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Published in:
IEEE Photonics Journal

Link to article, DOI:
10.1109/JPHOT.2015.2510330

Publication date:
2016

Document Version
Publisher's PDF, also known as Version of record

Link back to DTU Orbit

Citation (APA):

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Volume 8, Number 1, February 2016

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DOI: 10.1109/JPHOT.2015.2510330
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Manuscript received November 16, 2015; accepted December 14, 2015. Date of publication December 17, 2015; date of current version January 12, 2016. This work was supported in part by the Brazilian agencies CAPES, CNPq, and FAPESP. Corresponding author: B.-H. V. Borges (e-mail: benhur@sc.usp.br).

Abstract: In this paper, new analytical formalisms to evaluate the packet throughput of multiservice multirate slotted ALOHA optical code-division multiple-access (OCDMA) networks are proposed. The proposed formalisms can be successfully applied to 1-D and 2-D OCDMA networks with any number of user classes in the system. The bit error rate (BER) and packet correct probability expressions are derived, considering the multiple-access interference as binomially distributed. Packet throughput expressions, on the other hand, are derived considering Poisson, binomial, and Markov chain approaches for the composite packet arrivals distributions, with the latter defined as benchmark. A throughput performance evaluation is carried out for two distinct user code sequences separately, namely, 1-D and 2-D multiweight multilength optical orthogonal code (MWML-OOC). Numerical results show that the Poisson approach underestimates the throughput performance in unacceptable levels and incorrectly predicts the number of successfully received packets for most offered load values even in favorable conditions, such as for the 2-D MWML-OOC OCDMA network with a considerably large number of simultaneous users. On the other hand, the binomial approach proved to be more straightforward, computationally more efficient, and just as accurate as the Markov chain approach.

Index Terms: Optical code-division multiple-access (OCDMA), throughput, bit error rate (BER), flexible networks, multiservice, multirate.

1. Introduction

The massive growth of bandwidth-intensive networking applications, such as interactive e-learning, e-health, cloud computing, and 3-D/4K ultra HD video streaming indicates that end-user demand for higher-bandwidth network services is at an all-time high. However, conventional low bandwidth-consuming services are expected to coexist with these multimedia applications through the foreseeable future. Therefore, emerging access network architectures with full service support and flexible bandwidth capabilities to handle such vast amounts and diversified patterns of data traffic have become paramount nowadays.
Optical code-division multiple-access (OCDMA) has emerged as an attractive architecture in flexible, robust and high capacity passive optical networks that straightforwardly support multi-service and multiple data rate transmissions at the physical layer via manipulation of the users' code parameters [1]–[17]. Moreover, OCDMA has several desirable features, like, for example, support for burst Internet protocol traffic, asynchronous tell-and-go multiple access capability (a key issue for practical network deployment), low-latency access, symmetric bandwidth support for up- and downlink, soft-capacity on-demand, statistical multiplexing, simplified network control, high data confidentiality, flexibility in code design, and support to random-access protocols [18]–[28].

Normally, in multiservice and multirate OCDMA, multiple users share the same network resources by means of uniquely assigned code sequences, where different code weights support multiservice transmissions (related to the bit error rate (BER) level) and different code lengths support multirate transmissions. Amongst several codes proposed especially for multiservice, multirate OCDMA networks [1], [2], [4], [5], [7], [8], [10]–[17], the so-called 1-D and 2-D multi-weight multi-length optical orthogonal code (MWML-OOC) [1], [2] have attracted a great deal of attention since they are flexible enough to allow both the code weight and length to be chosen arbitrarily, therefore achieving arbitrary levels of service differentiation and multirate transmissions simultaneously. In addition, both 1-D and 2-D MWML-OOC have maximum out-of-phase autocorrelation and cross-correlation values bounded by one. These correlation properties in turn reduce the multiple-access interference (MAI) among users, generally considered as the dominant noise source in OCDMA networks [19], [29].

The MAI has a serious impact not only on the BER, but also on the packet throughput performance of OCDMA networks. An expressive number of analytical approaches have been proposed to evaluating the packet throughput performance. However, all of them are limited to single service and single rate OCDMA networks, such as the approaches based respectively on Poisson [30], [31] and binomial [32]–[34] distributions to model the composite packet arrivals. On the other hand, a Markov chain approach was used in [35]–[40] to evaluate the packet throughput of single service and single rate networks.

In this paper, new analytical formalisms for evaluating the packet throughput performance of multiservice, multirate OCDMA packet networks that can be successfully applied to either 1-D or 2-D codes are proposed. These formalisms can be further applied to an arbitrary number of user classes in the network. New packet throughput expressions considering Poisson, binomial, and Markov chain approaches for the composite packet arrivals are derived, with the Markov chain approach assumed as benchmark against which the throughput performance can be compared. Moreover, expressions for BER and packet correct probability for throughput evaluation of a slotted ALOHA (S-ALOHA) OCDMA network are derived assuming binomial distribution for the MAI.

Then, a throughput performance evaluation of a two-class 1-D MWML-OOC S-ALOHA OCDMA packet network employing binomial, Markov and Poisson approaches is carried out, showing that users' class assigned with higher code weight have their throughput completely underestimated by the Poisson approach. Subsequently, the same parameters' set of this two-class 1-D MWML-OOC network is once again used, but this time only for a single class (class-1). The goal is to carry out a validation procedure defined here as the convergence of the proposed multiservice and multirate formalisms towards the well-known single service and single rate formalisms. Furthermore, a throughput performance evaluation of a 1-D MWML-OOC OCDMA packet network employing binomial, Markov and Poisson approaches is carried out, showing that the latter does not converge towards the Markov approach. In contrast, the binomial approach shows complete agreement with the Markov chain approach.

Finally, the throughput performance of a 2-D MWML-OOC OCDMA packet network is also investigated. Results showed that the Poisson approach underestimates the packet throughput in unacceptable levels and incorrectly predicts the number of successfully received packets for most values of offered load. Even under favorable conditions, such as for the 2-D MWML-OOC with a considerably large number of simultaneous users, the Poisson approach does not show
accurate results for most of the offered load range. On the other hand, the binomial approach besides converging to the Markov chain approach, it is also computationally more efficient and more convenient for numerical evaluation by virtue of its simpler mathematical formalism.

2. Multiservice, Multirate Packet Network Description

The multiservice, multirate slotted OCDMA network considered here is arranged in a star topology connecting all users to the multiple access channel via optical fibers as illustrated in Fig. 1. This time-slotted network consists of J-class users that share the same optical medium. The input power at the passive star coupler is split and transmitted equally among all J-class users. Normally, in a slotted packet network the time is divided into equal slots, where each user transmits at the beginning of a slot [36]. Without loss of generality, deleterious sources such as channel impairments and time jitter are neglected since the main focus is on the MAI between the users' codes.

A unique optical code sequence is assigned to each user, where each user transmits one packet through the network in a single time slot. The users are divided into classes according to their required QoS and transmission rates. In the analysis the following assumptions are made: 1) Different users' classes coexist in the same time slot, where packet transmission errors can occur due to MAI, 2) the transmitter can independently determine the success of a packet transmission, and schedule a packet for retransmission when it is received with errors, and 3) the overhead required by error detection techniques at the receiver is neglected for the sake of simplicity [30]. In addition, the S-ALOHA protocol is employed to provide medium access control in the MAC layer [33], [36] and the network transmits on-off pulses employing on-off keying (OOK) modulation.

In the multiservice multirate J-class OCDMA slotted network, for each user class \( j \in \{1, 2, \ldots, J\} \) the users' data rate is defined according with its given code length in a manner that high rate users have smaller code length and low rate users have longer code length such that \( L_1 < L_2 < \cdots < L_j < \cdots < L_J \), and \( L_j = T_j/T_c \) where \( T_j \) and \( T_c \) are the bit and chip period, respectively [1]. \( T_c \) is assumed constant and the same for all classes \( J \), and the transmission power of all users \( U \) in all classes \( J \) is normalized to unity. In addition, the network supports multiservice (differentiated services, or equivalently QoS) transmission through specific code weights \( W_j \) in a manner that the larger the code weight, the higher the QoS supported. Moreover, without any loss of generality, it is assumed that the desired user is the first user in the desired class denoted
as \( j' \) [3]. The total number of users in the network is \( U = \sum_{j=1}^{J} U_j \), where \( U_j \) is the number of class-\( j \) users. The bit transmission rate and the packet length of class-\( j \) is defined, respectively, as \( R_j = 1/T_j \) and \( H_j \). Then, it is evident the packet period of class-\( j \) is

\[
T_{p_j} = \frac{H_j}{R_j} = T_p. \quad (1)
\]

Similarly, the bit transmission rate and the packet length of class-\( j' \) is defined as \( R_{j'} = 1/T_{j'} \) and \( H_{j'} \), respectively, resulting in the following expression for the packet period of class-\( j' \)

\[
T_{p_{j'}} = \frac{H_{j'}}{R_{j'}} = T_p. \quad (2)
\]

It can be clearly noted from (1) and (2) that the packet period of both classes is the same (see also Fig. 2). It is worth pointing out that each class has a different \( T_j \) and, consequently, a different \( R_j \). Then a viable way of keeping the packet period \( (T_p) \) fixed is by allocating a different amount of bits according to each specific class so as to guarantee that \( T_p \) is the same for all classes. Therefore, in a multirate network the amount of bits transmitted in each packet class can be different from each other. Assuming \( H_J \) as the amount of bits in class-\( J \), then the amount of bits in class-\( j \) can be simply calculated as

\[
H_j = \left\lfloor \frac{L_j}{T_j} H_J \right\rfloor \quad (3)
\]

where \( \lfloor . \rfloor \) denotes the integer part. Independently of the number of classes present in the network, (3) allows one to calculate the amount of bits in all classes. An example of a three-class network, namely class-1 with \( L_1 = 4 \) (class-\( j \)), class-2 with \( L_2 = 5 \) (class-\( j+1 \)), and class-3 with \( L_3 = 8 \) and \( H_3 = 2 \) bits (class-\( J \)) is provided in Fig. 2. It can be noted that a packet from class-1 has twice the amount of bits than a packet of class-3 \( (H_1 = H_3 = 4, \text{ refer to (3)}) \). Notice further that the bit period of class-3 \( (T_{j=3}) \), and consequently \( L_3 \) is twice as long as the bit period of class-1 \( (T_{j=1}) \), and consequently \( L_1 \). Accordingly, class-3 transmits half the amount of bits of class-1 while keeping the same packet period \( (T_p) \) for both classes [36], [41]. On the other hand, class-2 transmits one bit more than does class-3 \( (H_2 = H_3 = 3, \text{ refer to (3)}) \) because \( T_{j=2} \) (and consequently \( L_2 \)) is smaller than \( T_{j=3} \) (and consequently \( L_3 \)) while keeping the same packet period \( (T_p) \).

### 3. Multiservice, Multirate Packet Network Performance Evaluation

In order to evaluate the throughput of the multiservice network, the BER and packet correct probability \( P_C(j') \) of the \( J \)-class system are required prior to deriving the packet throughput expressions. Thus one should start with the calculation of the BER\( j' \) and \( P_C(j') \) of the 1-D and 2-D MWML-OOC OCDMA, described in Section 3.1. Next, the network throughput based on Poisson and binomial approaches are demonstrated in Sections 3.2. and 3.3., respectively, while the network throughput based on Markov chain approach is demonstrated in Section 3.4.
The Poisson and binomial approaches are chosen because they are largely employed in single-service single-rate networks [30]–[34] while the Markov chain is chosen due to its high accuracy in the evaluation of the throughput performance [35].

3.1. BER and Packet Correct Probability

This subsection deals with the evaluation of the BER of a multiservice, multirate OCDMA packet network capable of supporting QoS and multiple data rate transmissions. Current available BER expressions are not directly applicable to the multiservice multirate network investigated in this paper. For example, [1] deals with multiservice multirate networks based only on 1-D MWML-OOC codes. Further, a Poisson distribution was used to model the MAI, which may underestimate the BER performance in inadmissible levels [6]. On the other hand, [3] deals exclusively with single-service multirate, optical fast frequency hopping code-division multiple-access (OFFH-CDMA) networks. Therefore, a new BER expression is required to assess the throughput performance of multiservice multirate OCDMA packet networks.

The network employs an OOK intensity-modulated incoherent structure, where each user transmits its assigned code sequence for data bit "1", otherwise no signal is transmitted for data bit "0". The BER expression is derived for the J-class system assuming that MAI has a binomial distribution, since the output interference of any OCDMA network is considered to be binomially distributed [1]–[3], [6]. Normally, the MAI is addressed as the most dominant noise source in OCDMA networks [19], [29]. In addition, the MAI is assumed to be the only source of performance degradation while ignoring other deleterious sources (such as time jitter, channel impairments, non-linear effects, and temperature variation effects). Moreover, it is considered that both 1-D and 2-D MWML-OOC employed here are strict codes, which means they have maximum nonzero shift autocorrelation and cross-correlation bounded by one [1], [2]. Thus, each interfering user contributes with only one chip overlapping on the desired user's code.

It is further assumed that transmissions among users are chip-synchronous, which reflects the worst possible scenario for the network analysis [42], [43] as well as users transmit data bits "0" and "1" with equal probability 1/2. Furthermore, setting the threshold value to the code weight $W_j$, and also taking into account the additive and positive properties of the optical channel, no decision error will occur when a data bit "1" is sent, which means that bit "1" is always decoded correctly, i.e., $P(\text{error}|1) = 0$. Therefore, the BER of class-$j'$ users can be expressed as

$$\text{BER}(j') = \frac{1}{2} [P(\text{error}|0) + P(\text{error}|1)] = \frac{1}{2} P(\text{error}|0) = \frac{1}{2} \sum_{i=1}^{U-1} \left( \frac{U-1}{i} \right) P'(1-P)^{U-1-i}$$

where $P'$ is the multiservice, multirate probability of interference defined as [3]

$$P = \sum_{j=1}^{J} \frac{N_j p_{jj}}{(U-1)}$$

and $\mu$ is the threshold value of the decision device, $N_j$ is the number of interfering users in class-$j$, and $p_{jj}$ is the probability of interference caused by a code of class-$j$ on a code of class-$j'$ written as $p_{jj} = W_j W_j / 2 L_j F$, where $F$ is the number of available wavelengths (it also accounts for the wavelength dimension of the 2-D code design). Note that $F = 1$ for the 1-D MWML OOC [1]. For a single service and single rate network, the probability of interference is reduced to $p_{jj} = W^2 / 2 LF$, where $W = W_j$ and $L = L_j$ [3]. It is noteworthy that (4) not only permits the evaluation of an arbitrary number of user classes in a simple way, but it also imposes low computational effort even when the number of classes becomes very large.

After being transmitted over the channel, several packets might be received corrupted due to MAI. Thus, considering the bits of a given packet as independent of each other, the probability
of receiving a packet without errors during the transmission of class-$j'$ users, denoted packet correct probability, can be written as

$$P_C(j') = [1 - \text{BER}(j')]^{H_j}.$$  

(6)

3.2. Throughput Evaluation Based on Poisson Approach

Under the assumption that several packets might be received incorrectly due to MAI, the complement of the packet correct probability, denoted packet error probability, with $m$ simultaneous packets transmissions from all users’ classes in the channel can be defined as $P_E(j') = 1 - P_C(j')$.

Subsequently, let’s consider $M$ a random variable representing the number of total simultaneous packets transmissions in a time slot. Also, consider that errors in different packets can occur independently of one another [33]. Thus, the conditional distribution of successfully received packets $S$ becomes [30]

$$P(S = s|M = m) = \binom{M}{s} P_C^s(j') P_E^{M-s}(j').$$  

(7)

The steady-state network throughput $\beta$, defined as the expected number of successful packets transmissions per time slot, can be expressed as

$$\beta(j') = E[S] = E\{E[S|M]\}$$

$$= E\left[ \sum_{s=0}^{M} s \binom{M}{s} P_C^s(j') P_E^{M-s}(j') \right].$$  

(8)

The summations in (8) can be rearranged as follows:

$$\sum_{s=0}^{M} s \binom{M}{s} P_C^s(j') P_E^{M-s}(j') = \sum_{s=1}^{M} s \binom{M}{s} P_C^s(j') P_E^{M-s}(j')$$

(9)

where $s \binom{M}{s}$ can be easily demonstrated to be equal to $M \binom{M-1}{s-1}$. Then, (9) can be cast as follows:

$$\sum_{s=1}^{M} s \binom{M}{s} P_C^s(j') P_E^{M-s}(j') = \sum_{s=1}^{M} M \binom{M-1}{s-1} P_C^s(j') P_E^{M-s}(j')$$

$$= MP_C(j') \sum_{s=1}^{M} \binom{M-1}{s-1} P_C^{s-1}(j') P_E^{M-s}(j')$$

$$= MP_C(j') \sum_{i=0}^{M-1} \binom{M-1}{i} P_C^i(j') P_E^{M-1-i}(j')$$

$$= MP_C(j') [P_C(j') + P_E(j')]^{M-1}$$

$$= MP_C(j').$$  

(10)

Finally, after substituting (10) into (8), the general network throughput in packets per slot of class-$j'$ can be rewritten as

$$\beta(j') = E[MP_C(j')]$$

$$= \sum_{m=1}^{\infty} m P_C(j') f_M(m)$$  

(11)

where $f_M(m)$ is the general steady-state probability distribution of composite arrivals (new and retransmitted packets) [35].
It is worth pointing out that (11) allows the evaluation of the network throughput for the three approaches considered here for composite arrival distributions \( f_M(m) \). In order to calculate \( f_M(m) \) is assumed a packet flow model with two modes of operation, namely, origination and backlogged mode [35]. Users in the origination mode transmit new packets in a time slot with probability \( p_o \). When these new packets’ transmissions fail, the affected users enter into backlogged mode and then try to retransmit the packets after a random delay with probability \( p_r \).

Initially, the composite arrival distribution of class-\( j' \) packets can be considered Poissonian when \( p_o = p_r \rightarrow 0 \) and \( U \rightarrow \infty \) [35]. Therefore, the steady-state Poisson probability distribution of class-\( j' \) composite arrivals, \( f_{POI}(m_j) \), is given by

\[
f_M(m) = f_{POI}(m_j) = \frac{G_j^m}{m_j!} e^{-\lambda_j}
\]

where \( m_j \in \{0, 1, \ldots, U_j\} \) is the number of simultaneous class-\( j' \) transmitting users, and \( G_j \) is the offered load of class-\( j' \) defined as the average number of generated packets from class-\( j' \).

Accordingly, the Poisson probability distribution of class-\( j \) composite arrivals \( f_{POI}(m_j) \) is given as

\[
f_M(m) = f_{POI}(m_j) = \frac{G_j^m}{m_j!} e^{-\lambda_j}
\]

where \( m_j \in \{0, 1, \ldots, U_j\} \) is the number of simultaneous class-\( j \) transmitting users, and \( G_j \) is the offered load of class-\( j \). Subsequently, employing (12) and (11), the throughput of class-\( j' \) can be obtained as

\[
\beta_{POI}(j') = \sum_{m_j=1}^{\infty} m_j P_C(j') f_{POI}(m_j).
\]

Furthermore, for a two-class multishade network [41], where for example the desired class-\( j' \) is the first class and the interfering class is the second one, the throughput will be

\[
\beta_{POI}(1) = \sum_{m_2=0}^{\infty} \sum_{m_1=1}^{\infty} m_1 P_C(j') f_{POI}(m_1) f_{POI}(m_2)
\]

\[
= \sum_{m_2=0}^{\infty} \sum_{m_1=1}^{\infty} m_1 P_C(j') \prod_{j=1}^{2} \frac{G_j^m}{m_j!} e^{-\lambda_j}.
\]

Further, for a three-class multishade network [44], where for example the desired class-\( j' \) is the first class and the interfering classes are the second and the third classes, the throughput will be

\[
\beta_{POI}(1) = \sum_{m_3=0}^{\infty} \sum_{m_2=0}^{\infty} \sum_{m_1=1}^{\infty} m_1 P_C(j') f_{POI}(m_1) f_{POI}(m_2) f_{POI}(m_3)
\]

\[
= \sum_{m_3=0}^{\infty} \sum_{m_2=0}^{\infty} \sum_{m_1=1}^{\infty} m_1 P_C(j') \prod_{j=1}^{3} \frac{G_j^m}{m_j!} e^{-\lambda_j}.
\]

Without any loss of generality of the Poisson properties \( U \rightarrow \infty \), as well as for convenience sake, the summation for \( J \)-class can be generalized as follows:

\[
\sum_{m_{j-1}=0}^{U_{j-1}} \sum_{m_{j-2}=0}^{U_{j-2}} \cdots \sum_{m_{1}=0}^{U_{1}} \sum_{m_{0}=0}^{U_{0}} \{ \ldots \} = \sum_{j=1}^{J} \sum_{m_{j-1}=0}^{U_{j-1}} \sum_{m_{j-2}=0}^{U_{j-2}} \cdots \sum_{m_{1}=0}^{U_{1}} \sum_{m_{0}=0}^{U_{0}} \{ \ldots \}.
\]
Therefore, one can finally obtain the total throughput of class-$j'$, $\beta_{\text{POI}}(j')$, for a general number of classes, i.e., $J$-class, as

$$
\beta_{\text{POI}}(j') = \sum_{j=1}^{J} \sum_{m=0}^{\infty} \sum_{m_j=1}^{\infty} m_j \cdot P_C(j') \prod_{j=1}^{J} f_{\text{POI}}(m_j).
$$

### 3.3. Throughput Evaluation Based on Binomial Approach

This subsection concerns with the derivation of the throughput equation based on binomial approach. The same assumptions adopted in the previous subsection up until (11) as well as the packet flow model with origination and backlogged modes are considered here. The composite arrival distribution of class-$j'$ packets is considered binomial when $p_o = p_r$ and the number of users in the network is finite [35]. Therefore, the steady-state binomial probability distribution of class-$j'$ composite arrivals $f_{\text{BIN}}(m_j)$ becomes

$$
f_M(m) = f_{\text{BIN}}(m_j) = \left( \frac{U_j}{m_j} \right) \left( \frac{G_j}{U_j} \right)^{m_j} \left( 1 - \frac{G_j}{U_j} \right)^{U_j - m_j} \quad \text{(19)}
$$

where $U_j$ is the number of users in the desired class-$j'$. Accordingly, the binomial probability distribution of class-$j$ composite arrivals $f_{\text{BIN}}(m_j)$ is

$$
f_M(m) = f_{\text{BIN}}(m_j) = \left( \frac{U_j}{m_j} \right) \left( \frac{G_j}{U_j} \right)^{m_j} \left( 1 - \frac{G_j}{U_j} \right)^{U_j - m_j} \quad \text{(20)}
$$

After substituting (19) into (11), the following expression for the throughput of class-$j'$ is obtained:

$$
\beta_{\text{BIN}}^p(j') = \sum_{m_j=1}^{U_j} m_j \cdot P_C(j') \cdot f_{\text{BIN}}(m_j) \quad \text{(21)}
$$

Thus, considering all classes $J$ of the multiservice multirate network and following the same steps as in the previous subsection, one can finally obtain the total throughput of class-$j'$ in packets per slot as

$$
\beta_{\text{BIN}}(j') = \sum_{j=1}^{J} \sum_{m=0}^{\infty} \sum_{m_j=1}^{\infty} m_j \cdot P_C(j') \prod_{j=1}^{J} f_{\text{BIN}}(m_j) \quad \text{(22)}
$$

### 3.4. Throughput Evaluation Under Markov Chain Approach

In this subsection, the throughput formalism is developed based on the discrete-time Markov chain [35]. The Markov chain approach can take any arbitrary value of $p_o$ and $p_r$ with a finite number of users in the network. A Markov chain $X_t$ has a finite state-space $\{0, 1, 2, \ldots, n, U_j\}$ with a transition matrix defined as $P = [P_{nm} : n, m \in U_j]$, where each state $n$ corresponds to the number of $n$ backlogged users. $P_{nm}$ denotes the probability that $m$ backlogged users will be present in the next state given that $n$ are present in the current state. This one-step transition probability from state $n$ to state $m$ can be written as

$$
P_{nm} = \Pr\{X_{t+1} = m|X_{t} = n\} \quad \text{(23)}
$$

Notice that when leaving state $n$ the chain must move to one of the $m$ states, with $m \in U_j$ knowing that each row of the transition square matrix sums to one. Next, a transition from state $n$ to
state \( m \) is determined by the difference between the number of unsuccessful new transmissions, denoted as \( U - NTX \), and successful retransmissions, denoted as \( S - RTX \). Then, the system state can change only due to unsuccessful new transmissions or successful retransmissions. In a given time slot, there are \( NTX = \xi_0 \) new transmissions and \( RTX = \xi_r \) retransmissions of packets with probabilities of \( U - NTX \) and \( S - RTX \) defined, respectively, as [35]

\[
\begin{align*}
\Pr\{ U - NTX = l | NTX = \xi_0, RTX = \xi_r \} &= b[l, \xi_0, P_E(\xi_0 + \xi_r)], \quad \xi_0 \geq l \\
\Pr\{ S - RTX = k | NTX = \xi_0, RTX = \xi_r \} &= b[k, \xi_r, P_C(\xi_0 + \xi_r)], \quad \xi_r \geq k
\end{align*}
\]  

(24)  

(25)

where \( b(\delta, \alpha, p) = \binom{\alpha}{\delta} p^\delta (1 - p)^{\alpha - \delta} \) is the probability mass function of the binomial distribution. The probability of \( \xi_0 \) attempted new transmissions and \( \xi_r \) attempted retransmissions (where \( \xi_0 \) and \( \xi_r \) are independent Bernoulli random variables) given that \( n \) backlogged users are present in the current state are, respectively, \( b(\xi_0, U_f - n, p_0) \) and \( b(\xi_r, n, p_r) \) [35]. Therefore, employing these definitions, (24), and (25), the joint probability distribution of \( U - NTX \) and \( S - RTX \) becomes

\[
\Pr\{ S - RTX = k, U - NTX = l | X_t = n \} = \sum_{\xi_0 = l}^{\xi_0 = k} b[l, \xi_0, P_E(\xi_0 + \xi_r)] \cdot b[k, \xi_r, P_C(\xi_0 + \xi_r)] \cdot b(\xi_0, U_f - n, p_0) \cdot b(\xi_r, n, p_r).
\]  

(26)

It is noteworthy that when \( m \geq n \) a transition from state \( n \) to \( m \) occurs if \( U - NTX \) exceeds \( S - RTX \) by \( (m - n) \). Conversely, when \( m \leq n \) a transition occurs if \( S - RTX \) exceeds \( U - NTX \) by \( (n - m) \). Thus, the one-step state transition probabilities of the Markov chain \( X_t \) can be expressed as [35]

\[
P_{nm} = \begin{cases} 
\sum_{\xi_0 = m}^{\xi_0 = m - v} \Pr\{ S - RTX = v, U - NTX = m - n + v | X_t = n \}, & m \geq n \\
\sum_{\xi_0 = 0}^{\xi_0 = m - n} \Pr\{ S - RTX = n - m + v, U - NTX = v | X_t = n \}, & m \leq n.
\end{cases}
\]

(27)

The finite state-space Markov chain \( X_t \) is irreducible and aperiodic with a transition matrix \( P \), which implies the existence of a long-term state occupancy probability distribution \( \mu \) [45]. This probability distribution vector \( \mu \) can be computed by solving the following linear equation system, with normalization constraint \( \sum_{n = 0}^{U_f} \mu(n) = 1 \)

\[
\mu^T = \mu^T P, \quad \text{where} \quad \mu^T = [\mu(0), \mu(1), \ldots, \mu(U_f)].
\]

(28)

After calculating the matrix of transition \( P \) given by (27), it is desired to solve (28). Generally, if the matrix is a \( U_f \times U_f \) square matrix, it has \( U_f \) eigenvalues and eigenvectors. A neat approach to calculate (28) is to firstly compute the eigenvalues and eigenvectors of the transpose of \( P \). Then, find the index of the eigenvalue element equal to its largest entry. Then, normalize the eigenvector matched to this same index by its own summation in order to sum to 1. Finally, after this procedure, the desired eigenvector is the transpose of the normalized eigenvector.

Next, the steady-state composite arrival distribution for \( n \) backlogged users in the current time slot \( X_t = n \) is given by [35]

\[
f_M(m|n) = \sum_{v = \max(m-n,0)}^{\min(m, U_f - n)} \Pr\{ NTX = v, TRX = m - v \} = \sum_{v = \max(m-n,0)}^{\min(m, U_f - n)} b(v, U_f - n, p_0) \cdot b(m - v, n, p_r).
\]

(29)
Therefore, the steady-state packet throughput based on Markov chain approach can be found employing (11), the normalization \( P_{U_0} = 1 \), the solution of (28), as well as (29), which gives

\[
B(j') = \sum_{m=1}^{U_{ij}} m P_C(j') f_M(m) = \sum_{m=1}^{U_{ij}} m P_C(j') \sum_{n=0}^{U_{ij}} \mu(n) f_M(m)
\]

\[
= \sum_{m=1}^{U_{ij}} m P_C(j') \left\{ \sum_{n=0}^{U_{ij}} \mu(n) f_M(m|n) \right\}.
\]

Finally, the offered load of class-\( j' \) in packets per slot assuming that \( p_o = p_r = p \) is given as \( G_{j'} = U_{ij} p \). The delay and expected drift evaluation of multiservice multirate networks is beyond the scope of this work and will be published elsewhere.

4. Numerical Results

The mathematical formalisms developed in the previous section are now applied to investigate the throughput performance of multiservice, multirate networks. The three approaches are employed under different network packet traffic conditions. Also, the throughput performance of a single rate one-class OCDMA employing only the first class’ parameters (class-1) of a multirate two-class 1-D MWML-OOC OCDMA network is evaluated in order to prove convergence between the proposed formalisms and the well-known single rate throughput formalisms [30], [32].

The analyses carried out in this section adopt well-known network scenarios for convenience sake. The first network scenario is based on [30], while the second and third scenarios are based on [6].

Consider first a two-class S-ALOHA OCDMA network employing 1-D MWML-OOC [1] with code length and weight, number of users, and packet length parameters, respective to each class, as given by \( L_1 = 600 \), \( W_1 = 4 \), \( U_1 = 36 \), and \( H_1 = 1024 \) bits and \( L_2 = 1200 \), \( W_2 = 5 \), \( U_2 = 36 \), and \( H_2 = 512 \) bits. Note that \( H_1 \) is calculated with (3). Then, class-1 has low-QoS and high-transmission rate, and class-2 has high-QoS and low-transmission rate. Also, the cardinality of each user class of the 1-D MWML-OOC is given in [1, Eq. (6)]. The 1-D MWML-OOC employs a single wavelength, then \( F = 1 \). The packet throughput of the desired class is plotted as a function of the offered load of the desired class (for both classes) in Fig. 3(a) for Poisson (diamonds), binomial (squares) and Markov (circles) approaches. All curves were directly obtained respectively from (18), (22), and (30). It is worth pointing out that the Markov approach is considered here as the benchmark.

As can be seen from this figure, both the binomial and Markov-based results completely overlap whereas the Poisson one shows convergence only for class-1 and under high offered load conditions. Nonetheless, it is highly desirable to have the throughput as linear as possible since this implies lower probability of receiving packets with errors. In this sense, class-2 not only has better throughput performance but higher throughput peak than class-1 as well. Class-1, by its turn, has a poor throughput performance since this class has lower code weight making this users’ class more susceptible to the effects of MAI. On the other hand, the lower codes weight of class-1 causes the Poisson results to be more accurate than those for class-2 [see dotted vertical lines in Fig. 3(a)].

It can still be seen from Fig. 3(a) that the Poisson approach produces unacceptable results for most offered load values and underestimates the throughput peak for class-1 and class-2 users in around 59% and 73%, respectively. Furthermore, it underestimates the general network performance in unacceptable levels and incorrectly predicts the throughput of class-2 for any given offered load. Therefore, the Poisson approach can only be considered sufficiently accurate for the throughput performance evaluation of class-1 as long as more than 30 simultaneous users (offered load) are present in the network, and it does not produce accurate results for any offered load in class-2. On the other hand, the binomial approach shows complete agreement with the Markov chain approach for both classes. In fact, this
can be regarded as a validation of the derived binomial expression (22) since the Markov approach is considered here a benchmark against which the throughput performance can be compared.

Now, the packet throughput performance of a single rate, single service OCDMA network is investigated for validation purposes. The idea is to guarantee the convergence of the proposed multiservice/multirate formalisms towards the well-known single service/rate formalisms. The same set of parameters adopted in the previous case (and also in [30]) is now used exclusively for class-1, namely, $L = 600$, $W = 4$, $U = 36$, and $H = 1024$ bits, where $L = L_1$, $W = W_1$, $U = U_1$, and $H = H_1$.

Then, the throughput of the single rate single service network versus the offered load for both the Poisson [30] and binomial [32] composite arrival distributions is plotted in Fig. 3(b). Also plotted in this figure are the throughput performances using the proposed Poisson (18), binomial (22), and Markov (30) approaches and considering no users in class-2. Observe that the throughput formalisms proposed here are in good agreement with the expected values for single rate networks. Moreover, notice that the binomial-based throughput is more accurate than the Poisson-based one when the offered load is high.

Next, a two-class multirate OCDMA network employing 1-D MWML-OOC $(F = 1)$ is investigated for the three proposed approaches, i.e., Poisson, binomial, and Markov chain. The following parameters are assumed: $L_1 = 400$, $W_1 = 5$, $U_1 = 21$, and $H_1 = 3072$ bits for class-1 and $L_2 = 1200$, $W_2 = 6$, $U_2 = 26$, and $H_2 = 1024$ bits for class-2. The throughput of class-1 versus the offered load of class-1 is plotted in Fig. 4(a), where is evident the overlap of both binomial and Markov chain approaches. As a result, the former can be employed as a reliable approach to accurately assess the throughput performance of multiservice OCDMA networks.

The Poisson approach, by its turn, underestimates the throughput performance for most offered load range, peaking at approximately $G_1 = 10$ where it produces a throughput around 64% lower than that predicted with the binomial/Markov approach [see dotted vertical line in Fig. 4(a)]. This means that under the Poisson approach the network would receive more packets with errors than it actually should. In fact, the Poisson curve only tends to converge towards the binomial/Markov one after $G_1 \geq 18$.

Subsequently, the throughput of class-2 versus the offered load of class-2 is plotted in Fig. 4(b), again for the three proposed approaches. Differently from the Poisson approach, both the binomial and Markov curves are in good agreement. The Poisson approach significantly underestimates the packet throughput performance. For example, at $G_2 = 8$, the throughput is underestimated in around 75% [see dotted vertical line in Fig. 4(b)], and this issue becomes even more significant for a higher offered load. At $G_2 = 25$, the binomial approach
predicts a throughput around 15 packets/slot, while the Poisson-based one predicts only around 7 packets/slot. This represents an underestimation around 120% in the number of successfully received packets [see dotted vertical line in Fig. 4(b)]. Therefore, the Poisson distribution cannot be considered a reliable approach to evaluate the throughput performance of class-1 and class-2 for this network scenario.

Finally, the performance of a multiservice, multirate OCDMA network employing 2-D MWML-OOC is now assessed. This code scheme requires a simple code construction algorithm and is capable of supporting a large number of users [2]. The two-class S-ALOHA 2-D MWML-OOC OCDMA packet network is defined as follows: 

\[ L_1 = 100, W_1 = 4, U_1 = 70, H_1 = 2048 \text{ bits for class-1}, \quad L_2 = 200, W_2 = 5, U_2 = 50, \quad \text{and} \quad H_2 = 1024 \text{ bits for class-2}. \]

Then, class-1 has low-QoS and high-transmission rate, and class-2 has high-QoS and low-transmission rate. Also, the cardinality of each user class of the 2-D MWML-OOC is given in [2, Eq. (15)]. Further, all users’ code share the same number of available wavelengths utilized by the 2-D MWML-OOC, \( F = 11 \) in (5). The throughput of class-1 for the three proposed approaches is plotted in Fig. 5(a) versus the offered load of class-1.

Once more, the results obtained via binomial and Markov approaches are in good agreement. However, the binomial approach proved to be numerically more straightforward, more
convenient, and computationally faster than the Markov chain approach. Note that the throughput expression (22) only depends on the packet correct probability and on the (binomial) probability distribution itself. On the other hand, the Markov throughput expression (30) relies on a state transition matrix for each user and, consequently, on the conditional statements, summations and binomial coefficients of (24)–(27). These equations require more computational effort to evaluate the packet throughput. In addition, the larger the number of users, the larger the square transition matrix and, consequently, the larger the computational effort required to evaluate the throughput. For example, a $70 \times 70$ and a $50 \times 50$ square transition matrix (plus their eigenvalues and eigenvectors) are required to obtain the Markov chain approach results of Fig. 5(a) and (b), respectively. Therefore, the Markov numerical simulation demands a heavy computational effort. The Poisson approach, by its turn, considerably underestimates the packet throughput as shown in Fig. 5(a). For example, the predicted throughput of class-1 is around 63% less than that predicted with binomial/Markov chain approach at $G_1 = 20$ packets/slot. A fairly good agreement among the three approaches only occurs at $G_1 \geq 50$. Therefore, the Poisson approach can only be considered sufficiently accurate under this network scenario as long as above 50 simultaneous users in class-1 are present in the network [see dotted vertical line in Fig. 5(a)]. On the other hand, this approach fails to predict correctly the throughput performance of class-2 for any number of offered load (simultaneous users’ transmissions) as shown in Fig. 5(b).

The throughput of class-2 versus the offered load of class-2 is plotted in Fig. 5(b). Notice once more that both binomial and Markov chain approaches have good agreement whereas the Poisson approach poorly predicts the packet network behavior. In such case the Poisson approach diverges even under high offered load conditions, where the throughput underestimation can be as high as 141% at $G_2 = 50$. Therefore, the Poisson based-approach cannot be recommended for throughput estimation of multiservice, multirate OCDMA networks. Furthermore, the throughput performance depends on the BER, and consequently, depends on the code's parameters as well as the code family employed, and the number of users present in the network. In this sense, the larger the code weight of a user class, the lower the probability of interference caused on the desired user, and then the better the BER and throughput performance. The number of simultaneous users, by its turn, does influence the BER and throughput performance since OCDMA networks are considered as statistical multiplexing systems [46].

Though a three-class network performance has not been addressed in this paper, one should expect the same performance behavior observed for the two-class networks investigated here. In this sense, the class defined with the highest code weight will have underestimated results in terms of throughput performance for the Poisson approach, while the results for the class defined with the lowest code weight will converge to the binomial/Markov ones, as long as a large number of simultaneous users are present in the network.

### 5. Conclusion

In this paper, new mathematical formalisms for evaluating the packet throughput of multiservice, multirate S-ALOHA OCDMA networks have been proposed. These new formalisms can be successfully applied to 1-D and 2-D OCDMA networks distinctively with any number of user’s classes in the system. In addition, it can be further applied to other family of codes or modulation formats (such as phase-shift keying) as long as the BER is provided. Moreover, the throughput expressions can be further used to assess the performance of multiservice multirate networks either where the MAI is the only degradation source or with both MAI and several other deleterious sources (such as time jitter and channel impairments), as long as the BER is provided. A validation criterion was defined to assess the new mathematical formalisms. Numerical results have shown that the proposed formalisms converge towards the well-known single service and single rate formalisms. The BER and packet correct probability expressions have been derived, addressing the main deleterious source in incoherent OCDMA, namely MAI. Other figures of
merit, such as packet delay, efficiency, and expected drift of multiservice, and multirate networks, will be addressed in a future publication.

Furthermore, a throughput performance investigation of multiservice multirate 1-D and 2-D MWML-OOC OCDMA packet networks employing Poisson, binomial, and Markov chain approaches was also carried out. The Poisson-based approach proved to be a poor approximation for incorrectly predicting the number of successfully received packets for most offered load values, especially for user classes with high code weight. Even under favorable conditions, such as for the 2-D MWML-OOC with a considerably large number of simultaneous users, the Poisson approach does not provide accurate results. The binomial approach, by its turn, proved to be more convenient and computationally faster than the Markov chain approach since its final mathematical expression only depends on the packet correct probability and the (binomial) probability distribution itself, whereas the Markov approach relies on a state transition matrix. Then, the binomial approach not only is simpler to be implemented but just as accurate as the Markov approach as well. Therefore, the binomial approach has proven to be a more appropriate choice for throughput performance evaluation of multiservice, multirate S-ALOHA OCDMA networks.

Acknowledgment
The first author would like to thank R. A. Castrequini for his valuable comments and suggestions.

References
IEEE Photonics Journal Evaluation of Multiservice Multirate OCDMA


