text{ meV}$.

EPR experiments performed for NaCl:Cu$^{2+}$ at $T = 77 \text{ K}$ involving strains up to $\sim 3 \times 10^{-3}$ prove the existence of a quasi-JT regime. In the absence of an external strain there are three equally populated centres whose $C_4$ axis is parallel to the X, Y or Z axis of the crystal because the ground state energy of such centres, $E_I$ (I = X, Y, Z) is the same$^{12,28}$. Although this degeneracy is broken by the action of the applied strain the EPR spectra are however close to those observed for $e_{zz} = 0$ characteristic of a JT effect. The similarities and differences with respect to $e_{zz} = 0$ are the following$^{28}$: (1) For a general orientation of the applied magnetic field, $H$, the signals from the three distortions with the $C_4$ axis parallel to the X, Y or Z axes are all observed in EPR spectra. (2) The values of the EPR parameters corresponding to each centre are essentially those measured at zero-strain as they are practically insensitive to applied strains smaller than $\sim 10^{-2}$ $^{50}$. (3) Nevertheless, the population of the three centers is now different as $E_Z$ is no longer equal to $E_X$ or $E_Y$ due to the action of the applied strain.

Under an external strain, $e_{zz} < 0$, the difference between $E_Z$ and $E_X$ or $E_Y$ is simply given by$^{2,12}$

$$E_Z - E_X = (3/2)V_2e_{zz} \quad (9)$$

Thus, if $e_{zz} \sim -3 \times 10^{-3}$ and $V_2 \sim -2 \text{ eV/strain}$ then $E_Z - E_X \sim 10 \text{ meV}$. As this quantity is comparable to $kT = 7 \text{ meV}$ if $T = 77 \text{ K}$, this explains that the three centres with the $C_4$ axis
parallel to the X, Y, Z axes in NaCl:Cu$^{2+}$ are still observed under a $e_{zz} \sim 3 \times 10^{-3}$ strain although with a different intensity. A quasi-JT regime has also been observed for CsCdF$_3$:Cu$^{2+}$ and NaF:Ag$^{2+}$ under an applied uniaxial stress$^{24-25,31}$.

The present discussion on the quasi-JT effect points out that the ground state of a system displaying a static JT effect can be modified when subject to a $e_{zz}$ strain. This means that if the d$^9$ JT complex is elongated with a principal axis parallel to OZ axis then both the ground state and the equilibrium geometry can be changed under an axial $e_{zz} < 0$ strain, especially if the barrier, B, is low. Preliminary calculations carried out on KZnF$_3$:Cu$^{2+}$, where B is only equal to 8 meV, supports that such a change can be reached for $e_{zz} \sim 5 \times 10^{-3}$.

It is worth noting now that in any real crystal there are random strains produced by the unavoidable presence of dislocation and point defects. Such random strains destroy the equivalence of three adiabatic distortions which appear in a $E \otimes e$ JT system being thus responsible for the observation of a static JT effect at low temperatures$^{2,12,49}$. Nevertheless, as in the case of random strains $|e_{zz}|$ is of the order of $10^{-4}$ the associated perturbation cannot alter the nature of the ground state in a static JT effect.

**Extrinsic and intrinsic contributions: a general view**

The present results prove the important role played by the internal field $E_R(r)$ for a correct understanding of the anomalous ground state observed for Ba$_2$ZnF$_6$:Cu$^{2+}$ as well as its associated d-d transitions. Indeed, simply looking at the shape of $V_R(r)$ (Figure 3) we realize that it opens a gap between the two $b_{1g}(\sim x^2-y^2)$ and $a_{1g}(\sim 3z^2-r^2)$ levels even if the CuF$_6^{4-}$ complex is perfectly octahedral. Moreover, this gap mainly comes from the electronic density lying in the ligand region rather than close to the copper site where $E_R(0) = 0$.

Obviously, the internal electric field has to play a relevant role for other transition metal impurities placed in layered insulators. A direct proof of this assertion appears looking at the properties displayed by the CrF$_6^{3-}$ complex placed in the tetragonal K$_2$MgF$_4$ lattice with remote charge compensation$^{51}$. Both the experimental g-tensor ($g_\parallel = 1.9727$, $g_\perp = 1.9743$) and the existence of a zero-field splitting ($D = 0.042$ cm$^{-1}$) observed in EPR spectra$^{51}$ support a clear tetragonal point symmetry for this center although the calculated $R_{eq}$ and $R_{ax}$ values are identical within 0.3%$^{52}$. Again the observed tetragonal symmetry in EPR spectra has been shown to arise from the effects of $E_R(r)$ that induces a gap between $b_{1g}(\sim x^2-y^2)$ and $a_{1g}(\sim 3z^2-r^2)$ levels leading to a splitting of 120 meV in the $4^T_2$ first excited state$^{52}$.

According to the present analysis the internal electric field, although often ignored, cannot be discarded in any right understanding of spectroscopic properties due to transition metal ions in insulating lattices. So, $E_R(r)$, aside from being responsible for the different color in ruby (Al$_2$O$_3$:Cr$^{3+}$), emerald (Be$_3$Al$_2$Si$_6$O$_{18}$:Cr$^{3+}$) and alexandrite
(BeAl₂O₄:Cr³⁺)³⁹, can also influence the gap, 10Dq, between t₂₈(xz, yz, xy) and e₈(x²-y², 3z²-r²) levels of perfect octahedral CrO₆³⁻ complexes embedded in cubic lattices⁴⁶. In other words, similarly to Eq. (3) we can write for these cases

\[ 10Dq = 10Dq_{\text{int}}(R) + 10Dq_{\text{ext}} \]  

where the intrinsic contribution 10Dq_{\text{int}}(R) is sensitive to the actual value of the metal-ligand distance, R.

The existence of an extrinsic contribution for LiBaF₃⁴⁰, displaying a cubic inverted perovskite structure, explains why MF₆⁴⁻ complexes (M = Mn, Ni, Co) exhibit a 10Dq value which is systematically −0.2 eV higher⁵³ when embedded in that lattice than in the normal perovskite KMgF₃. In the same vein, the identical 10Dq = 2 eV measured for MgO:Cr³⁺⁵⁴ and the emerald⁵⁵-⁵⁶ has been shown to come from an extrinsic contribution 10Dq_{\text{ext}} = 0.2 eV for the former system while it is practically null for emerald⁴⁶.

When in the cubic MgO lattice the host cation Mg²⁺ is replaced by a transition metal impurity like Cr³⁺, with remote charge compensation, the lattice symmetry implies that the internal electric field at the metal cation is zero. This situation, somewhat similar to that previously discussed for Ba₂ZnF₆:Cu²⁺, stresses that the effects of E_R(r) upon the observed spectroscopic properties cannot be understood neglecting the electronic density present in the ligand region.

Furthermore, the actual value of the intrinsic contribution, 10Dq_{\text{int}}(R), as well as its dependence upon the metal-ligand distance, R, have been shown to arise from the different covalent bonding in the antibonding t₂(z, yz, xy) and e(x²-y², 3z²-r²) levels of an octahedral or tetrahedral complex⁵⁷. Along this line for complexes like MnF₆⁴⁻, CrF₆³⁻ or FeO₄²⁻ 10Dq_{\text{int}}(R) depends on R⁻ⁿ (n ≈ 5) and this strong dependence has been shown to come mainly from the hybridization of 3d(e) levels with 2s ligand levels⁵,⁵⁸. Although the rough crystal-field theory, which treats the ligands as point charges, predicts a dependence of 10Dq_{\text{int}}(R) ∝ R⁻⁵ this agreement has to be viewed only as a fortuitous coincidence because that theory also leads to 10Dq values which are one order of magnitude smaller than experimental ones.¹⁷ For this reason, although there are still papers claiming that an experimental dependence 10Dq ∝ R⁻ⁿ (n ≈ 5) proves the validity of crystal-field theory⁵⁹ that assertion is simply meaningless.

Bearing in mind that electron localization is a fingerprint of insulating materials⁶⁰-⁶¹ it can be expected that the internal electric field also plays a relevant role for understanding the ground state of pure insulating compounds. Along this line it has been shown that in La₂CuO₄ E_R(r) forces the hole to be in the x²-y² while in K₂CuF₄ the unpaired electron lies in an orbital with a dominant 3z²-r² character as a result of the internal electric field⁶². Other examples showing the importance of the internal field, E_R(r), for understanding the orbital ordering in pure non-cubic transition metal compounds are given in Ref.².⁶²
CONCLUSIONS

The present results prove the absence of symmetry breaking for CuF_6^{4-} in Ba_2ZnF_6 as even when R_{ax} = R_{eq} the complex already exhibits a D_{4h} symmetry due to the presence of the anisotropic internal field, E_{int}(r). Furthermore, they also demonstrate that E_{int}(r) in Ba_2ZnF_6, leading to Δ_{ax} ≈ 0.35 eV, can destroy the pattern characteristic of a JT effect. This means that, contrary to what is found for systems like NaCl:Cu^{2+} or CsCdF_3:Cu^{2+} under ε_2 ~ 10^{-3} strains, we cannot understand the properties displayed by CuF_6^{4-} in the tetragonal lattices Ba_2ZnF_6 or K_2ZnF_4 on the basis of a quasi-JT effect.

The structural and electronic properties of compounds containing d^9 ions are very often discussed assuming the existence of a JT effect. The present results stress however, that if the host lattice is not cubic that assumption may, in general, not to be right. The construction of correct models on these systems necessarily involves fully assuming the low-symmetry of the system, including changes in the splitting of the levels, forces, frequencies, etc. that are not present in a JT model and that are closely related to the internal field E_{int}(r). Moreover, the appearance of the three minima typically associated to a JT problem now involves the vibronic coupling between non-degenerate levels and thus if orthorhombic minima were present, which is not assured, it could only be explained through the use of a full pseudo Jahn-Teller model.

At the time of publication we found a recent paper claiming that the properties of K_2ZnF_4:Cu^{2+} can be explained on the basis of the JT effect in the same way as KZnF_3:Cu^{2+}. This assertion is supported in Ref. indicating that the absolute value of the experimental splitting between b_{1g}(-x^2-y^2) and a_{1g}(-3z^2-r^2) levels in KZnF_3:Cu^{2+} (|Δ| = 0.62 eV) is nearly identical to that for K_2ZnF_4:Cu^{2+} (|Δ| = 0.64 eV). However, the later statement raises questions at various levels: (1) As it was previously explained, K_2ZnF_4:Cu^{2+} and Ba_2ZnF_6:Cu^{2+} do not need to consider as JT systems because their experimental EPR data do not exhibit the characteristic fingerprint of a static JT system that is, however, well observed for KZnF_3:Cu^{2+}. (2) As the character of the ground state in both systems is different, Δ is positive for K_2ZnF_4:Cu^{2+} and Ba_2ZnF_6:Cu^{2+} (Table 1) but negative for KZnF_3:Cu^{2+} (see also Figure 2). This key fact is not explained in Ref. while its actual origin is well understood through the present work. (3) The value |Δ| = 0.62 eV considered in Ref. for KZnF_3:Cu^{2+} does not correspond to any experimental measurement. In this respect, optical absorption data on KZnF_3:Cu^{2+} show that no absorption occurs in the range 0.5-0.94 eV after which the e_g(xz;yz) → b_{1g}(x^2-y^2) transition is observed. Moreover, first-principle calculations match all observed bands and are consistent with this fact since they predict Δ = -0.40 eV, well below the value |Δ| = 0.62 eV given in Ref.

The study carried out in this work shows the importance of the internal field E_{int}(r) for a quantitative understanding of spectroscopic properties displayed by transition metal cations in insulating lattices. This implies that models which try to explain such
properties based solely on isolated complexes are, in general, not correct\textsuperscript{17, 34, 39, 52}. For this reason, in the interpretation of experimental data, the use of a simple model should be avoided unless all its assumptions are well justified. In this regard first principle calculations have shown to be a valuable tool to check the validity of simple model's assumptions.

It is worth noting now that $E_R(r)$ is actually determined by a few shells of ions lying close to the considered complex as further ions lead to a potential $V_R(r)$ essentially flat in the complex region where active electrons reside. The contribution of different shells of ions to the total $V_R(r)$ potential has been explored for systems like ruby\textsuperscript{67}, MgO:Cr$^{3+}$ \textsuperscript{46} or LiBaF$_3$:Mn$^{2+}$ \textsuperscript{40}.

We have seen in the present work that the internal electric field induces an extrinsic contribution, $\Delta_{\text{ext}}$, to the $a_{1g}(\sim 3z^2-r^2) - b_{1g}(\sim x^2-y^2)$ gap of Ba$_2$ZnF$_6$:Cu$^{2+}$ of only $\sim 0.35$ eV. Nevertheless, very recently it has been proved that for the square-planar CuO$_4$ complex formed in CaCuSi$_4$O$_{10}$\textsuperscript{68} the internal electric field produces even higher shifts ($\Delta_{\text{ext}} \sim 1$ eV)\textsuperscript{69}, a crucial fact which is behind the blue color displayed by the Egyptian Blue pigment. According to this result it could be expected that, in comparison to what is found for CuF$_6^{4-}$ in Ba$_2$ZnF$_6$:Cu$^{2+}$, higher extrinsic contributions also appear in the case of square-planar CuF$_4^{2-}$ complexes formed in fluoride lattices\textsuperscript{35, 70}. Work along this line is now underway.

Acknowledgments

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References

42. CRYSTAL basis sets {http://www.crystal.unito.it/basis-sets.php} (accessed 02/09/2017).


Table 1. Experimental values of Zn$^{2+}$-F$^-$ distances in the perfect host lattice for K$_2$ZnF$_4$ and Ba$_2$ZnF$_6$ layered perovskites. The axial Zn$^{2+}$-F$^-$ distance, $R_{ax}^0$, is that along the crystal $c$ axis (Figure 1) while $R_{eq}^0$ corresponds to the layer plane and $R_{av}^0$ is the average value. The gap energy, $\Delta$, between the $b_{1g}$($\sim x^2-y^2$) and $a_{1g}$($\sim 3z^2-r^2$) levels measured for both K$_2$ZnF$_4$:Cu$^{2+}$ and Ba$_2$ZnF$_6$:Cu$^{2+}$ is also reported.

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Table 2. Comparison of $a$ and $c$ lattice parameters, and the axial, $R_{ax}^0$, and equatorial, $R_{eq}^0$, Zn$^{2+}$-F$^-$ distances corresponding to pure Ba$_2$ZnF$_6$ derived through periodic and cluster calculations with those obtained from X-ray diffraction data. All distances are given in pm units.

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<td>207.5</td>
<td>191.9</td>
<td>15.6</td>
<td>202.3</td>
<td>-0.65</td>
</tr>
<tr>
<td></td>
<td>Cluster</td>
<td>$B_{1g}$</td>
<td>201.4</td>
<td>207.8</td>
<td>-6.4</td>
<td>203.5</td>
<td>-0.09</td>
</tr>
<tr>
<td></td>
<td>Cluster</td>
<td>$b_{1g}^{1.5}a_{1g}^{1.5}$</td>
<td>205.0</td>
<td>199.6</td>
<td>5.4</td>
<td>203.2</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 3. Calculated values of equilibrium axial, $R_{ax}$, and equatorial, $R_{eq}$, Cu$^{2+}$-F$^-$ distances, for CuF$_6^{4-}$ complexes formed in Ba$_2$ZnF$_6$ and K$_2$ZnF$_4$ host lattices using periodic and 49 ion cluster calculations. Periodic calculations always converge to the ground state $A_{1g}$. Cluster calculations in Ba$_2$ZnF$_6$:Cu$^{2+}$ have been performed for the ground state $A_{1g}$, the excited state $B_{1g}$ and also the reference state with electronic configuration $b_{1g}^{1.5}a_{1g}^{1.5}$. Values of the tetragonal distortion, $R_{eq} - R_{ax}$, and the mean distance, $R_m = (R_{ax} + 2R_{eq})/3$, are also given for comparison. All distances are in pm. $\Delta$E is the calculated energy difference (in eV) with respect to the reference $b_{1g}^{1.5}a_{1g}^{1.5}$ state.
<table>
<thead>
<tr>
<th>Transition</th>
<th>Isolated CuF$_6^{4-}$</th>
<th>CuF$_6^{4-}$ under $E_R(r)$</th>
<th>Experimental</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_{1g}(x^2-y^2) \rightarrow a_{1g}(3z^2-r^2)$</td>
<td>0.53</td>
<td>0.93</td>
<td>0.80</td>
</tr>
<tr>
<td>$e_g(xz,yz) \rightarrow a_{1g}(3z^2-r^2)$</td>
<td>1.15</td>
<td>1.24</td>
<td>1.18</td>
</tr>
<tr>
<td>$b_{2g}(xy) \rightarrow a_{1g}(3z^2-r^2)$</td>
<td>1.31</td>
<td>1.61</td>
<td>1.61</td>
</tr>
</tbody>
</table>

**Table 4.** Energy values (in eV) of the three $d$-$d$ transitions calculated for the tetragonal CuF$_6^{4-}$ unit formed in Ba$_2$ZnF$_6$ at the equilibrium geometry. In addition to results derived for the isolated complex those obtained including the effects of the internal electric field, $E_R(r)$, are displayed as well. The experimental optical transitions measured for CuF$_6^{4-}$ in Ba$_2$ZnF$_6$ are also enclosed for comparison.
Figure 1. Unit cell of Ba$_2$ZnF$_6$ layered perovskite showing the Cu$^{2+}$ centre I corresponding to a CuF$_6^{4-}$ complex.
Figure 2. Adiabatic Potential Energy Surface (APES) obtained in DFT calculations on a 57 ion cluster simulating KZnF$_3$:Cu$^{2+}$ and on a 49 ion cluster simulating Ba$_2$ZnF$_6$:Cu$^{2+}$ by constraining the occupation of the orbitals to obtain $A_{1g}$ ($b_{1g}^2a_{1g}^{-1}$ configuration) and $B_{2g}$ ($a_{1g}^2b_{1g}^{-1}$) states. Calculations have been carried as a function of a JT distortion coordinate $\eta$, defined by $R_{eq} = R_{eq}(av) \pm \eta \ y R_{ax} = R_{ax}(av) \mu 2\eta$, where $R_{eq}(av)$ and $R_{ax}(av)$ are the optimized Cu$^{2+}$-F$^-$ distances for the average reference state with configuration $b_{1g}^{1.5}a_{1g}^{1.5}$. Note the different scales used for the two systems. Dotted lines correspond to the geometry and energy of the average reference states.
Figure 3. Behavior of the energy \((-e)V_R(r)\) associated with the electrostatic potential, \(V_R(r)\), produced on a 7 atom CuF\(_6^4^-\) complex by the rest of ions on the cubic KZnF\(_3\) lattice and the tetragonal Ba\(_2\)ZnF\(_6\) lattice. In this later case the quantity \((-e)V_R(r)\) is depicted along [100] and [001] crystalline directions.
KZnF$_3$:Cu$^{2+}$
Jahn-Teller
$\eta$ (pm)
0.01 eV

Ba$_2$ZnF$_6$:Cu$^{2+}$
Jahn-Teller?
$\eta$ (pm)
0.7 eV