Technical and economic working domains of industrial heat pumps: Part 1 - single stage vapour compression heat pumps

Ommen, Torben Schmidt; Jensen, Jonas Kjær; Markussen, Wiebke Brix; Reinholdt, Lars; Elmegaard, Brian
Published in: International Journal of Refrigeration

Link to article, DOI: 10.1016/j.ijrefrig.2015.02.012

Publication date: 2015

Document Version
Peer reviewed version

Link back to DTU Orbit

Citation (APA):
Technical and economic working domains of industrial heat pumps: Part 1 - vapour compression heat pumps

Torben Ommen a, *, Jonas Kjær Jensen a, Wiebke Brix Markussen a, Lars Reinholdt b, Brian Elmegaard a

a Technical University of Denmark, Kgs. Lyngby, DK-2800, Denmark
b Danish Technological Institute, Aarhus, DK-8000, Denmark

Abstract

A large amount of operational and economic constraints limit the applicability of heat pumps operated with natural working fluids. The limitations are highly dependent on the integration of heat source and sink streams. An evaluation of feasible operating conditions is carried out considering the constraints of available refrigeration equipment and a requirement of a positive net present value of the investment. The considered sink outlet temperature range is from 40 °C to 140 °C, but for the six heat pump systems considered in this paper, the upper limit of their working domain is at 120 °C. For each set of heat sink and source temperatures the optimal solution is determined. At low sink temperature glide, either R717 or R600a heat pumps are optimal depending on the sink outlet temperature. At higher sink temperature glide the transcritical R744 also becomes important in a limited domain.

Keywords: Industrial heat pumps, working domain, economic evaluation, natural working fluids

1. Introduction

Large vapour compression heat pumps (VCHP) are increasingly used, both in industry and for heat production in district heating networks. The type and configuration of the previously installed heat pumps typically depend on the local legislation as well as the layout of the sink/source process streams. For industrial applications most installed systems are in the range of sink temperatures between 50 °C and 90 °C (Annex-21, 1995). This sink temperature span may likely be due to limitations for the heat pump technology, rather than a limited demand at higher temperatures. Brunin et al. (1997) investigate the working domain of many working fluids in the temperature range up to 200 °C. The main part of the investigated working fluids are banned today (or will be shortly) throughout most of Europe, especially when considering large industrial scale systems. For the natural alternatives equipment limitations are significant, resulting in operating conditions which are not covered by the proposed working fluids. The study does not consider economics as such, but employs two physical constraints to represent economic feasibility. The constraints are coefficient of performance (COP) and volumetric heating capacity (VHC).

This study seeks to reveal the possible working domains for several natural working fluids when considering temperature lift and sink temperatures, using the one stage vapour compression heat pump cycle. Five heat pump systems utilising natural working fluids are compared in terms of technical, thermodynamic and economic constraints, in order to include the heat pump performance and investment into the consideration. The considered natural refrigerants are: R290, R600a, R744 and R717, where the latter utilises both low and high pressure components. One HFC working fluid (R134a) is included in the study for comparison of the feasibility of natural working fluids. The considered sink temperature range is from 40 °C to 140 °C. In part 2 of
this paper (Jensen et al., 2014), the ammonia-water absorption-compression heat pump is studied and compared to the results of the VCHP determined in this paper.

2. Method

Examination of the working domain is carried out for six single stage VCHP systems. The comparison is carried out without considering possibilities for improved performance by e.g. internal heat exchangers. A model of each heat pump is implemented in Engineering Equation Solver (F-Chart, 1992).

The heat pumps are compared using both economical and technical constraints. The technical constraints considered are typically caused by the behaviour of the working fluid and by limitations in the development of suitable components. This is further discussed in section 2.3.

A few effects have been neglected as they are assumed of similar magnitude between the investigated heat pumps. Such effects include pressure drop in pipes, the extent of non-useful superheat and subcooling, and compressor heat losses. Only full load steady state operation is considered.

2.1. Vapour compression heat pump

For industrial processes heat is often transferred by heat transfer fluids, which may be oil or water based. In this study it is assumed to be pure water, which is pressurized to avoid evaporation of the secondary working fluid at elevated temperatures. Pinch point temperature difference is used to model heat exchange with both sink and source media (Nellis and Klein, 2009). A principle sketch of a VCHP, and a temperature - heat load diagram for an azeotropic working fluid, are presented in Fig. 1. In the condenser, the working fluid is sub-cooled until it reaches the pinch temperature difference at
the sink entrance.
The performance of the VCHP is calculated using constant efficiencies for compressor and electrical motor, as well as fixed temperature differences in the heat exchangers. The used values are presented in Table 1.

For the case of R744, where heat rejection from the working fluid is at supercritical pressure, the heat pump performance is affected by the gas cooler pressure as presented in Nekså et al. (1998) and further discussed in Cecchinato et al. (2010). By changing the heat exchange process of the gas cooler, also the investment of the heat pump system is affected. For all the considered temperature configurations of transcritical heat pumps, the heat rejection pressure allowing the optimal net present value (further explained in section 2.4.1) is determined and used. This methodology is proposed for easy comparison with all configurations allowing additional degrees of freedom in the system design.

2.2. Estimation of plate heat exchanger area and pressure drop

Heat exchange processes are important in any VCHP and a significant part of the physical system, with high influence to the investment and the derived heat cost. Detailed heat transfer correlations for both evaporators and condensers are implemented and used in moving boundary models of the heat exchangers.

Chevron type plate heat exchangers are considered, as they appear to be cost efficient and are typically used in such systems today. The correlations used for the analysis are presented in table 2, while the dimensions are for a fixed type corresponding to pressure level and working fluid constraints (SWEP International AB, 2013).

The heat pump systems are fitted with liquid receivers at an intermediate pressure for both subcritical and supercritical systems (Corberan, 2011; Kim et al., 2004). By using this option, it is possible to obtain the desired subcooling in the condenser, and operate the system freely at various conditions such as part-load and at start-up.

2.3. Compressors and operating conditions

In order to achieve the considered temperatures using vapour compression heat pumps, specially designed compressors are used. The pressure limit of the high pressure side may in many cases dictate the achievable sink temperatures, but also the suction pressure may pose limitations. Similarly for transcritical processes, where the pressure is not directly dependent on the sink temperature, the pressure limit has an effect on the heat pump performance. The condensing temperature and pressure may in rare cases be lower than that of the sink stream leaving the condenser. This is possible in the case where a high fraction of heat dissipation is from superheated vapour.

Oil degradation may pose limitations due to high temperatures in the compressor. To reduce wear and excess degradation, the compressor discharge temperatures are limited to 180 °C.

Compressors from large international manufacturers were investigated, where both price and operation limits were available. Five different types have been identified according to different working fluid properties, flammability and availability. The data for these are presented in Table 3.

Compressor types 1 - 3 are similar, where type 1 is applicable for HFC working fluids, type 2 is prepared for flammable environments and type 3 is equipped for R717 specifically. Additionally two high pressure compressors are included, where type 4 is for R717 and type 5 is for transcritical R744 processes. The investigation has not been constrained to individual compressor technologies, but due to data availability considering the compressor cost, the below results represent reciprocating piston compressors.

2.4. Economic evaluation

The economic evaluation of the heat pumps is based on the economic method presented by Bejan et al. (1996), where individual component costs are used to account for the overall collected system. The method requires detailed cost data for components presented in a process flow diagram.

As mentioned in section 2.3, coherent and comprehensive data for specific components are not straightforward to uncover. To aggregate from the data at hand, it is assumed that:

- Purchased Equipment Cost (PEC) for an open type compressor is solely dependent on the type (specified in Table 3) and the swept volume of the compressor.
- PEC for an electrical motor with a fixed efficiency is dependent on the shaft power.
Figure 1: (a) Principle sketch of the vapour compression HP. (b) temperature - heat load diagram for condenser and evaporator where temperature variation of both sink and source is 10 K.

Table 1: Operating point and performance of VCHP and natural gas boiler

<table>
<thead>
<tr>
<th>Type of data</th>
<th>Value</th>
<th>Unit</th>
<th>Designation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Efficiency</td>
<td>0.8</td>
<td>-</td>
<td>Compressor isentropic efficiency</td>
</tr>
<tr>
<td></td>
<td>0.8</td>
<td>-</td>
<td>Compressor volumetric efficiency</td>
</tr>
<tr>
<td></td>
<td>0.95</td>
<td>-</td>
<td>Compressor electric motor efficiency</td>
</tr>
<tr>
<td>Temperature</td>
<td>5 K</td>
<td></td>
<td>Pinch point temperature difference in condensors, gascoolers and evaporators</td>
</tr>
<tr>
<td>Temperature</td>
<td>5 K</td>
<td></td>
<td>Compressor suction superheat</td>
</tr>
</tbody>
</table>

Table 2: Applied heat transfer and pressure drop correlations for the evaporator, condenser and gascooler

<table>
<thead>
<tr>
<th>Component</th>
<th>Media</th>
<th>Heat transfer</th>
<th>Pressure drop</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>two-phase:</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>transcritical:</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>liquid only:</td>
<td></td>
</tr>
<tr>
<td>Evaporator</td>
<td>Rxxx</td>
<td>two-phase:</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>vapour only:</td>
<td></td>
</tr>
</tbody>
</table>

Table 3: Available compressor technology with current operating limits

<table>
<thead>
<tr>
<th>Working fluid</th>
<th>Type</th>
<th>Pressure limit</th>
<th>Lubrication</th>
<th>Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Bar</td>
<td>max. temp.</td>
<td>(1500 RPM)</td>
</tr>
<tr>
<td>R134a</td>
<td>1</td>
<td>28</td>
<td>180</td>
<td>5-280</td>
</tr>
<tr>
<td>R290</td>
<td>2</td>
<td>28</td>
<td>180</td>
<td>5-280</td>
</tr>
<tr>
<td>R600a</td>
<td>2</td>
<td>28</td>
<td>180</td>
<td>5-280</td>
</tr>
<tr>
<td>R717-LP</td>
<td>3</td>
<td>28</td>
<td>180</td>
<td>5-180</td>
</tr>
<tr>
<td>R717-HP</td>
<td>4</td>
<td>50</td>
<td>180</td>
<td>90-200</td>
</tr>
<tr>
<td>R744</td>
<td>5</td>
<td>140</td>
<td>180</td>
<td>6-25</td>
</tr>
</tbody>
</table>
PEC for a heat exchanger is a function of the heat exchange area and pressure limit.

PEC of an intermediate pressure receiver is a function of volume and pressure limit.

The PECs of expansion valve and oil separator are neglected.

Capital investment of a component is 4.16 higher than PEC of the component. This is done to account for additional cost related to new investment (Bejan et al., 1996) at an existing facility.

Electricity and natural gas prices correspond to the market cost for industrial consumers in the year 2012 according to Danish Energy Agency (2013).

For the comparison with natural gas burner investment costs are neglected, as the heat pump may replace existing installations.

Source heat is readily available as a process stream.

Interest and inflation rate is fixed (7% and 2% respectively) and the plant is assumed to operate for a technical lifetime of 15 years.

Cost correlations are assumed to be valid for heat pump capacities between 100 kW and 2 MW. Restrictions are due to data availability for component correlations.

The component prices are based on prices from intermediate Danish trade business such as H. Jessen Jørgensen A/S (2013) and individual manufactures, eg. Sørensen (2013) for R717 HP equipment.

When comparing technical solutions the economically optimal solution is often preferred. Two different profitability evaluations are used for the analysis, the (simple) payback period (PBP) and the detailed net present value (NPV). The method for determination of the two factors is explained in the following sections.

2.4.1. Net present value

As indicated by its title, this profitability evaluation calculates the present value of the heat pump system, by considering the time value of cash flow streams during the lifetime of the plant. The present value of the proposed system is then compared to the present value of the existing solution. The capital-recovery factor (CRF), Eq. (1), is used to calculate the levelized cost rates over the lifetime of the plant $n$:

$$\text{CRF} = \frac{r^{\text{eff}}(1 + r^{\text{eff}})^n}{(1 + r^{\text{eff}})^n - 1}$$  \hspace{1cm} (1)

Where $r^{\text{eff}}$ is the effective interest rate over the lifetime of the system, calculated as seen in Eq. (2), where $r$ is the interest rate, and $i_L$ is the inflation rate.

$$r^{\text{eff}} = \frac{1 + r}{1 + i_L} - 1$$  \hspace{1cm} (2)

The annual fuel cost of the plant can then be calculated according to Eq. (3) for a heat pump or Eq. (4) for natural gas burner.

$$\text{FC}_{\text{hp}} = \dot{W}_{\text{hp}} \cdot c_{f,\text{elec}} \cdot h_{\text{op}}$$  \hspace{1cm} (3)

$$\text{FC}_{\text{ng}} = \dot{Q}_{\text{ng}} \cdot c_{f,\text{ng}} \cdot h_{\text{op}}$$  \hspace{1cm} (4)

where $\dot{W}_{\text{hp}}$ is the power required by the heat pump to produce the desired heat rate at the condenser $\dot{Q}$. The natural gas burner efficiency is denoted $\eta_{\text{ng}}$. $c_f$ denotes the fuel price for electricity or natural gas and $h_{\text{op}}$ is the annual amount of operating hours.

The present value of the expenses associated with the project $j$ can be calculated as presented in Eq. (5).

$$\text{PV}_j = \text{CI}_j + \frac{\text{FC}_j}{\text{CRF}} + \text{OMC}_j$$  \hspace{1cm} (5)

where $\text{CI}_j$ is the capital investment of all required components for system $j$. $\text{OMC}_j$ is the operation and maintenance cost (OMC) (not including fuel cost) for system $j$, which is assumed to be a fixed factor (20 %) of $\text{CI}_j$. For natural gas burners the capital investments are considered as sunk cost ($\text{CI}_{\text{ng}} = 0$) and thus the OMC costs are similarly neglected.

The NPV of the heat pump projects are calculated as presented in Eq. (6) for the case where a heat pump is proposed to replace a natural gas burner in a facility.

$$\text{NPV}_{\text{hp}} = \text{PV}_{\text{ng}} - \text{PV}_{\text{hp}}$$  \hspace{1cm} (6)

If the NPV is positive, the proposed heat pump is a cost effective alternative to the already installed burner.
2.4.2. Pay back period
The PBP determines the length of time required for the proposed installation to fully recover the CI by reduction in FC compared to the existing solution. PBP is often used as a measure for feasibility of technical projects, although this factor does not include the time value of money or include cash flows that occur after the calculated period.

\[
PBP_{hp} = \frac{CI_{hp}}{FC_{ng} - FC_{hp}}
\]  

(7)

3. Results
The working domains of the individual heat pumps are calculated for a given amount of operating hours and heat pump capacity. A parametric variation of the two quantities is presented in Fig. 2a. Contours of two different simple pay back periods (PBP) and the net present value of zero (meaning that both solutions are equally feasible for the technical lifetime of the heat pump system) are plotted. All three economic constraints, NPV=0 as well as PBP=4 years and PBP=8 years will be printed in each plot, allowing the reader to decide the appropriate economic constraint.

It is worth noticing, that all solutions in Fig. 2a represent COP=5.1 and VHC=5.9 MJ/m³, which are within the recommended levels defined by Brunin et al. (1997). Although within the limits, a large part of the solutions are found to be infeasible compared to the fuel cost of a natural gas boiler, which indicate that the use of semi-economical constraints may not include the required level of detail.

Based on the results of Fig. 2a the heat pump working domains are investigated for a case of 3500 operating hours yearly, and \( Q_{HP} = 1000 \) kW. Fig. 2b presents a parametric variation of significant assumptions for calculation of the PV at this operation point. The PV of the considered heat pump is most sensitive to the assumptions of electricity price and isentropic efficiency. The observed sensitivity is assumed not to limit the validity of the economic comparisons between the technologies, as offsets would result in similar changes for all the considered systems.

The technical and economic limitations for heat pump operation are investigated for four different sink and source temperature glides. The investigated temperature glides are: (sink/source): 10K/10K, 20K/10K, 20K/20K and 40K/10K. In the study the working domains are established by considering the operational boundaries for each considered heat pump type for 10K/10K (Fig. 3) and 40K/10K (Fig. 4). The remaining temperature glides (20K/10K and 20K/20K) are presented in appendix A as Fig. A.1 and Fig. A.2 respectively.

The hatched areas of the individual plots are areas where the considered technology is not appropriate. One area is caused if \( T_{source.out} \) is below 0 °C, which requires another source media to be considered, and thus influences the heat transfer characteristics. This could be feasible if a brine was applied, but it would imply cooling the heat source below ambient temperatures, which may only be relevant if a cooling demand is satisfied. The second area corresponds to a case where \( T_{sink.in} < T_{source.in} \). In such a heat transfer processes, heat should be exchanged directly between the two streams before a heat pump is integrated in the process (Annex-21, 1995).

The economic constraints are presented as green lines (NPV) and turquoise / dashed turquoise lines for PBP. Red lines indicates high discharge temperatures, whereas blue lines show the pressure constraints for the considered systems. Each individual plot shows the constraints for the specific working fluid, showing both the components and the economic restrictions. By examination of Fig. 3 it can be seen, that combination of the four natural working fluids allows for a large working domain. The NPV constraints for R717LP and R717HP include a large fraction of the considered temperature span, but the technology is restricted by high compressor discharge temperatures, as well as the allowable high pressure. For the four remaining heat pumps, the discharge temperatures are below the limit in all cases, but at the same time it may be seen that the NPV constraint is at a lower \( \Delta T_{lift} \) compared to the two ammonia configurations.

Specifically for all R744 configurations (e.g. Fig. 3a) the critical pressure of the working fluid limits the working domain at high \( T_{sink.out} \), as two-phase evaporation is not possible in some of the considered combinations. Furthermore, the dashed pressure constraint indicates where optimal pressure is above the maximum allowable for the compressor. Beyond this line the high pressure side is kept at the maximum allowable pressure, which results in decreased performance and changed slope of the economic constraints.

For 40K/10K (Fig. 4) similar plots are presented.
At higher sink temperature glide, the high discharge temperature for R717 decreases the area of feasible solutions. At the same time, transcritical R744 is economically and technically feasible at even higher temperature lifts, than for the case 10K/10K. It can be seen that an area exist where R744 increases the allowable $\Delta T_{\text{lift}}$, in the areas where R717 is constrained by discharge temperature. None of the considered heat pumps for sink/source glide of 40K/10K in Fig. 4a - 4f allow a PBT below 4 years.

From investigation of Fig. 3 and 4 it is found that the trends for NPV are similar for all subcritical systems, although at different magnitude for $\Delta T_{\text{lift}}$. In order to further understand the mechanisms, an investigation of the thermodynamic performance and investment has been performed for one case (corresponding to R600a at 10K/10K sink/source glide in Fig. 3e). The trends for COP and investment are presented in Fig. 5a and 5b respectively. High COP is obtained for low temperature lifts resulting in low $F_{C_{hp}}$. The investment $C_{hp}$ is lowest at high sink outlet temperatures and medium (e.g. 30-50 K) temperature lift. At lower lift the investment required for the condenser is increased, while at low sink outlet temperature and high temperature lift the PEC of the compressor is significantly increased, as the suction volume of the heat pump is high.

High temperature glide in the source is typically used in the cases where the capacity of the source at low temperature glide does not fully match the heat demand. Optimal heat production may then be considered as a tradeoff between capacity and performance. By inspection of 20K/20K (Fig. A.2), it is clear that the working domains of the heat pumps are closely related to those of 20K/10K (Fig. A.1). Because of the increased pressure ratio between evaporator and condenser, the R717 discharge temperature constraint is moved below a temperature lift of 40 K. Similarly, the economic constraints are moved towards a lower temperature lift.

A full comparison of working domains for all heat pump systems is presented in Fig. 6 and Fig. 7 for all of the considered sink and source glides. Fig. 6a presents the combination of boundaries for each individual working fluid for sink/source glide of 10K/10K and Fig. 6b highlights the economically best (NPV) solutions in the entire working domain. Similar plots are presented for the alternative glides. Fig. 6c-d correspond to 20K/20K glide. Fig. 7a-b and Fig. 7c-d are for 20K/10K and 40K/10K respectively.

For all of the sink/source glides, it is found that R717LP configurations are optimal within the established technical and economical boundaries for the system. For increasing sink temperatures, R717HP is cost optimal in the area constrained by discharge temperatures and high pressure limit. For further increase in sink temperatures, R600a configurations are the only feasible option until the limitations on pressure at approximately 115°C. The R600a configurations are furthermore limited by an economic constraint at $\Delta T_{\text{lift}}$ between 30 to 40 K for most glides, and 40 to 50 K for the 40K/10K glide (Fig. 7d).

Transcritical R744 is the optimal configuration at
Figure 3: Working domains for five different heat pumps - $\Delta T_{\text{sink}}=10$ K / $\Delta T_{\text{source}}=10$ K
Figure 4: Working domains for five different heat pumps - $\Delta T_{\text{sink}} = 40$ K / $\Delta T_{\text{source}} = 10$ K
4. Discussion

The economic correlations used in the investigation are based directly on data from Danish suppliers. Reasonable price correlations are not easily found. Producers and intermediate trade businesses seem reluctant to reveal the true prices. Unlisted component cost or sealed price agreements result in what appears to be uncompetitive heat pump systems, as the heat cost is increased. From Fig. 2b a 12% decreased PV is found for a 30% decrease in component cost. Such a change would significantly increase the feasible area for all the considered heat pump types. It is expected that the cost reductions are similar for the investigated technologies, and as such the analysis would not change the relationship significantly between the types, but only between heat pumps and natural gas installations.

The results presented here stem solely from simulation results by verified models but are not validated with experimental results. However, the same approach has been applied for all investigations and thus it is assumed that if any fault occurs it will mainly effect the absolute values.

The heat transfer correlations used for condensation and evaporation are derived from experiments with R134a, but used in the evaluation for all working fluids, assuming that the individual transport properties of the fluids allow the required detail for calculating the correct heat transfer coefficient. An updated experimental investigation of the correlation at the correct temperature levels is being pursued at the time of writing. The used correlations have been compared to data from a few manufacturers with promising results. Investigation of the parametric variations in Fig. 2b shows that the effect of changed heat transfer coefficient is between -12% to -5% for the total heat transfer coefficient on PV.

5. Conclusion

From a variation of operation hours and heat production of a heat pump, it is found that the net present value or payback period shows higher detail in the feasibility of investigated solutions than the use of semi-economic parameters such as COP and VHC.

All of the six considered heat pumps show working domains where net present value is positive, when compared to the fuel cost of a natural gas burner. Four of the heat pumps are advantageous when considering all of the presented constraints. These heat pumps are LP R717, HP R717, R600a and transcritical R744. In areas covered by the working domain of several configurations, the R717LP and R717HP allow the optimal NPV.
Figure 6: Compilation of working domains (a & c) and optimal solutions based on NPV (b & d) for selected heat pumps -
(a & b) $\Delta T_{\text{sink}}=10 \text{ K} / \Delta T_{\text{source}}=10 \text{ K}$
(c & d) $\Delta T_{\text{sink}}=20 \text{ K} / \Delta T_{\text{source}}=20 \text{ K}$
Figure 7: Compilation of working domains (a & c) and optimal solutions based on NPV (b & d) for selected heat pumps -
(a & b) $\Delta T_{\text{sink}} = 20$ K / $\Delta T_{\text{source}} = 10$ K
(c & d) $\Delta T_{\text{sink}} = 40$ K / $\Delta T_{\text{source}} = 10$ K
By investigation of the four sink/source working domains for vapour compression heat pumps it is found, that sink temperatures of up to 115 °C and temperature lifts up to 40 K can be achieved by four common heat pump systems. For both LP and HP R717 heat pump systems, the discharge temperature is a significant limitation for expansion of the working domains, while as for the remaining technologies the economic constraints restrict feasible solutions.

Acknowledgements

This work was supported by DONG Energy, Copenhagen Cleantech Cluster (CCC) and Teknologisk Institut

References


Sørensen, K., 2013. HPO R717 compressor cost. Private communication.


A. Appendix: Technical and economical operating boundaries for 20/10 and 20/20

Figure A.1: Working domains for five different heat pumps - $\Delta T_{\text{sink}}=20$ K / $\Delta T_{\text{source}}=10$ K
Figure A.2: Working domains for five different heat pumps - $\Delta T_{\text{sink}}=20 \text{ K} / \Delta T_{\text{source}}=20 \text{ K}$