Robust Return Algorithm for Anisotropic Plasticity Models

Tidemann, L.; Krenk, Steen

Published in:
Proceedings of the 30th Nordic Seminar on Computational Mechanics (NSCM-30)

Publication date:
2017

Document Version
Publisher's PDF, also known as Version of record

Link back to DTU Orbit

Citation (APA):

General rights
Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.
ROBUST RETURN ALGORITHM FOR ANISOTROPIC PLASTICITY MODELS

L. TIDEMANN∗† AND S. KRENK∗

∗Department of Mechanical Engineering, Technical University of Denmark
DK-2800 Kongens Lyngby, Denmark
†Mærsk Olie & Gas
DK-6700 Esbjerg, Denmark
e-mail: lastid@mek.dtu.dk

Key words: Return algorithms, Computational methods, Cyclic plasticity.

1 INTRODUCTION

Plasticity models can be defined by an energy potential, a plastic flow potential and a yield surface. The energy potential defines the relation between the observable elastic strains γe and the energy conjugate stresses τe and between the non-observable internal strains γi and the energy conjugate internal stresses τi, where the internal stresses control the various hardening mechanisms. Plasticity models may be defined either in terms of traditional stresses and strains τ = [σ11, σ22, · · · ]T and γ = [ε11, ε22, · · · ]T or generalized stresses and strains, e.g. τ = [N, My, · · · ]T and γ = [ε, κy, · · · ]T, the latter typically used in plastic analysis of frame structures. To have a compact notation in the following τe and τi are arranged in a common vector ˜τT = [τeT , τiT ] and correspondingly γe and γi are arranged in the common vector ˜γT = [γeT , γiT ].

In traditional stress-based analyses the stress is evaluated at a material point, where a single plastic mechanism may be active, whereas in the case of frame structures each beam may have two active plastic mechanisms, in the form of a yield hinge in each end as illustrated in Fig. 1. In general multiple plastic mechanisms may be active for different types of elements. Each plastic mechanism has a yield surface described by a yield function Fj and a flow potential Gj describing the plastic flow evolution by its gradient and these potentials are conveniently collected in the vectors f = [F1( ˜τ), ..., Fn( ˜τ)]T and g = [G1( ˜τ), ..., Gn( ˜τ)]T respectively.

The key to developing a general and robust return algorithm for anisotropic plasticity models is the strain evolution equation. The strain evolution equation is obtained from maximizing the dissipation rate under the assumption that the material is described by the flow potential

$$\dot{\gamma} = \begin{bmatrix} \dot{\gamma}_e \\ 0 \end{bmatrix} - \sum_j \partial_\tau G_j \dot{\lambda}_j = \begin{bmatrix} \dot{\gamma}_e \\ 0 \end{bmatrix} - (\partial_\tau^T g)^T \dot{\lambda},$$

(1)
where \( \dot{\gamma}_t \) is the total increment in the observable strains. The plastic multipliers \( \dot{\lambda} \) are determined by the consistency condition that during plastic loading, the stress state \( \tau_e \) must stay on the yield surfaces defined in \( f_y \).

2 RETURN ALGORITHM

The strain evolution equation (1) is reformulated to finite increments and is assumed to be satisfied in the final generalized stress state\(^3\). For non-trivial yield surfaces the strain evolution equation will not initially be satisfied and a residual is formed

\[
r_{\dot{\gamma}} = \begin{bmatrix} \Delta \gamma_t \\ 0 \end{bmatrix} - \Delta \dot{\gamma} - (\partial^T \dot{\tau})^T \Delta \lambda.
\] (2)

The final state where \( r_{\dot{\gamma}} = 0 \) is obtained by a first order variation of the residual (2) combined with the consistency condition that the final stress state must be on the yield surface. The first order variation is formulated entirely in terms of \( \delta \dot{\tau} \) and \( \delta \lambda \) with use of the constitutive relation

\[
\delta \dot{\tau} = \tilde{K} \delta \dot{\gamma},
\] (3)

where the tangent stiffness matrix \( \tilde{K} \) contains the double derivatives of the energy potential. The resulting equation system to solve is

\[
\begin{bmatrix}
\tilde{K}_A^{-1} \\
\frac{\partial}{\partial \dot{\tau} f_y} \\
0
\end{bmatrix}
\begin{bmatrix}
(\partial^T \dot{\tau} f_y) \\
\partial^T \dot{\tau} \dot{\lambda} \\
\delta \lambda
\end{bmatrix} =
\begin{bmatrix}
\dot{\gamma} \\
-\dot{\tau}
\end{bmatrix},
\quad
\tilde{K}_A^{-1} = \tilde{K}^{-1} + \sum_j \frac{\partial^2 G_j}{\partial \dot{\tau} \partial \dot{\tau}} \Delta \lambda_j,
\] (4)

where \( \tilde{K}_A \) is the consistent algorithmic stiffness matrix. Instead of solving (4) directly it is solved sequentially by eliminating \( \delta \dot{\tau} \) in the first equation and determining \( \delta \lambda \) from the second equation and back-substituting the result into the first equation. Anisotropic
plasticity models may have yield surfaces with regions with large curvature, Fig. 1, leading
to large changes in the direction of the gradient of the yield surface and the plastic flow
potential. Though $\delta \tilde{\tau}$ is a linear function of the residual $r_{\gamma}$ and the value of the yield
function $f_y$ according to (4) the function is non-linear as $K_1^{-1}$, $\partial_{\tilde{\tau}}^T g$ and $\partial_{\tilde{\tau}}^T f_y$ in general are
non-linear. The increment $\delta \tilde{\tau} = \delta \tilde{\tau}(\xi r_{\gamma}, \xi f_y)$ is therefore represented by a second order
approximation

$$
\delta \tilde{\tau}(\xi) = \xi \frac{\partial(\delta \tilde{\tau})}{\partial \xi} \bigg|_{\xi=0} + \frac{1}{2} \xi^2 \frac{\partial^2(\delta \tilde{\tau})}{\partial \xi^2} \bigg|_{\xi=0},
$$

where the constant term is zero for $\xi = 0$ and the two derivatives are given by

$$
\frac{\partial(\delta \tilde{\tau})}{\partial \xi} = K_r r_{\gamma} - K_f f_y, \quad \frac{\partial^2(\delta \tilde{\tau})}{\partial^2 \xi} = \frac{\partial}{\partial \xi} \left(K_r r_{\gamma} - K_f f_y\right) \simeq \frac{\Delta K_r}{\Delta \xi} r_{\gamma} - \frac{\Delta K_f}{\Delta \xi} f_y.
$$

The differences $\Delta K_r$ and $\Delta K_f$ are determined by making half a step, i.e. setting $\xi = 1/2$ and
determining the matrices in the updated state by the gradients $\partial_{\tilde{\tau}}^T g$ and $\partial_{\tilde{\tau}}^T f_y$ as well
as the second order derivatives $\partial^2 G_j / (\partial \tilde{\tau}^T \partial \tilde{\tau})$. These are combined with the solution of
the equation system (4) to form $K_1^{1/2}$ and $K_f^{1/2}$. Inserting the results into (6) and (5)
with $\Delta \xi = 1/2$ and setting $\xi = 1$ gives the relation

$$
\delta \tilde{\tau} = K_r^{1/2} r_{\gamma} - K_f^{1/2} f_y.
$$

This is analogous to a method used in explicit stress integration\(^3\) where a midpoint is
found and the elasto-plastic stiffness at the midpoint is used for a full step.

### 3 NUMERICAL EXAMPLES

The robustness of the return algorithm is illustrated by deformation of a beam with
plastic hinges, Fig. 1, described by a cyclic plasticity model\(^1\) in terms of the normalized
section forces\(^2\) $n = N/M_y$ and $m = M/M_y$. The yield surface is slightly rounded in
comparison to the one shown in Fig. 1. The energy potential consists of two quadratic
terms uncoupling $\tau_e$ and $\tau_i$ whereby $\tilde{K}$ becomes a block diagonal matrix with $K_e$ and
$K_i$ in the diagonals. The yield surface is kinematic hardening and is tailored for cyclic
plasticity models with general hardening behaviour\(^1\). The beam is modelled with param-
eters representing ideal-plastic behaviour, Fig. 2(a) and (b), and parameters representing
non-linear hardening plastic hinges, Fig. 2(c) and (d). Both beams are subjected to a
large strain increment with an equivalent estimated elastic stress state with $n = 14$ and
$m_y = 4$ at one hinge and $n = 14$ and $m_y = 0$ at the other hinge.

The estimated elastic stress state is located far away from the yield surface in a region
with relatively large curvature of the yield surface and two plastic mechanisms. Never-
theless the algorithm returns the stress state to the yield surfaces in just 10 iterations
in the ideal plasticity case as illustrated in Fig. 2(a) and (b). About half the number of
iterations is used to get to the neighbourhood of the final state and the remaining half is
to ensure $\tilde{r}_\gamma = 0$. Hardening typically eases return and as shown in Fig. 2(c) and (d) it does in the present case as well, as the return is made in only 7 steps. It is noted that a traditional single-step return algorithm fails to converge for the predicted stress states shown in Fig. 2. In general the method presented here is more robust, e.g. if half the deformation increment used above is applied a traditional single-step return algorithm will converge in the ideal-plastic case but not in the hardening case.

REFERENCES

