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An attempt to define critical wave and wind scenarios leading to capsize in beam sea

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Abstract

The IMO Weather Criterion has proven to be the governing stability criterion regarding minimum GM for e.g. small ferries and large passenger ships. The formulation of the Weather Criterion is based on some empirical relations derived many years ago for vessels not necessarily representative for current new buildings with large superstructures. Thus it seems reasonable to investigate the possibility of capsizing in beam sea under the joint action of waves and wind using direct time domain simulations. This has already been done in several studies. Here it is combined with the First Order Reliability Method (FORM) to define possible combined critical wave and wind scenarios leading to capsize and corresponding probability of capsize. The FORM results for a fictitious vessel are compared with Monte Carlo simulation and good agreement is found at a much lesser computational effort. Finally, the results for a large container vessel and a small ferry will be discussed in the light of the current weather criterion.

Keywords

Weather criteria; wind loads; wave loads; FORM; capsize; design load scenarios

Introduction

Recently the International Maritime Organization (IMO) has initiated a thorough revision of the intact stability rules in the framework of Goal Based Design, e.g. Peters et al. (2013). Several draft guidelines have been issued, e.g. SDC 1/NF.8 by United States and Japan discussing in details the requirements for the hydrodynamic software to be applied including qualitative and quantitative assessment procedures. The focus so far has been on failure modes related to the change of righting lever in waves, notably parametric rolling and pure loss of stability; whereas the dead ship behavior in beam sea still is based on the existing Weather Criterion issued by IMO (1985) as Resolution A.562. This criterion is based to a large extent on model tests of older hull forms and does not provide any probability of capsize for a given vessel, just a pass/no pass result. Furthermore, the wave environment is not explicitly specified in the criterion thus leading to the same requirement whether the ship is sailing in restricted areas or not. A very detailed and precise description of the drawbacks in the IMO Weather Criterion, as applied to modern ships, is given in Bulian and Francescutto (2004).

The reason for the current revision of the intact stability rules is obviously related to the damage cases reported for ships. Here parametric rolling and pure loss of stability have been in focus for some years, and still loss of containers due to excessive roll motions happens quite often. The dead ship condition in beam sea is not as frequent a scenario. However, the restriction on the transverse metacentric height (GM) imposed by the Weather Criterion can be rather severe and be the governing criterion including damage stability criteria for ships with large superstructures. This is for instance so for a number of small ferries operating in Danish water, Erichsen et al. (2015), where the Weather Criterion adds one to two meters on top of the GM required from all other intact and damage stability criteria. The required increase in GM here leads to rather small roll natural periods with possible discomfort for the passengers and crew. Similar observations are seen for large passenger ships, Francescutto et al. (2001). The latter paper also contains a very relevant discussion on the assumptions in the Weather Criterion.

Previously, the Weather Criterion has been investigated by e.g. Bulian and Francescutto (2004) using a linearized one degree-of-freedom system. Thereby, the statistics of the roll response is easily obtained. A very detailed description of the wind load including wind admittance factors is given and comparison with Monte Carlo simulations shows favorable agreement for the two example vessels. In another study, Vassalos et al. (2003) use a state-of-the-art six degree-of freedom hydrodynamic software and a time domain analysis is performed. For the example vessel the roll period is chosen such that it is the wave action rather than the wind force that leads to rolling. Estimation of the probability of capsizing is done using Order Statistics, based on the rather limited time domain simulations performed. An interesting conclusion is that the wind and roll scenario up to the onset of capsize is in fair agreement with what is assumed in the Weather Criterion; but also that the probability of the scenario is low, possibly due to the low probability of simultaneously occurrence of the stipulated wave and wind forces. Other interesting studies along the same lines have been published by Umeda and colleagues, e.g. Paroka et al. (2006).

Reliable statistics of capsize in dead ship conditions require long time domain simulations. This can be very time consuming when a detailed hydrodynamic model is
applied. Therefore, other statistical procedures than Monte Carlo simulations (MCS) are worth looking at. In the present paper the First Order Reliability Method (FORM) will be considered. It has previously been found useful for extreme value prediction of stationary stochastic time domain processes, e.g. Der Kiureghian (2000), Jensen (2007) and Jensen (2014). Stationary stochastic wave and wind loads can be considered simultaneously, Kogiso and Murotsu (2008), Jensen et al. (2011) and hence readily applicable for an investigation of the Weather Criterion. Kogiso and Murotsu (2008) actually deal with the Weather Criterion using a simplified one degree-of-freedom model with a linearized wind load model. They find for an example ship that the capsize probability from the FORM analysis disagree by a factor of 30 from the Monte Carlo simulations. Due to the choice of roll period the main load leading to capsize comes from the wind force. They try to identify the reason for the disagreement and suggest that multiple failure scenarios with the same probability of occurrence are the main reason. It is not clear from their presentation exactly how the statistical predictions from the Monte Carlo simulations are made. Furthermore, the reliability index of 2.466 found from the FORM analysis is fairly low and, hence, some deviations from the Monte Carlo simulations can be expected, e.g. Jensen (2014). Clearly, for a linear system the failure surface will be linear with only one most probable failure point.

In the present paper a FORM analysis is performed using also a one degree-of-freedom system. Differences compared to the analysis of Kogiso and Murotsu (2008) are the use of more wave and wind components, a non-linear wind load description and a different wave exciting moment.

The aim of the analysis is partly to discuss whether FORM can provide accurate probabilities of capsize at low probability levels and partly to compare the capsize scenarios in terms of most probable time domain variations of wave elevation, wind speed and roll angles leading to capsize with those assumed in the IMO Weather Criterion.

**Mathematical formulation**

The analysis of a dead ship condition in beam sea will be made using a single-degree-of-freedom model adapted from Jensen (2007). Hence, the roll angle \( \phi(t) \) as function of time \( t \) is determined by the solution of the equation of motion:

\[
\ddot{\phi} + 2 \xi_1 \omega_\phi \dot{\phi} - \xi_2 \phi = \frac{g \cdot GZ(\phi)}{\omega_\phi^2} + \frac{M_c}{J_z} \tag{1}
\]

where \( \xi_1, \xi_2, \xi_3 \) are non-dimensional damping coefficients, \( g = 9.81 \text{ m/s}^2 \), \( M_c \) the roll exciting moment from wind and waves and \( r_z \) the roll radius of gyration in water. The roll mass moment of inertia \( J_z \) is related to the mass of the vessel \( \Delta \) through \( J_z = r_z^2 \Delta \) and the roll natural frequency \( \omega_\phi = \sqrt{gGM / r_z} \). The stationary stochastic long-crested wave \( H(t) \) at the position of the vessel is given as

\[
H(t) = \sum_{j=1}^{\infty} h_j(\omega_j, t, \theta)
\]

where \( h_j(\omega_j, t, \theta) = \sqrt{S_\omega(\omega_j)d\omega_j} (u_j \cos(\omega_j t) - \overline{u_j} \sin(\omega_j t)) \)

and the roll angle is fairly low, hence, some deviations from the Monte Carlo simulations are the main reason. It is not clear from their presentation exactly how the statistical predictions from the Monte Carlo simulations are made. Furthermore, the reliability index of 2.466 found from the FORM analysis is fairly low and, hence, some deviations from the Monte Carlo simulations can be expected, e.g. Jensen (2014). Clearly, for a linear system the failure surface will be linear with only one most probable failure point.

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The wind-induced roll moment is modelled as

\[
M_{\text{wind}}(t) = \sum_{j=1}^{\infty} \frac{\rho_w A_{\text{wind}} C_{\text{wind}} (U_{\text{mean}} + U_{\text{gust}}) \cdot S_\omega(\omega_j)d\omega_j}{\omega_j} (u_j \cos(\omega_j t) - \overline{u_j} \sin(\omega_j t)) \tag{4}
\]

The response amplitude operator \( RAO_\omega(\phi) \) will in the first part of the paper, dealing with comparison between FORM and MCS analyses, be taken as closed form expressions, Jensen et al. (2004), for a triangular shaped prismatic beam. Later, in the application section, the wind-induced roll moment will be determined by six degree-of-freedom state-of-the-art hydrodynamic software.

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\]

where \( S_\omega(\omega) \) is the wave spectrum as function of wave frequency \( \omega \) and \( u_j, \overline{u}_j \) statistical independent and standard normal distributed variables. The frequency discretization is taken with constant increment \( d\omega_j = \omega_j - \omega_{j-1} \). Second order waves can easily be included albeit with a significant increase in computational effort, Jensen (2014).

The wave-induced roll exciting moment is assumed linearly dependent on the wave slope:

\[
M_{\text{wind}}(t) = \sum_{j=1}^{\infty} \sqrt{S_\omega(\omega_j)d\omega_j} (u_j \cos(\omega_j t) - \overline{u_j} \sin(\omega_j t)) \tag{6}
\]

The wind-induced roll moment is modelled as

\[
M_{\text{wind}}(t) = \frac{\rho_w A_{\text{wind}} C_{\text{wind}} (U_{\text{mean}} + U_{\text{gust}}) \cdot S_\omega(\omega_j)d\omega_j}{\omega_j} (u_j \cos(\omega_j t) - \overline{u_j} \sin(\omega_j t)) \tag{4}
\]

and \( GZ(\phi) \) is the righting lever as function of the roll angle. In the present study \( GZ(\phi) \) is modelled as

\[
GZ(\phi) = (GM - A_s) \sin(\phi) + A_s \phi + A_s \phi^3 + A_s \phi^5 \tag{2}
\]

where \( A_s, A_s, A_s \) are determined by curve-fitting to the actual \( GZ(\phi) \).

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and the roll angle is fairly low, hence, some deviations from the Monte Carlo simulations are the main reason. It is not clear from their presentation exactly how the statistical predictions from the Monte Carlo simulations are made. Furthermore, the reliability index of 2.466 found from the FORM analysis is fairly low and, hence, some deviations from the Monte Carlo simulations can be expected, e.g. Jensen (2014). Clearly, for a linear system the failure surface will be linear with only one most probable failure point.

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The wind-induced roll moment is modelled as

\[
M_{\text{wind}}(t) = \frac{\rho_w A_{\text{wind}} C_{\text{wind}} (U_{\text{mean}} + U_{\text{gust}}) \cdot S_\omega(\omega_j)d\omega_j}{\omega_j} (u_j \cos(\omega_j t) - \overline{u_j} \sin(\omega_j t)) \tag{5}
\]

Here \( \rho_w, A_{\text{wind}}, C_{\text{wind}}, U_{\text{mean}}, U_{\text{gust}}, \) \( z_{\text{wind}} \) are the mass density of air \( (1.225 \text{ kg/m}^3) \), the lateral wind area, the wind coefficient, the mean wind speed, the gust wind speed and, the wind lever, respectively. The gust wind speed is assumed to be a stationary stochastic process:

\[
U_{\text{gust}}(t) = \sum_{j=1}^{\infty} \sqrt{S_\omega(\omega_j)d\omega_j} (u_j \cos(\omega_j t) - \overline{u_j} \sin(\omega_j t)) \tag{6}
\]

where \( S_\omega(\omega) \) is the gust wind spectrum and \( u_j, \overline{u}_j \) again statistical independent and standard normal distributed variables. Several gust wind spectra have been formulated; see e.g. Bec (2010) for a summary. Here the Davenport and Kaimal spectra have been considered.

For each realization of \( \{\overline{u}_j, \overline{u}_j\} = \{u_j, \overline{u}_j; i = 1,2,...,m + n\} \) the solution of Eq. (1) yields a time domain response only dependent on the initial conditions. Due to the stationary stochastic properties assumed the statistics of
the response do not depend on the absolute value of time. Hence, if a time \( t_0 \) is chosen sufficiently far away from the initial conditions, the statistical properties of the response will not depend on this value. In the present context \( t_0 \) is chosen as 300s, since the hydrodynamic memory effect is less than 150s in typical roll motion simulations, excluding parametric rolling. The realization, which exceeds a given threshold \( \phi_0 \) at time \( t = t_0 \) with the highest probability, is sought. This can be formulated as a limit state problem, Der Kiureghian (2000), within time-invariant reliability theory:

\[
G(\phi_0, \{u, \overline{u}\}) = \phi_0 - \phi(t_0, \{u, \overline{u}\}) = 0
\]  

Due to the statistical properties of all \( u, \overline{u} \) the most probable realization of waves and gust wind is the one where

\[
[u^2(\phi_0)] = \sum_{i=1}^{m+n} (u_i^2 + \overline{u}_i^2) 
\]  

has its minimum value. This realization \( \{u^*, \overline{u}^*\} \) can be considered as a kind of design or critical wave and wind scenario leading to exceedance of the prescribed response \( \phi_0 \). Due to the non-linearity in Eq. (1) the limit state surface, Eq. (7), is not linear in \( \{u, \overline{u}\} \) and an effective search routine is needed when \( m+n \) is large. An excellent summary of optimization algorithms is given by Liu and Der Kiureghian (1991) and in the present study the so-called modified Hasofer-Lind procedure is implemented. It is based on the original work by Hasofer and Lind (1974), but extended with a line search where a merit function containing the error in both the limit state function and its derivative is minimized. In all cases considered in the present paper this procedure has provided the design point \( \{u^*, \overline{u}^*\} \) for all values of the roll limiting angle \( \phi_i \) up to \( GZ(\phi_i) = 0 \). The calculations are most effectively performed by choosing a moderate value of \( \phi_0 \) and then use the design point for this roll angle as the starting point \( \{u, \overline{u}\} \) for the search for the design point for a next higher roll angle. Typical values of starting point and increment are here 0.2 rad and 0.05 rad, respectively. When design point has been determined the point-wise probability of exceedance the response \( \phi_0 \) can be estimated by the FORM approximation:

\[
P_{\text{FORM}}[\phi > \phi_0] = \Phi(-\beta(\phi_0))
\]  

where \( \Phi \) is the standard normal distribution function and with the reliability index \( \beta \) defined as

\[
\beta(\phi_0) = \sqrt{\min_{\phi_0} [u^2(\phi_0)]}
\]  

Mean up-crossing rates can be determined within the FORM approximation from the design point information, Jensen and Capul (2006), and used to estimate the probability of exceedance during a time range \( T \). In most cases, the result can be simplified to

\[
P[t \max \phi(t) > \phi_0 | \phi_0 > \phi_i] = 1 - \exp(-N \exp(-0.5(\beta(\phi_i)^2))
\]  

where \( N \) is the number of peaks above the mean (close to static) roll angle \( \phi_i \) during the time range \( T \). The static roll angle follows from Eq. (1) by omitting all dynamic terms and, furthermore, \( N = T \alpha_y / 2\pi \).

Due to the linearization of the limit state surface around the exact design point the probability information, Eqs. (10) and (12), is not exact, but holds only asymptotically for large values of the reliability index \( \beta \). Therefore, Monte Carlo simulations will be performed to validate the FORM results. The Monte Carlo simulation also uses Eq. (1) and collects for each simulation only the value \( \phi(t_0) \). After \( M \) simulations the results are ordered: \( \phi_{\text{max}} \leq \phi_i \), \( i = 2, 3, \ldots, M \). The point-wise probability of exceedance is then determined as

\[
P_{\text{MCS}}[\phi > \phi_i] = 1 - \frac{i}{M+1}; \quad i = 1, 2, \ldots, M
\]  

with the corresponding reliability index

\[
\beta_{\text{MCS}}(\phi) = -\Phi\left(1 - \frac{i}{M+1}\right)
\]  

A comparison of the results from Eqs. (10) and (14) can then be used to estimate the accuracy of the FORM approximation. This is done in the next section.

**FORM versus Monte Carlo simulations**

This section evaluates the FORM approach considering a fictitious prismatic vessel using a closed form expression for the wave-induced roll moment, Jensen et al. (2004), for triangular shapes sections. A comparison of this expression with experimental results by Vugts (1968) shows reasonable agreement, albeit with large variations over the frequency range considered. Furthermore, \( r_s = 0.4B \) is assumed where \( B \) is the waterline breadth. The parameters in Eqs. (1)-(5) are chosen somewhat arbitrarily as: \((\xi_1, \xi_2, \xi_3) = (0.012, 0.40, 0.42) \) (Bulian (2005)), \( B = 32.2m \), draft \( D = 10.5m \), \( (A_1, A_2, A_3) = (5, 1, -10)m \), block coefficient \( C_{\text{wind}} = 1 \), \( A_{\text{wind}} = 4LD \), \( U_{\text{mean}} = 26m/s \). The ship length \( L \) cancels out for a prismatic beam. The assumed wind area is quite large, somewhat between what is seen for container vessels and large cruise ships. The wind lever is taken as

\[
z_{\text{wind}}(\phi) = 0.5L \left(1 + \frac{A_{\text{wind}}}{LD}\right) (0.3 + 0.7 \cos^3 \phi)
\]  

where the variation of the lever with roll angle is taken from Vassalos et al. (2003). The waves are modelled by a JONSWAP spectrum with significant wave height \( H_s = 11m \) and zero upcrossing period \( T_z = 12s \). The gust wind spectrum is of the Davenny type with a variance equal to \( 6KU_{\text{mean}}^2 \), where
$K = 0.003$. Also the Kaimal spectrum with the same variance as the Davenport at $U_{\text{mean}} = 26 \text{m/s}$ has been applied, but even if the frequency variation of the two spectra is different, especially at low frequencies, the results regarding capsize probabilities were quite similar. The frequency range for the wave spectrum is taken as $[\pi / T_z, 3\pi / T_z]$ whereas for the gust wind speed the range used is $[0.05 \text{s}^{-1}, 0.6 \text{s}^{-1}]$.

The total number of frequencies $m + n = 50$, but different numbers of wave and wind components are considered to investigate the sensitivity to this choice.

The time $t_0 = 300 \text{s}$ is found large enough to avoid notable influence from the initial condition and short enough to avoid repetition of the stochastic wave and wind loads with the current discretization; see also Nielsen and Jensen (2009). In addition $t_0 = 420 \text{s}$ has been applied without any change in results. The time step in the Runge-Kutta solution procedure of Eq. (1) is taken as 0.5s. Finally, GM values between 1.5m and 4.5m will be considered. Fig. 1 shows the $GZ$ curve for $GM = 2.5m$ using Eq. (2). For the other GM values similar variations of $GZ$ with roll angle are achieved, albeit with changing maximum value and angle of vanishing stability $GZ = 0$.

A comparison between FORM and Monte Carlo simulations (MCS) for different values of GM is given in Figs. 2-6. The MCS are performed with 100,000 simulations implying a maximum reliability index $\beta_{\text{MCS}} = -\Phi^{-1}(10^{-3}) = 4.265$. The Coefficient of Variation (COV) of the MCS is roughly equal to $1/\sqrt{E}$ where $E$ is the number of results above the target value. Thus, with a prescribed COV of, say, 0.1, the number $E = 100$ which means that MCS is only accurate here up to $\beta_{\text{MCS}} = -\Phi^{-1}(10^{-3}) = 3.09$. This is in agreement with a visual inspection of Figs. 2-6.

For a linear system $\beta = (\phi - \phi_0) / s_\phi$ where $s_\phi$ is the standard deviation of the roll angle. It is seen from Figs. 2-6 that the non-linearity in Eq. (1) influences $\beta$ to some extent for $\phi_0$ greater than 0.25rad. For the MCS $\beta = 0$ corresponds to $\phi_0 = \phi_0$.

The case with $GM = 1.5m$ has the highest probability of capsize implying that four MCS realizations diverge, i.e. capsize ($GZ = 0$) at 0.70rad, before reaching the time $t_0 = 300 \text{s}$, as indicated in Fig. 2. FORM shows the expected flattening behavior up to 0.70rad. For the particular set of results in Fig. 2, $m = 10, n = 40$ as the wind force, controlled by $n$, in this case is the dominant load leading to roll. For Figs. 3-6 $m = 30, n = 20$ are used. The wave and wind spectra are shown in Fig. 7 for this discretization.

Generally, the FORM results are seen to be slightly conservative compared to the MCS results for lower values of the reliability index $\beta$. However, when $\beta$ exceeds about three the FORM results seem to approach the MCS results, albeit the MCS results here are scarce with only 100,000 simulations. Therefore one case with $GM = 4m$ is considered in detail, see Fig. 8. Here additional MCS with 1,000,000 realizations are included as well as results where the number of frequency compo-
tems \((m,n)\) in the wave and gust wind spectra is changed: \(m = 45, n = 5\) and \(m = 20, n = 30\). It is seen that neither the FORM nor the MCS results are sensitive to this choice. The two FORM calculations cannot be separated from each other in Fig. 8. This is also so for all \(GM\) values considered in Figs. 2-6. Moreover, the results are seen to be invariant of the selected sampling time, 300s vs. 420s.

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The variation of the reliability index \(\beta\) and the corresponding probability of exceedance during 3 hours exposure, Eq. (12), to the prescribed stationary wave and wind systems is shown in Figs. 10 and 11, respectively. Two target roll angles are considered: 0.65rad and 0.70 rad, where the latter angle is close to the angle of vanishing stability for the lowest \(GM\) values. Only the FORM results are shown since MCS results are not obtained for \(\phi_0 \geq 0.65\) rad.

An interesting behavior is noticed from these two figures, namely that the probability of exceeding a specified roll angle (defining ‘capsize’) is not a monotonic decreasing function of \(GM\). Due to the different peak periods of the wind and wave spectra and the dependence of the roll natural period on \(GM\), capsize is more probable in this example for \(GM\) around 2.5-3m than for \(GM\) around 2m. Of course, this behavior depends on the ship parameters and the model of especially the wave roll exciting moment. This will be considered in
more details in a later section dealing with a specific real-world vessel. It should also be mentioned that the heave acceleration $\ddot{w}$ might be important through replacement of $g$ with $g - \ddot{w}$ in Eq. (1). This can be evaluated by using, say, the closed form expression for heave in Jensen et al. (2004). If heave acceleration is considered and setting $GM = 2.5$ m and a target roll angle $\phi_0 = 0.65$ rad, for instance, the reliability index $\beta$ changes from 4.58 to 4.16 and the corresponding probability of exceedance during 3 hours operation changes from 0.018 to 0.109.

**Fig. 11:** Probability of exceedance the target roll angles $\phi_0 = 0.65$ and 0.7 rad during 3 hours as function of $GM$

### Critical wave and wind scenarios

With the given set of parameters used for the present fictitious vessel the IMO Weather Criterion, IMO (1985), prescribes a minimum $GM = 2.5$ m. Hence it is found interesting to compare the deterministic scenario postulated in IMO (1985) with the critical wave and wind scenario obtained from the FORM analysis for this $GM$ value. The set of FORM results is found by substituting the design point value $\{\bar{u}', \bar{\omega}'\}$ into Eqs. (3) and (6). The design scenario and its associated results are shown in Figs. 12-13 with the corresponding roll variation in Fig. 14. The target roll angle is taken as $\phi_0 = 0.65$ rad being close to capsize. The $GZ$ curve is the one shown in Fig. 1. For this case, the roll natural frequency $\omega_y$ is 0.38 rad/s implying that the wave-induced roll motion dominates over the wind-induced roll moment, see Fig. 7.

**Fig. 12:** Most probable wave scenario at the position of the ship leading to a roll angle of 0.65 rad for $GM = 2.5$m

The probabilities of capsize with $GM = 2.5$ m during 3 hours of operation are around 0.02 and 0.003 using 0.65 rad and 0.70 rad (40 deg), respectively, as the angles leading to capsize, see Fig. 11. These values might be reasonable numbers in this severe sea state but, unfortunately, cannot be compared to anything in IMO (1985). It is noted that the probability of capsize depends significantly on the angle defining capsize.

The first 120s of the time signals are not shown as they are influenced by the choice of initial conditions. The remaining parts, shown in Figs. 12-14, are unaffected by the initial conditions. It can be observed from Fig. 14 that the angle the ship attains to windward from the static wind heel position before “capsize” here is 22.6 deg which is very close to the similar angle (denoted $\phi_1$) in IMO (1985): 22.7 deg. In addition, the maximum wind speed (30.7 m/s from Fig. 13) is close to the assumed maximum wind speed in IMO (1985): $\sqrt{1.5 U_{mean}} = 31.8$ m/s. The maximum roll angle occurs, as expected, when the wave slope is large at the position of the ship after the passing of a wave trough, see Fig. 12 and 14.

It is seen that the most probable wave episode in Fig. 12 has a nearly constant period close to the roll natural period of 16 s, even if the zero-crossing period of the sea state is 12 s. This reflects the resonance behavior of the roll response and the ability of the FORM procedure to identify this critical, and most probable, wave scenario leading to capsize.

**Fig. 13:** Most probable wind scenario leading to a roll angle of 0.65 rad for $GM = 2.5$m

**Fig. 14:** Most probable roll scenario leading to a roll angle of 0.65 rad for $GM = 2.5$m

Generally, the analysis of this fictitious vessel yields high confidence in the calculation procedures and therefore, in the next section, a real-world vessel will be considered and compared with the regulations by IMO (1985).

### Application examples

First a large container vessel (length: 350 m, beam:
48.2m, draft: 14.5m, wind area: 8705m², GM=1.71m) is considered. The damping coefficients are taken as those used in the previous section. The roll period becomes 29.6s, which makes the gust wind the dominant stochastic roll excitation. The result is a reliability index $\beta$ in excess of 10 for roll angles above 10 degrees. Thus the probability of capsize in dead ship conditions for this vessel is extremely small and of no significance for the intact stability verification.

The second application deals with a small ferry designed for inland transportation in Danish waters. The pertinent data are: length: 45m, beam: 13.1m, draft: 2.7m, wind area: 501.7m². The required GM is 2.285m without taking account for the IMO weather criterion and 4.072m when the weather criterion is applied. The large influence of the weather criterion on the required GM has been observed before for small ferries, Erichsen et al (2015), and the observation makes it interesting to quantify the probability of capsize inherent in this regulation. The analysis for the ferry is done for both GM values using the corresponding GZ curves as input together with the same damping coefficients as before. The roll exciting moment (Froude-Krylov and diffraction) is calculated for the actual hull by using the 3D hydrodynamic WAMIT® procedure (wamit.com). Compared to the closed form expression a significant smaller wave-induced roll excitation is found. The stationary wave condition is specified as a JONSWAP spectrum with significant wave height $H_s = 3.5$m and zero upcrossing period $T_z = 5$s which is chosen as the most severe realistic operational condition for the vessel. The wind spectrum is the same Davenport spectrum as used in previous sections. The spectra and the roll natural periods are shown in Fig. 15. It is seen that the waves are the most important roll excitation source in this case.

The reliability index $\beta$ as function of target roll angle is shown in Fig. 16 and the corresponding probability of exceeding the target roll angle during three hours of exposure to this dead ship condition is given in Fig. 17. The target (limiting) roll angle for the ferry is 0.6rad (35 deg.) due to down-flooding points and thus for the case with GM = 2.285m the probability of exceedance becomes 0.24, i.e. the return period for the exceedance of 0.6rad is roughly 12 hours in this severe sea state. With GM = 4.072m as required from the IMO weather criterion the probability of exceedance roll angles above 0.5rad becomes nearly zero as seen from Fig. 17. This seems reasonable results and also a support for the need for a weather criterion for this kind of vessels. However, the weather criterion in its present formulation might be rather conservative for small ferries.

Conclusions

The paper addresses the search for second generation of intact stability criteria for ships in dead ship conditions subjected to stochastic wave and wind forces. The physical model for the roll motion is a standard 1D model, but the paper suggests a new application of the FORM procedure for estimation of the extreme value distribution of the roll angle and subsequent capsize probability.

The results indicate that the FORM procedure is a feasible tool for extreme value prediction of the roll motion and that the current IMO weather criterion is reasonable in the sense that the required GM from this criterion for a small ferry implies a nearly zero probability of capsize. However, the lack of an accepted target probability of failure makes a quantitative evaluation impossible and the present criterion might be too conservative for small ferries as judged from the present results.

Further studies considering more vessels, operational conditions and a coupling between ship motions components are obviously needed together with an IMO specification of the allowable probability of failure.
References


