Market Design and Strategy Making for Proactive Distribution Grid with DERs

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Distributed energy resources (DERs) tend to occupy a high share in the distribution-level network. In a deregulated environment, this stimulates the distribution company (DISCO) to preferentially procure DERs’ generations at comparable prices.

In the U.S., the recent initiative named the New York Reforming Energy Vision (NY REV) has addressed the regulatory changes to liberate a distribution-level market for cost-effective use of DERs. To this end, the DISCO gets an opportunity to strategically engage in the transmission-level markets by rationally purchasing electricity from the distribution-level DERs. In this situation, the market framework becomes more complex.

To handle these small-size and dispersed DERs, this thesis proposes a methodology to optimize the procurements of the proactive distribution company (PDISCO) trading in the presented distribution-level market. Particularly, taking the demand response (DR) resource to represent a type of DERs, the PDISCO’s procurement strategies can cover real-time market transactions and aggregator-based DR.

On the other hand, to maximize profit, the PDISCO is also eager to participate in the transmission-level markets. To achieve this goal, the PDISCO has to make decisions on procuring DERs’ portfolios in distribution-level market, and strategically submits offers/bids to the transmission-level markets, simultaneously. Crossing the day-ahead and real-time markets, the transactions between PDISCO and markets are characterized in a bidirectional fashion.
In order to capture the PDISCO’s trading strategies mentioned above, the PDISCO trading within markets can be formulated as one-leader multi-follower game models, realizing in differing bi-level structures.

Pertaining to the solving algorithm, the primal-dual approach is applied to reformulate each proposed bi-level model to a solvable single-level mathematical program with equilibrium constraints (MPEC).

The effectiveness of the proposed models are verified by individual numerical analyses.
Distribuerede energikilder (DERs) bidrager væsentligt til energiproduktionen i distributionsnetværket. I et dereguleret system stimulerer dette distributionsfirmaer (DISCO) til preferentiel at købe strøm fra DERs, til priser sammenlignelige med centraliserede kilder.

I USA har New York Reforming Energy Vision (NY REV), et nyligt startet initiativ, imødegået de nødvendige regulatoriske ændringer for at danne et liberaliseret distributionsnetværk marked, der udnytter DERs på en kost-effektiv måde. For at opnå dette, får DISCO mulighed for strategisk at deltage i marked på transmissionsniveau ved at købe de underliggende DERs production på distributionsnetværket-niveau. Dette gør markedssammen mere kompleks.

For at håndtere et stort antal små og vidt udbredte DERs, foreslår denne afhandling en metodiologi for at optimere indkøbet hos et proaktivt distributionsfirma (PDISCO), der håndterer de i afhandlingen undersøgte distributionsnetværkemarked. Specifikt vises det, at behandles responsiv efterspørgsel (DR) som en DER, kan PDISCO’ens strategier håndtere både de nødvendige markeshandlinger i realtid og aggregeringsrollen for DR.

På den anden side er PDISCO’en ivrig efter at deltage i标记er på distributions netværket-niveau, idet PDISCO’ens profit dermed kan øges. For at opnå dette skal PDISCO’en simultant vælge, hvilke underliggende DER-portfolio der skal indkøbes, og hvilke udbud- eller efterspørgselsbud der skal afgives i marker på transmissionsniveau-niveau. Eftersom disse transaktion sammenfletter markerer day-ahead og i real-time, karakteriseres de ved en interaktion mellem disse to markerer.
PDISCO’ens strategier kan i denne sammenhæng repræsenteres som et enkelt- leder, multi-følger spil, hvilket giver anledning til bi-niveau optimiseringsmodell er.

En primal-dual tilgang (primal-dual approach) anvendes på hver foreslåede bi- niveau model, hvilket reformulerer hver model til et enkelt-niveau program med ligevægtsconstraints (MPEC).

Effektiviteten af de foreslåede modeller verificeres ved numerisk analyse.
This thesis was prepared at the Center for Electric Power and Energy (CEE) at the Technical University of Denmark (DTU) in fulfilment of the requirements for acquiring a Ph.D. degree.

The thesis deals with the distribution-level market design and strategy making for the proactive distribution grid with DERs. The content mainly focuses on the procurement strategies in distribution-level market and the trading strategies in the transmission-level markets.

The thesis consists of the modeling of the proposed distribution-level market and solving algorithms, plus the presented trading structure and clearing model for a strategic proactive distribution company (PDISCO) in the transmission-level markets.

Lyngby, 14-November-2015

Chunyu Zhang
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Patent Related with the Ph.D. Study

The following patents have been prepared during the course of Ph.D. study. They are not included in this thesis since they have been taken over by DTU, processing in the application and commercialization.


Publications

Publications Related with the Thesis


Other Publications not Related

The following publications are omitted in this thesis because they are contributed to the patents or other topics during the course of the Ph.D. study.


Contents

Summary (English) i
Summary (Danish) iii
Preface v
Acknowledgements vii
Patents ix
Publications xi

1 Introduction 1
1.1 Objective and Approach 2
1.2 Contributions 4
1.3 Thesis Outline 5

2 Proactive Distribution Grid 7
2.1 Market Liberalization 8
2.2 Proactive DISCO (PDISCO) 10
2.3 Rational Aggregator (RA) 10
2.4 Modeling and Solving Approach 11
2.5 Conclusion 16

3 PDISCO Procurement Strategies with DERs 17
3.1 Market Design 20
3.2 Problem Formulation 22
3.2.1 Assumptions 22
3.2.2 Bi-level Model 24
xvi CONTENTS

3.2.3 MPEC ....................................................... 28
3.3 33-bus Distribution Network ...................................... 29
  3.3.1 Data ...................................................... 29
  3.3.2 PDISCO Procurement Strategies .............................. 32
  3.3.3 RA Real-time Trading ....................................... 32
  3.3.4 Impact of Elasticity Limit .................................. 36
  3.3.5 Impact of Inelasticity Control Factor ........................ 36
  3.3.6 Impact of Consumption Control Factor ....................... 36
  3.3.7 Impact of Profit Guarantee Factor ........................... 37
  3.3.8 Impact of RA Procurement Price ............................ 37
  3.3.9 Impact of RA Aggregation ................................. 39
  3.3.10 DR Comparison ($\alpha, \beta$) vs. ($\alpha$) ....................... 40
  3.3.11 Network Congestion .................................... 41
3.4 69-bus Distribution Network ...................................... 44
3.5 Conclusion ...................................................... 48

4 PDISCO Trading Strategies in Wholesale Markets 51
  4.1 Market Design ............................................... 54
  4.2 Problem Formulation .......................................... 55
    4.2.1 Assumptions .............................................. 56
    4.2.2 Markets ................................................ 58
    4.2.3 PDISCO .................................................. 60
    4.2.4 MPEC ................................................... 64
  4.3 14-bus Distribution-level Network with 9-bus Transmission-level Network ....................................................... 66
    4.3.1 Data ...................................................... 66
    4.3.2 PDISCO Trading Strategies ................................ 68
    4.3.3 Generators .............................................. 73
    4.3.4 Cases ................................................... 73
  4.4 Conclusion ...................................................... 76

5 Conclusions and Perspectives ...................................... 77
  5.1 Conclusion Overview .......................................... 77
  5.2 Research Perspectives ........................................ 78

A 33-bus Distribution Network: Base Parameters 81

B 69-bus Distribution Network: Base Parameters 83

C 14-bus Distribution Network: Base Parameters 87

D 9-bus Transmission Network: Base Parameters 89

Bibliography ........................................................... 91
Chapter 1

Introduction

In the modern power systems, high penetration and deployment of distributed energy resources (DERs), such as demand response (DR), distributed generation (DG), etc., motivate the distribution-level systems to be more active than the traditionally passive networks. Distribution companies (DISCOs), as load serving entities (LSEs), have started to get more engaged in electricity market transactions. In this situation, a DISCO has to deal with both market participants in the transmission-level markets and the distribution-level resources. For instance, in the U.S., recent initiatives led by the New York Public Service Commission have addressed regulatory changes to promote utilization of distributed resources mentioned above, i.e., the New York Reforming Energy Vision (NY REV) [1]. One of the main purposes of NY REV is to establish a distribution-level market platform where all distribution resources can transact and trade with each other, which also highly motivates this work.

From the long-term perspective, DERs are seen as more efficient energy resources, which can critically improve the security of energy supply by drawing upon sustainable natural sources and reducing environmental impacts [2]. In Denmark and other European Union countries, various distribution-level flexibility services are proposed and demonstrated the technical feasibility of turning DER stochastic productions into valuable supplements for power systems' management and reinforcement [3, 4].
For the short-term operation, as reviewed in [5], DERs are normally small-scale, varied and dispersed, which are difficult to handle system-wide. To this end, nearly all the applications [6, 7, 8] and discussions [9, 10] are concentrated on aggregated DR or DG participating in the existing markets at the transmission level, e.g., trading as a virtual plant in the day-ahead market or real-time market. However, derived from the demand side, DER is a natural candidate to directly trade with a distribution company (DISCO) in the local area. Such an advantage may stimulate DR providers or DG owners to play essential roles in the emerging trading structures, which in turn improve the competitiveness of the markets and facilitates the DISCO’s electricity procurements among differing resources. In this situation, the DISCO’s market structure becomes more complex.

1.1 Objective and Approach

Emphasizing on the short-term transactions, this thesis aims to present optimization tools to efficiently trade DERs in the proposed frameworks between liberalized distribution-level market and existing transmission-level wholesale markets. At distribution level, as the resource providers, DERs are restricted as DR and stochastic DGs in this thesis. The proactive DISCO (PDISCO) is identified as a new market player, who is assumed to own and operate the local distribution network, and interconnect the transmission-level network with a main substation.

Interfacing the distribution-level and transmission-level markets, to maximize the profit, the PDISCO has the strategy making issue oriented from the procurement with various DERs and trading in the wholesale markets. As a business entity, the procurements of the PDISCO cover the transactions by power exchanging and purchases through DER productions. To engage in the transmission-level markets, the PDISCO performs strategically in the day-ahead and real-time markets by the surplus of DER purchases after satisfying the distribution network constrains. Thus, a hierarchical market structure is achieved, i.e. the transmission-level markets regard the PDISCO as a bidirectional transactor, while the PDISCO considers transmission-level markets and distribution-level DERs as power providers.

Assuming the transmission-level market information is known, to illustrate the PDISCO procurement in the distribution-level market, the DR resource can be taken as an instance to show the trading strategies and market performance. A real-time trading setup is presented along with a newly defined rational aggregator (RA) in this thesis. Compared with the small-scale DRs, a RA represents a group of smaller DR resources to bid to the PDISCO. In other words, in
1.1 Objective and Approach

In this competitive environment, each RA tries to fulfill the PDISCO’s requirement (procurement volumes and offering prices) by rationally putting forward its kW quantities and bidding prices to maximize its profit. For a PDISCO, based on its purchase in the day-ahead market, it has to make optimal decisions on the procurements in the real-time market to adjust its position and maximize its own profit through real-time exchanging, RA virtual generation, possible load-shedding, in addition to electricity sales revenue. In particular, this thesis assumes the PDISCO has the ability to purchase or sell active power according to the real-time market price and conduct bidirectional power exchanging.

To model load shaving and load shifting, this thesis assumes all of the elastic demand is shiftable among the hours, and further divides the elastic demand into shavable and unshavable components, which can capture the DR resource characteristics in a flexible and comprehensive fashion. In order to achieve an appropriate trade-off among real-time market transactions, DISCO-RA trading, and others, a multi-period AC power flow formulation is used to accurately represent the underlying physics of the power networks.

Taking into account the listed considerations above, the optimal procurement problem of a PDISCO with RAs can be formulated with a bi-level structure. At each time \( t \), the upper-level problem indicates the PDISCO’s optimal procurements for maximizing profit, the lower-level problems describe multiple RAs’ decisions for rational bidding, one per RA.

In practice, the well-developed smart grid technology turns the bidirectional power exchanging between the distribution and transmission networks into a reality. Thus, the PDISCO also gets an opportunity to strategically engage in the transmission-level markets by purchasing the electricity from the distribution-level DERs.

To participate in the day-ahead and real-time markets, for each time \( t \), the PDISCO has to make a trade-off on acquiring DERs’ portfolio and trading strategy (offer/bid) to maximize its profit. Crossing the two-stage markets, the transactions between PDISCO and markets are characterized in a bidirectional fashion, implying the PDISCO behaves as an active producer when providing offers, but as an active consumer when submitting bids. On the contrary, the PDISCO’s trading strategies (offering/bidding prices and power quantities) are endogenously interrelated with the markets’ outcomes (Locational marginal prices (LMPs) and production/consumption quantities). Thus, the trading between the PDISCO and markets follows a typical gaming structure.

In order to capture the PDISCO’s trading strategies, the PDISCO trading within markets can be formulated as a one-leader multi-follower game model, realizing in a bi-level structure. Market-clearing procedures are indicated as the stage-
based lower-level problems through DC power flow. The lower-level day-ahead market problem is to maximize the transmission-level social welfare. Particularly, in the real-time process, scenario-based method [11] can be used to embody the stochastic outputs of individual DERs. On the basis of this, a lower-level real-time market problem seeks to minimize the transmission-level operation cost per scenario \( \omega \). The upper-level problem represents the profit maximization of the PDISCO, with the strategic offers/bids constrained by AC power flow.

Note that the individual bi-level models presented above are linear and convex regarding the lower-level problems, while the upper-level problem is non-linear and non-convex due to the involvement of AC power flow. As addressed in [12], this kind of bi-level problem can be transformed into a solvable single-level problem, in which the lower-level problems can be replaced by their first-order optimality conditions. Particularly, in view of the linearity of lower-level problems, the first-order conditions are formulated by a primal-dual approach, containing primal and dual constraints, and the strong duality requirements. This approach is equivalent to the broadly utilized Karush-Kuhn-Tucker (KKT) conditions, but of high computational efficiency and tractability [12]. Then this reformulated problem renders a mathematical program with equilibrium constraint (MPEC).

1.2 Contributions

In view of the context above, focusing on short-term transactions, the contributions of this thesis are fivefold:

1) Present a distribution-level market framework for the PDISCO to strategically procure DERs and transact with the transmission-level markets. The PDISCO and RA are defined as the new market players at the distribution level.

2) Propose a one-leader multi-follower bi-level model for simulating the PDISCO’s optimal procurements between power exchanging and RAs’ bids, gaming with each RA’s bidding in a competitive environment.

3) Define a DR formulation for actuating the real-time load shaving and load shifting, simultaneously.

4) Present a one-leader multi-follower bi-level model for a PDISCO to make continuous offering and bidding strategies to trade within the transmission-level
1.3 Thesis Outline

day-ahead and real-time markets.

5) Reformulate the proposed individual game-theoretic models to MPECs by replacing the lower-level problems with the primal-dual approach, respectively.

In addition, the flexibility market designs on the basis of varied long-term bilateral contracts (between individual DERs and the DISCO) are also contributed in the Ph.D. study, presenting in [2, 3, 4]. These are not included in chapters since the topics are put forward for the passive distribution grid without strategy making concerns, while this thesis makes efforts to address the strategic behaviors of the PDISCO trading in the short-term distribution-level and transmission-level markets.

1.3 Thesis Outline

The rest of this thesis is organized as follows:

Chapter 2 reviews the market liberalization in electricity sector, and presents a basic market architecture with newly defined participants (i.e. PDISCO and RA) to interrelate with the transmission-level markets.

Chapter 3 presents a methodology for the PDISCO to make procurement strategies between the transmission-level market transactions and distribution-level RA bids. In particular, taking real-time DR to represent DERs to participate in a proposed real-time trading framework. A one-leader multi-follower bi-level model is presented to formulate the PDISCO’s strategy making problem. Case studies are carried out on a 33-bus and a 69-bus distribution network to show the effectiveness and scalability of the presented approach, respectively.

Chapter 4 presents a methodology to identify the trading strategies of a PDISCO participated in the transmission-level day-ahead and real-time markets. Stochastic DERs are considered as the distribution-level resources pertaining to the PDISCO’s procurement. A one-leader multi-follower bi-level model is used to formulate the game between the PDISCO and transmission-level markets. In particular, the PDISCO problem is constrained with AC power flow, while the markets’ problems are indicated through DC power flow. The model’s effectiveness is verified by a 14-bus distribution-level network interconnected with a 9-bus transmission-level network.

Chapter 5 summarizes the relevant conclusions and perspectives of this thesis.
Introduction
Chapter 2

Proactive Distribution Grid

The presented distribution-level market architecture and newly defined market players turn the traditional passive distribution grid into a proactive fashion. The liberalization of a distribution-level market is an essential part in the whole chain of trading electricity, transacting with the wholesale market and acquiring differing DERs. Meanwhile, the distribution grid is still responsible for the power delivery from the transmission level to the consumers. Thus, the physical network constraints also perform and impact the behaviors of the transmission-level and distribution-level markets.

This chapter briefly reviews the liberalization of the electricity markets, and further illustrates the function of the distribution-level market by presenting a basic framework to interact with the transmission-level markets. Considering the distribution grid comprises commercial and physical characteristics, the PDISCO is set as a new market participant, which is assumed to own and operate the distribution grid. As a broker, each RA is defined to upwards bid profitably to the PDISCO and downwards schedule the contractual DERs, which regard DR and DG as the major components. As the interface actor, the PDISCO’s procurement in the distribution-level market and trading in the transmission-level markets are both exhibited as hierarchical structures, which can be formulated with bi-level game-theoretic models, respectively. The corresponding modeling and solving approaches are summarized in the end.
2.1 Market Liberalization

Till now, at the transmission level, electricity markets are normally recognized as wholesale markets and realized as pools. For short-term transactions, the electricity can be traded in the day-ahead and real-time markets. For the day-ahead market, a two-sided auction framework is commonly applied to find a trade-off between the offers and bids submitted by producers and consumers, respectively. Each offer/bid is specified as a set of price-quantity pairs. The operator seeks to clear the market with a uniform pricing mechanism for the following day, one per hour. This determines the day-ahead schedule for each producer. Throughout the day, the real-time market (with hour or minutes base) aims to ensure the power balance between the production and consumption through an auction. The upward/downward regulation power is traded at this stage to handle the positive/negative imbalance from the actual consumption and the day-ahead scheduled production. The pay-as-bid (PAB) pricing or uniform pricing can be used to associate with the market-clearing process [5]. On the other hand, if the transmission network is considered in a corresponding market-clearing procedure, instead of a uniform price, a LMP occurs at each bus, covering line congestion and line losses [5,12].

For the purposes of environmental protections and sustainable developments, DERs (e.g., small-size wind turbines and photovoltaic systems) increasingly penetrate the power systems, especially at the distribution level. To cost-effective utilize DERs, a distribution-level market is considered as an essential role to mobilize and trade these distributed resources with the local DISCO [2].

Few papers are available to discuss the DISCO trading in the transmission-level/distribution-level markets. A day-ahead distributed company acquisition market (DCAM) is proposed in [13], in form of the pool market and bilateral contracts. The DISCO purchases electricity according to the offers from DG units, customers, the wholesale market, and contracted load-shedding options. The load, DG units and DCAM objectives are all stipulated in quadratic functions, while the model is used for a static simulation. Further developed in [14], the DGs and interruptible loads are seen as DRs, and a DISCO-ISO bi-level model is presented. The upper-level problem represents individual DISCO’s profit maximization with its own DGs, and the lower-level problem indicates ISO’s day-ahead market clearing model for minimizing generation costs and load-shedding compensation. However, the DG output and load-shedding price are fixed and the network constraints are not included. To evaluate the optimal contract pricing between DISCO and DG owners, a bi-level model is also considered in [15], the upper-level objective is to maximize the DG owners’ profit (without any physical constraints), the lower-level depicts the DISCO network constraints. To avoid the non-convexity of the constraints, the paper only con-
In the U.S., the recent initiative called NY REV [1] has addressed the regulatory changes to further liberalize a distribution-level market. This stimulates the DISCO to preferentially procure DERs’ generations at comparable low prices. To this end, the DISCO also gets an opportunity to strategically trade in the transmission-level markets. In this situation, the market structure becomes more complex.

In view of the context mentioned above, a basic distribution-level market framework is proposed in this chapter to incorporate in the transmission-level markets. As shown in Fig. 2.1, the distribution-level and transmission-level markets are bidirectionally transacted with each other and physically linked by the DISCO, which is assumed to own and operate the distribution grid towards a proactive manner. At the distribution level, new market participants are characterized as the PDISCO and RA, which are fully indicated in the following sections 2.2-2.3, respectively.

![Figure 2.1: Transmission-level and distribution-level markets.](image-url)
2.2 Proactive DISCO (PDISCO)

From the management perspective, a concept of active distribution network (ADN) has been widely discussed in the technical literature. ADN is defined as a distribution system, in which DGs are actively controlled by suitable energy management system (EMS) to achieve specific operation objectives \[16, 17\]. To exploit DG benefits, centralized or distributed control schemes have been proposed, together with additional communication, monitoring and control infrastructure, so as to manage DG output and other potentially controllable network elements, e.g., remotely controlled switches, on-load tap changing transformers, etc. \[18\].

For clarity, in order to identify the strategic trading behaviors of a DISCO, the proactive DISCO (PDISCO) concept presented in this thesis is derived from the market perspective. To take part in the markets as described in Fig. 2.1, the strategy marking issues of a PDISCO can be mainly focused on the following aspects:

1) Procurement with various DERs: As a profit-driven company, the PDISCO’s procurements cover transmission-level market transactions and DER purchases. In particular, besides supplying the local demands, the DISCO can even execute ambitious schemes to procure excessive DERs to sell to the transmission-level markets, performing as an active electricity producer at the transmission level. The PDISCO’s procurement strategy is comprehensively analyzed in Chapter 3.

2) Trading in the wholesale markets: To maximize profit, the PDISCO procures estimated capacities from individual RAs (i.e. DER productions) at the day-ahead stage, while balances the deviations of stochastic DER productions through real-time power exchanging. Across the two-stage transmission-level markets, the PDISCO can strategically offer/bid as an active producer/consumer according to the transmission-level LMPs. The PDISCO’s offers and bids result a continuous strategic performance during the whole timespan (e.g., 24 hours), respectively for the day-ahead market and real-time market. The PDISCO’s trading strategy is investigated in detail in Chapter 4.

2.3 Rational Aggregator (RA)

Over the past decade, the concept of aggregator (or virtual power plant) is proposed to enable DERs to participate in the existing transmission-level markets,
especially for providing different kinds of ancillary services \[19, 20\]. However, 
originated from the demand side, small-scale DERs have the superiority to be 
aggregated and directly traded with a PDISCO in the local area. In contrast, 
with the contractual DERs, each distribution-level aggregator competes against 
the others and bids rationally, serving as a candidate resource.

As shown in Fig. 2.1 at the distribution level, the new defined RA is a virtual 
business entity, who has no physical integration with the system network, but 
has the commercial and technical abilities to behave rationally as follows:

1) For the sake of harvesting DER generations, for each time \( t \) at both day-
ahead and real-time stages, RA acquires individual DERs with contracts, and 
makes an optimal decision on pricing.

2) During the trading process, as a competitive market player, a RA satisfies 
the PDISCO’s request, self-evaluates the availabilities of the contractual DERs, 
sets up the bidding price and kW quantity, and bids to the PDISCO effectively.

3) After obtaining offers (procurement volumes and offering prices) from the 
PDISCO, RA mobilizes the corresponding DER portfolios to meet the require-
ments.

2.4 Modeling and Solving Approach

Recently, the bi-level game structure and complementarity theory have been 
increasingly adopted in electricity market modeling and analysis, typically re-
flecting the market outcomes with multiple strategic players competing in the 
decision-making process.

In the literature, to study the competitive behavior among individual generating 
companies, an incomplete information bi-level model is proposed in \[21\]. For 
strategically controlled microgrids (MGs) in a distribution network, a bi-level 
model for coordinated operation of the distribution network operator and MGs 
is presented in \[11\]. The equilibria obtained in an oligopolistic electricity pool 
with network constraints is presented in \[22\]. DC multi-period power flow is im-
plemented to simplify the transmission network constraints. In the same market 
setting, to investigate the wind power as a strategic producer, a stochastic bi-
level model is proposed in \[23\]. For the strategic generation investment problem, 
each strategic producer is represented using a bi-level model \[24\] \[23\], and the 
investment strategies are comprehensively investigated in various market sce-
narios. Pertaining to the DISCO operational issue, a bi-level model is employed
on purchasing dispatchable DG and interruptible loads. From the consumer perspective, the authors in [26] proposed an alternative day-head auction scheme for consumer payment minimization in the pool market, while a large consumer procurement strategy is implicitly modeled in [27]. From other perspectives, in [28], given wind power production as a set of correlated scenarios, a scenario-based bi-level model is presented to derive strategic offers for a wind producer within markets. To minimize the payment in Pool markets, a multi-period bi-level model [26] is proposed to address the consumer’s strategies in terms of LMPs.

As indicated in Section 2.2, the PDISCO procurement in the distribution-level market and trading in the transmission-level markets can be formulated as individual bi-level game-theoretic models. Furthermore, in order to explicitly address the revenue and impact of each DER, the physical constraints of the distribution-level and transmission-level networks are also included through multi-period AC and DC power flow, respectively.

In general, a bi-level optimization problem can be described by (2.1)-(2.4).

\[
\begin{align*}
\text{Minimize} & \{ x \} \cup \{ y^1, \ldots, y^k \} \cup \{ \mu^1, \ldots, \mu^k \} \\
& f \left( x, y^1, \ldots, y^k, \mu^1, \ldots, \mu^k \right) \\
\text{s.t.} & \\
& h \left( x, y^1, \ldots, y^k, \mu^1, \ldots, \mu^k \right) = 0 \\
& g \left( x, y^1, \ldots, y^k, \mu^1, \ldots, \mu^k \right) \leq 0, \\
& \begin{cases} 
\text{Minimize} & y^i c^T y^i \\
\text{s.t.} & A_i^t y^i \geq b^i : \mu^i \\
& y^i \geq 0 \\
i = 1, \ldots, k
\end{cases} 
\end{align*}
\] (2.1)

(2.2)

(2.3)

(2.4)

where vector \( x \in \mathbb{R}^{n_0} \) is the set of optimization variables which specifically belong to the upper-level problem, while vector \( y^i \in \mathbb{R}^{n^i} \) is the set of optimization variables constraining the lower-level problem \( i = 1, \ldots, k \), vector \( \mu^i \in \mathbb{R}^{m^i} \) is the set of the dual variables pertaining to the lower-level problem \( i \), the cost vector \( c^i \in \mathbb{R}^{n^i} \), the constraint matrix \( A_i^t \in \mathbb{R}^{m^i \times n^i} \) and the right-hand-side vector \( b^i \in \mathbb{R}^{m^i} \).

Under the assumption that KKT conditions are necessary and sufficient for optimality of the lower-level problems, the bi-level problem can be transformed into a single-level optimization problem. This is achieved by replacing the lower-level problems with their first-order optimality conditions, which renders an MPEC. Two alternative approaches are nominated for the reformulation:
2.4 Modeling and Solving Approach

1) KKT conditions, renders an MPCC (mathematical program with complementarity constraints) \[12\];

2) Primal-dual approach, renders an MPPDC (mathematical program with primal and dual constraints) (2.5)-(2.8).

\[
\begin{align*}
\text{Minimize}_{\{x\} \cup \{y^1,\ldots,y^k\} \cup \{\mu^1,\ldots,\mu^k\}} & \quad f(x, y^1, \ldots, y^k, \mu^1, \ldots, \mu^k) \\
\text{s.t.} & \quad h(x, y^1, \ldots, y^k, \mu^1, \ldots, \mu^k) = 0 \\
& \quad g(x, y^1, \ldots, y^k, \mu^1, \ldots, \mu^k) \leq 0,
\end{align*}
\]

Primal constraints:
\[
\begin{align*}
A^i y^i & \geq b^i \\
y^i & \geq 0
\end{align*}
\]

Dual constraints:
\[
\begin{align*}
A^{iT} \mu^i & \leq c^i \\
\mu^i & \geq 0
\end{align*}
\]

Strong duality equality:
\[
\begin{align*}
c^i y^i & = b^i \mu^i \\
i & = 1, \ldots, k
\end{align*}
\]

**Illustrative Example (Strategic offering for a producer in a market)**

In this example, a strategic producer seeks to derive hourly offers for profit maximization in a market, which is organized as an electricity pool and cleared with LMPs. For the sake of simplicity, a 3-bus network is considered to include a generator and a demand per bus \(i\), \(i=1,2,3\). The related network parameters can be found in \[12\].

Taking generator 1 as a strategic producer, its offering strategies can be formulated as the following bi-level model (2.9)-(2.19).

\[
\begin{align*}
\text{Minimize}_{\{K^G_P\} \cup \{P^G_P^1, P^G_P^2, P^G_P^3, P^P_P^1, P^P_P^2, P^P_P^3, \Theta_1, \Theta_2, \Theta_3\} \cup \Xi^{MO}} & \quad c^G_1 P^G_P^1 - \gamma_1 P^G_P^1 \\
\text{s.t.} & \quad \text{Minimize}_{\{P^G_P^1, P^G_P^2, P^G_P^3, P^P_P^1, P^P_P^2, P^P_P^3, \Theta_1, \Theta_2, \Theta_3\}} \\
& \quad K^G P^G_P^1 + \sum_{i=2}^{3} c^G_i P^G_i - \sum_{i=1}^{3} c^D_i P^D_i \\
& \quad P^G_P^1 - P^D_P^1 - B_1 (\theta_1 - \theta_2) - B_2 (\theta_1 - \theta_3) = 0 : \gamma_1
\end{align*}
\]
\[ P_2^G - P_2^D - B_1 (\theta_2 - \theta_1) - B_3 (\theta_2 - \theta_3) = 0 : \gamma_2 \quad (2.12) \]
\[ P_3^G - P_3^D - B_2 (\theta_3 - \theta_1) - B_3 (\theta_3 - \theta_2) = 0 : \gamma_3 \quad (2.13) \]
\[ 0 \leq P_i^G \leq \overline{P}_i^G : \phi_i^-, \phi_i^+, i = 1, 2, 3 \quad (2.14) \]
\[ 0 \leq P_i^D \leq \overline{P}_i^D : \varepsilon_i^-, \varepsilon_i^+, i = 1, 2, 3 \quad (2.15) \]
\[ -\overline{f}_1 \leq B_1 (\theta_1 - \theta_2) \leq \overline{f}_1 : \rho_1^-, \rho_1^+ \quad (2.16) \]
\[ -\overline{f}_2 \leq B_2 (\theta_1 - \theta_3) \leq \overline{f}_2 : \rho_2^-, \rho_2^+ \quad (2.17) \]
\[ -\overline{f}_3 \leq B_3 (\theta_2 - \theta_3) \leq \overline{f}_3 : \rho_3^-, \rho_3^+ \quad (2.18) \]
\[ \theta_3 = 0 : \phi \quad (2.19) \]

where \( \Xi^{MO} = \{ \gamma_i, \phi_i^-, \phi_i^+, \rho_i^-, \rho_i^+, \varepsilon_i^-, \varepsilon_i^+, \phi \} \) is the set of dual variables. \( P_i^G, P_i^D \) and \( \theta_i \) are the variables of the generation, demand and angle at bus \( i \). \( K_1^G \) is the strategic offering price of generator 1. The parameters are the line susceptance \( B_i \), transmission capacity limit \( \overline{f}_i \), generation limit \( \overline{P}_i^G \), consumption limit \( \overline{P}_i^D \), production cost of each generator \( c_i^G \), and utility of each demand \( c_i^D \).

The objective of the upper-level problem (2.9) seeks to minimize the minus profit of the strategic producer, including the revenue by selling energy to the market and the cost of electricity production. The LMP \( \gamma_1 \) is endogenously generated within the lower-level problem. The production \( P_1^G \) belongs to the feasible region identified by the lower-level problem. The upper-level problem has no constraints other than those from the lower-level problem. The goal of the lower-level problem (2.10) represents the market clearing by minimizing the minus social welfare. DC power flow is employed to carry out the physical network constraints. Constraints (2.11)-(2.13) enforces the power balance at each bus \( i \). Constraints (2.14) and (2.15) are the generation and demand bounds. Constraints (2.16)-(2.18) are the capacity limits of each line. Constraint (2.19) imposes bus 3 to be the reference bus.

Since the lower-level problem is linear and thus convex, then this bi-level problem can be recast as an MPEC by KKT conditions or the primal-dual approach.

1) MPEC with KKT conditions:

Minimize \( \{ K_1^G \} \cup \{ P_1^G, P_2^G, P_3^G, P_1^D, P_2^D, P_3^D, \theta_1, \theta_2, \theta_3 \} \cup \Xi^{MO} \)

(2.9)

s.t.
\[ P_1^G - P_1^D = B_1 (\theta_1 - \theta_2) + B_2 (\theta_1 - \theta_3) \quad (2.20) \]
\[ P_2^G - P_2^D = B_1 (\theta_2 - \theta_1) + B_3 (\theta_2 - \theta_3) \quad (2.21) \]
\[ P_3^G - P_3^D = B_2 (\theta_3 - \theta_1) + B_3 (\theta_3 - \theta_2) \quad (2.22) \]
\[ \theta_3 = 0 \quad (2.23) \]
2) MPEC with the primal-dual approach:

Minimize \( \{K_1^G \} \cup \{P_i^G, P_i^D, P_3^G, P_3^D, P_1^D, P_1^P, P_2^P, \theta_1, \theta_2, \theta_3 \} \cup \Xi^{MO} \)  

\( (2.9) \)

s.t.

Primal constraints: 
\( (2.11)-(2.19) \)

Dual constraints: 
\( (2.24)-(2.29) \)

\( \phi_i^- , \phi_i^+ \geq 0, i = 1, 2, 3 \)  

\( (2.40) \)

\( \epsilon_i^- , \epsilon_i^+ \geq 0, i = 1, 2, 3 \)  

\( (2.41) \)

\( \rho_i^- , \rho_i^+ \geq 0, i = 1, 2, 3 \)  

\( (2.42) \)

Strong duality equality:

\[
K_1^G P_1^G + \sum_{i=2}^{3} c_i^G P_i^G - \sum_{i=1}^{3} c_i^D P_i^G = -\sum_{i=1}^{3} P_i^G \phi_i^- - \sum_{i=1}^{3} P_i^D \epsilon_i^+ \\
- \sum_{i=1}^{3} J_i \rho_i^+ - \sum_{i=1}^{3} J_i \rho_i^- 
\]  

\( (2.43) \)
2.5 Conclusion

A basic distribution-level market framework is presented in this chapter to interrelate with the transmission-level markets. New players, i.e. PDISCO and RA, are identified with specific functions. This bidirectional market performance is carried out by the PDISCO’s procurement/trading strategies in the distribution-level/transmission-level markets. The bi-level programming can be used to model the PDISCO’s behaviors in these hierarchical markets for different purposes. KKT conditions and Primal-dual approach are addressed as the coherent solving methods.
A methodology is proposed to optimize the procurement of the PDISCO in real-time trading, which covers real-time market transactions and DER purchases. Since DR’s modeling and mobilization are more complex than the other types of DERs, in this chapter, DRs are selected to represent the distribution-level resources. However, the presented distribution-level market framework can be easily extended to include various DGs.

To highlight the real-time features and applications, the DR definition is characterized by coexistence of elasticity and inelasticity. Specifically, a real-time trading framework is further proposed on the basis of the market structure described in Section 2.1. The PDISCO’s decision-making problem is represented by a bi-level model. The upper-level problem (non-linear and non-convex) intends to maximize the PDISCO’s profit, while the lower-level (linear and convex) expresses the profit maximization per RA. To solve this complex problem, the proposed model is transformed into a solvable MPEC with the primal-dual approach as mentioned in Section 2.4. A modified 33-bus distribution network is utilized to thoroughly simulate the PDISCO decisions, RA profits and demand revenues. While a 69-bus distribution network is further investigated to verify the scalability of this approach. The numerical results demonstrate the

\[1\] The content of this chapter is mainly taken from [29, 30]
effectiveness of the proposed PDISCO’s procurement model.

**Nomenclature**

**Sets and Indices**

- $\mathcal{N}$: Set of system buses, indexed by $i$ or $j$.
- $\mathcal{B}$: Set of distribution feeders, indexed by $ij$.
- $\mathcal{K}$: Set of RAs, indexed by $k$.
- $\mathcal{L}$: Set of demands, indexed by $l$.
- $\mathcal{T}$: Set of time periods (e.g., hours per day), indexed by $t$.

- $\mathcal{M}_l$: Mapping of the set of demands onto the set of buses.
- $\mathcal{M}_{Agg}$: Mapping of the set of demands onto the set of aggregators.

**Variables**

- $\alpha_{lt}$: Consumption of elastic portion of demand $l$ at time $t$.
- $\beta_{lt}$: Virtual generation of elastic portion of demand $l$ at time $t$.
- $P_{kt}^{Agg}, Q_{kt}^{Agg}$: Active and reactive power produced by RA $k$ at time $t$.
- $\lambda_{kt}^{Agg}$: Marginal price for PDISCO purchasing production from RA $k$ at time $t$.
- $P_{lt}^{D}, Q_{lt}^{D}$: Real-time active and reactive power consumption of demand $l$ at time $t$.
- $Q_{lt}^{DE}$: Reactive power output along with virtual generation at demand $l$ at time $t$.
- $\lambda_{lt}^{RD}$: LMP at bus $i$ at time $t$.
- $P_{t}^{RT}, Q_{t}^{RT}$: Active and reactive power exchanging in real-time market at time $t$.
- $P_{lt}^{LS}, Q_{lt}^{LS}$: Active and reactive power of load-shedding at demand $l$ at time $t$. 
$P^{Flow}_{ijt}, Q^{Flow}_{ijt}$  
Active and reactive power flow through feeder $ij$ at time $t$.

$Q_{C_{it}}$  
Reactive power produced by shunt compensator at bus $i$ at time $t$.

$V_{it}, \theta_{it}$  
Voltage magnitude and phase angle at bus $i$ at time $t$.

**Parameters**

$P^{S}_{i_{t}}, Q^{S}_{i_{t}}$  
Active and reactive power purchased from day-ahead market at time $t$.

$P^{DSI}_{lt}, P^{DSE}_{lt}$  
Active power purchased from day-ahead market for inelastic and elastic portions of demand at bus $l$ at time $t$.

$Q^{DSE}_{lt}$  
Reactive power purchased from day-ahead market for elastic portion of demand at bus $l$ at time $t$.

$P^{DI}_{lt}$  
Real-time inelastic portion of demand at bus $l$ at time $t$.

$P^{Aggmin/max}_{kt}$  
Active power production bounds of RA $k$.

$Q^{Aggmin/max}_{kt}$  
Reactive power production bounds of RA $k$.

$\lambda_{kt}^{AggPro}$  
Contract price between demand and RA $k$ at time $t$.

$\lambda_{i_{t}}^{S}$  
Day-ahead market price at time $t$.

$\lambda_{i_{t}}^{RT}$  
Real-time price in the real-time market at time $t$.

$\lambda_{i_{t}}^{LS}$  
Load-shedding penalty price for PDISCO operation at time $t$.

$\lambda^{D}$  
Electricity sales price to the demand from PDISCO.

$Q^{C_{min/max}}_{i}$  
Reactive power limits of shunt compensator at bus $i$.

$V^{min/max}_{i}$  
Limits of voltage magnitude.

$\bar{S}$  
Capacity limit of main substation.

$S^{Flow}_{ij}$  
Capacity limit of feeder $ij$.

$G_{ij}, B_{ij}, b_{ij}$  
Conductance, susceptance and charging susceptance of feeder $ij$.

$\tau_{i}$  
Transformer tap ratio.

$\Gamma$  
Elasticity limit of real-time demand.
\[ \zeta_{\text{min/max}} \] Bounds of consumption control factor.

\[ \delta_{kt} \] Profit guarantee factor of RA \( k \).

\[ \varepsilon \] Inelasticity control factor of each demand.

### 3.1 Market Design

Traditionally, a DISCO seeks to supply the demands with the lowest possible operation cost. In order to fulfill this goal, the DISCO has to make appropriate decisions on procurements from the day-ahead and real-time electricity markets at the transmission level. Thus, the DISCO is exposed to volatile real-time prices and demand uncertainties. With the availability of DR resources, the DISCO has more flexibility from the demand side. However, small-scale DRs are allocated dispersely and heterogeneously in the distribution system, which makes it quite difficult to deal with. To address this issue, RA is employed to assemble and schedule the dispersed DR resources.

In the literature, the potential interests and feasible applications of real-time DR are illustrated in [31], in which the price uncertainty is accommodated through robust optimization, and the model formulation can be easily applied in a small utility. In [32], a bi-level model is proposed to maximize the retailer’s profit and optimize the consumer behavior under real-time prices. A vital conclusion shows that the real-time pricing is more effective in load shifting. However, this approach depends on a transactional model and lacks network constraints. In contrast, to enable the flexible demand participating in the transmission-level markets, a novel pool market mechanism is reported in [33], and further validated by [34]. Although the load shifting is realized by a Lagrangian relaxation based heuristic approach, the network constraints are still not modeled. From the energy management perspective, considering the interactions between DG and the main grid, a contract-based cluster [35] is promoted to initiate DR to purchase or sell energy at a proper time. While this approach also ignores the physical network impacts.

In this chapter, a demand is defined as the summation of both the inelastic portion and elastic portion. In the day-ahead market, demand \( l \) takes both these portions (\( P_{DST}^{l} \) and \( P_{DSE}^{l} \)) into account to make an electricity purchase. In real-time operation, as shown in Fig. 3.1, the inelastic portion \( P_{DST}^{l} \) is the indispensable consumption of demand \( l \), and can be deemed as the same quantity as \( P_{DST}^{l} \). In addition, the elastic portion can be assigned as \( \Gamma P_{DSE}^{l} \) \((\Gamma \geq 1)\) and further divided into two parts. The first part (referred to as \( P_{DSE}^{l} \)) is for the actual consumption of the elastic portion (CEP), which represents the shifting
3.1 Market Design

Figure 3.1: Definition of demand response.

The flexibility of real-time demand during time period $t$. This indicates the DR capability of load shifting. The second part (denoted as $\beta_{lt} P_{DSE}^{lt}$) expresses the shavable demand, which can be seen as virtual generation of the elastic portion (GEP) managed by a RA and sold to the PDISCO. This implies the DR function of load shaving. Thus, the total active power consumption of demand $l$ ($P_{lt}^{D}$) consists of $P_{lt}^{D1}$ and $\alpha_{lt} P_{DSE}^{lt}$.

Figure 3.2: Market framework for the PDISCO’s Procurements.

The aggregator-based DRs can offer the feasibility for prompt load adjustment with superior performance in terms of response time and cost. As shown in Fig. 3.2 as a profit-driven company, besides supplying the local demands, the PDISCO can even execute ambitious schemes to procure excessive DR to sell to
the real-time market, performing as an active electricity producer.

To minimize the payment, the PDISCO has to determine the amounts of electricity purchased from the day-ahead market, transactions with the real-time market and the aggregator-based DR. Note that, because the PDISCO has thorough knowledge of the day-ahead market prices, the hourly purchases \((\lambda^S_t, P^S_t)\) from the day-ahead market are fixed at the very beginning of the next day. Throughout the day, for each time \(t\), the real-time PDISCO trading occurs through the power exchanging (selling or purchasing by \(\lambda^{RT}_t, P^{RT}_t\)) in the real-time market, and the procurement \((\lambda^{Agg}_{kt}, P^{Agg}_{kt})\) with the aggregator-based DR.

Pay-as-bid (PAB) \([5]\) pricing is assumed to be the pricing mechanism between PDISCO and RAs, and one RA is stipulated to submit only one bid at each time \(t\). From a DR aggregator perspective, to compete with the others, an aggregator should behave rationally to meet the PDISCO’s request, and send the proper bids (kW quantity and bidding price) to the PDISCO, implying the target of its own profit maximization. At the same time, according to the available real-time market price, the dynamic active/reactive consumption, RA bids, and possible load-shedding, further constrained by the physical network and facility limits, the PDISCO has to optimize the real-time trading decisions in a complex situation.

Therefore, the interactions between PDISCO and RAs can be characterized by a game theoretical model formulation as discussed later in Section 3.2.

**3.2 Problem Formulation**

**3.2.1 Assumptions**

The mathematical formulation of the proposed PDISCO procurement model is based on the following assumptions.

1) The PDISCO’s distribution network physically connects to the transmission grid via only one main substation.

2) As for the real-time trading, multi-period AC power flow is adopted to represent the distribution network. Only the active power can be traded in the real-time market and between the PDISCO and RAs, since no uniform reactive power market has evolved.
3.2 Problem Formulation

3) In the real-time market, when the PDISCO is recognized as an active producer, its offering price is assumed to be the marginal price cleared at the interconnection point (main substation) with the transmission system, and its offering volume is based on the surplus of individual RAs’ bids after satisfying the distribution network constraints.

4) Each RA can explicitly predict the impact of its bids (bidding prices and kW quantities), versus the PDISCO offers (offering prices and procurement volumes). This is reflected as the linking variable $\lambda_{Agg}$ within this bi-level model.

5) For simplicity, regarding each RA’s demand contract, all the related demands are assumed to be paid with an identical price.

---

**Upper-level problem**

Minimize Minus-profit of the PDISCO  

\[ \text{Minimize} \quad \text{Minus-profit of the PDISCO} \]

\[ s.t. \]

1) Real-time consumption of each demand  
2) PDISCO offering prices  
3) Physical network constraints for the main substation  
   - Bus: AC power balance  
   - Compensator: capacity bounds  
   - Transformer: capacity limits  
   - Voltage angle and voltage value: reference  
4) Physical network constraints for the other buses  
   - Buses: AC power balance  
   - Feeders: capacity limits  
   - Compensator: capacity bounds  
   - Voltage angle and voltage value: bounds  
5) Active/reactive power control of GEP and load-shedding  
6) Total consumption control for the whole timespan

**Lower-level problems**

7) Bidding price and kW quantity of RA $k$  

\[ \text{Minimize} \quad \text{Minus-profit of each RA} \]

\[ s.t. \]

- Active/reactive power bidding bounds  
- GEP limits of each contractual demand

---

**Figure 3.3:** Bi-level structure of the proposed PDISCO procurement model.
3.2.2 Bi-level Model

In the DR environment, the PDISCO gets the opportunity to make an optimal procurement with RA-based DR (i.e. GEP), versus purchasing the power from the real-time market at the transmission level. While each RA \( k \) behaves rationally to pursue its profit maximization (minimizing minus-profit) to make decisions on bidding prices and kW quantities. Therefore, the proposed PDISCO procurement model follows the typical bi-level theoretical game structure [36], as shown in Fig. 3.3.

The upper-level problem represents the profit maximization (minimizing minus-profit) of the PDISCO, subject to the upper-level constraints 1)-6) and a set of lower-level problems 7). For each time \( t \), the upper-level constraints include real-time consumption of each demand, PDISCO offering prices, physical network constraints of the main substation (reference bus) and other buses, active/reactive power control of load-shedding and GEP, and total consumption control for the whole timespan. Meanwhile, a lower-level problem per RA expresses their bidding strategies for minimizing its own minus-profit, subject to active/reactive power bidding bounds, and GEP limits of each contractual demand.

Note that the upper-level and the lower-level problems are interrelated with each other. The bid price and quantity, put forward by the RAs from lower-level problems, impact the PDISCO’s procurement decisions in the upper-level. On the other hand, the upper-level problem determines the offering price and procurement volume, which directly influence the RAs’ profit in the lower-level problems. Therefore, the formulation of the proposed bi-level model is made up of two optimization levels, i.e. the upper-level (3.1)-(3.24) is for PDISCO procurement decisions, and the lower-level (3.25)-(3.30) is for the rational bidding of each RA.

\[
\begin{align*}
\text{Minimize} & \quad \Xi_{\text{Agg}} \cup \Xi_{\text{PDISCO}} \cup \Xi_{\text{Dual}} \left( \sum_{t,k} \lambda_{kt}^{Agg} P_{kt}^{Agg} + \sum_{t} \lambda_{t}^{RT} P_{t}^{RT} 
+ \sum_{t} \lambda_{t}^{S} P^{S} + \sum_{t,l} \lambda_{t}^{LS} P_{lt}^{L} - \lambda^{D} \sum_{t,l} P_{lt}^{D} \right) \\
\text{s.t.} & \quad 0 \leq \alpha_{lt} + \beta_{lt} \leq \Gamma, \forall l,t \\
& \quad P_{lt}^{DI} = P_{lt}^{DSI}, \forall l,t \\
& \quad P_{lt}^{D} = P_{lt}^{DI} + \alpha_{lt} P_{lt}^{DSE}, \forall l,t \\
& \quad \delta_{kt} \lambda^{AggPro}_{kt} \leq \lambda^{Agg}_{kt} \leq \lambda^{RT}_{kt}, \forall k,t
\end{align*}
\]
For the main substation (reference bus):

\[
P_t^S + P_t^{RT} + \beta_{1t} P_t^{DSE} + P_t^{LS} - P_t^D = \sum_{1j,j\neq 1} P_{ijt}^\text{Flow}, \forall t
\]  
(3.6)

\[
Q_t^S + Q_t^{RT} + Q_t^C + Q_t^{DE} + Q_t^{LS} - Q_t^D = \sum_{1j,j\neq 1} Q_{ijt}^\text{Flow}, \forall t
\]  
(3.7)

\[
\theta_{1t} = 0,
\]  
(3.8)

\[
V_{1t} = 1,
\]  
(3.9)

\[
(P_t^S + P_t^{RT})^2 + (Q_t^S + Q_t^{RT})^2 \leq \bar{S}^2, \forall t
\]  
(3.10)

For the other buses:

\[
\beta_{lt} P_{lt}^{DSE} + P_{lt}^{LS} - P_{lt}^D = \sum_{ij,j\neq i} P_{ijt}^\text{Flow}, : \lambda_{it}^{RD}
\]  
(3.11)

\[
Q_{it}^C + Q_{it}^{DE} + Q_{it}^{LS} - Q_{it}^D = \sum_{ij,j\neq i} Q_{ijt}^\text{Flow},
\]  
(3.12)

\[
P_{ijt}^\text{Flow} = -\tau_i V_{it}^2 G_{ij} + V_{it} V_{jt} [G_{ij} \cos (\theta_{it} - \theta_{jt})
\]
\[+ B_{ij} \sin (\theta_{it} - \theta_{jt})], \forall i, j, t
\]  
(3.13)

\[
Q_{ijt}^\text{Flow} = \tau_i V_{it}^2 B_{ij} - 0.5 b_{ij} + V_{it} V_{jt} [G_{ij} \sin (\theta_{it} - \theta_{jt}) - B_{ij} \cos (\theta_{it} - \theta_{jt})], \forall i, j, t
\]  
(3.14)

\[-\pi \leq \theta_{it} \leq \pi, \forall i, t
\]  
(3.15)

\[
V_{i\text{min}} \leq V_{it} \leq V_{i\text{max}}, \forall i, t
\]  
(3.16)

\[
(P_{ijt}^\text{Flow})^2 + (Q_{ijt}^\text{Flow})^2 \leq (S_{ijt}^\text{Flow})^2, \forall i, j, t
\]  
(3.17)

\[
Q_{it}^{C\text{min}} \leq Q_{it}^C \leq Q_{it}^{C\text{max}}, \forall i, t
\]  
(3.18)

\[
0 \leq P_{lt}^{LS} \leq P_{lt}^D, \forall l, t
\]  
(3.19)

\[
0 \leq Q_{lt}^{DE} \leq Q_{lt}^{DSE}, \forall l, t
\]  
(3.20)

\[
P_{lt}^{LS} Q_{lt}^D = P_{lt}^D Q_{lt}^{LS}, \forall l, t
\]  
(3.21)

\[
\beta_{lt} P_{lt}^{DSE} Q_{lt}^D = P_{lt}^D Q_{lt}^{DSE}, \forall l, t
\]  
(3.22)

\[
\sum_{l,t} P_{lt}^D - \sum_{l,t} P_{lt}^{LS} \geq \zeta_{\text{min}} \sum_t P_t^S,
\]  
(3.23)

\[
\sum_{l,t} P_{lt}^D - \sum_{l,t} P_{lt}^{LS} \leq \zeta_{\text{max}} \sum_t P_t^S,
\]  
(3.24)

\[
P_{kt}^{Agg}, Q_{kt}^{Agg}, \beta_{lt} \in \text{arg}
\]  
 Minimize

\[
\Xi_{\text{Agg}}
\]


\[
\left( \sum_t \lambda^{AggPro}_{kt} P^{Agg}_{kt} - \sum_t \lambda_{kt} P^{Agg}_{kt} \right)
\]

\[
\text{s.t.}
\]

\[
P^{Agg}_{kt} = \sum_{l:(l,k) \in M_{Agg}} \beta_{lt} P^{DSE}_{lt}, \forall k, t : \eta_{kt}
\]

\[
Q^{Agg}_{kt} = \sum_{l:(l,k) \in M_{Agg}} Q^{DE}_{lt}, \forall k, t : \mu_{kt}
\]

\[
P^{Aggmin}_{kt} \leq P^{Agg}_{kt} \leq P^{Aggmax}_{kt}, \forall k, t : \rho^-_{kt}, \rho^+_{kt}
\]

\[
Q^{Aggmin}_{kt} \leq Q^{Agg}_{kt} \leq Q^{Aggmax}_{kt}, \forall k, t : \sigma^-_{kt}, \sigma^+_{kt}
\]

\[
0 \leq \beta_{lt} \leq 1, \forall t, l : (l,k) \in M_{Agg} : \phi^-_{lt}, \phi^+_{lt}
\]

\[
\alpha_{lt}, \beta_{lt}, P^{Agg}_{kt}, Q^{Agg}_{kt}, \lambda^{Agg}_{kt}, P^{D}_{lt}, Q^{D}_{lt}, Q^{DE}_{lt},
\lambda_{it}^{RD}, P^{LS}_{lt}, Q^{LS}_{lt}, Q^{C}_{lt}, V_{lt}, \sigma^-_{kt}, \sigma^+_{kt}, \rho^-_{kt}, \rho^+_{kt}, \phi^-_{lt}, \phi^+_{lt},
\]

\[
P^{RT}_{lt}, Q^{RT}_{lt}, P^{Flow}_{ijlt}, Q^{Flow}_{ijlt}, \theta_{lt}, \eta_{kt}, \mu_{kt} : \text{free.}
\]

where \( \Xi^{Agg} = \{ P^{Agg}_{kt}, Q^{Agg}_{kt}, \beta_{lt} \} \) is the set of each lower-level problem variables.

\( \Xi^{PDISCO} = \{ \alpha_{lt}, \lambda^{Agg}_{kt}, P^{D}_{lt}, Q^{D}_{lt}, Q^{DE}_{lt}, P^{LS}_{lt}, Q^{LS}_{lt}, P^{RT}_{lt}, Q^{RT}_{lt}, P^{Flow}_{ijlt}, Q^{Flow}_{ijlt}, Q^{C}_{lt}, V_{lt}, \theta_{lt} \} \) is the set of upper-level problem variables. \( \Xi^{Dual} = \{ \lambda^{RD}_{lt}, \eta_{kt}, \mu_{kt}, \sigma^-_{kt}, \sigma^+_{kt}, \rho^-_{kt}, \rho^+_{kt}, \phi^-_{lt}, \phi^+_{lt} \} \) is the set of dual variables.

The objective (3.1) of the upper-level problem is to minimize the PDISCO’s minus-profit, which comprises the cost of purchasing the GEPs from RAs, exchanging power from the real-time market, acquiring active power from the day-ahead market, the penalty of possible load-shedding, and the minus revenue of electricity sales to demands.

For each time \( t \):

Constraints (3.2) enforce the bounds of the GEP/CEP produced/consumed by each demand. Constraints (3.3) impose the real-time quantity of inelastic active power is the same as purchased from the day-ahead market. Furthermore, constraints (3.4) indicate the consumed active power is composed of the inelastic portion \( P^{D}_{lt} \) and the CEP portion \( \alpha_{lt} P^{DSE}_{lt} \). From the perspective of the PDISCO, associated with the bid control factor \( \delta_{kt} \) \( (\delta_{kt} \geq 1, \forall k, t) \), a RA profit guarantee mechanism is yielded as the lower bound of constraints (3.5), which also emphasizes the acceptable RA’s bidding price should be no greater than the price from the real-time market (ceiling price). Here, AC power flow is employed
to simulate the real-time operation model. For the main substation (reference bus), constraints (3.6) and (3.7) represent the AC power balance, and the voltage angle and voltage value are retained at a constant level via constraints (3.8) and (3.9). The capacity limit of the main substation is specified in constraints (3.10). For the other buses, constraints (3.11), (3.12), (3.13) and (3.14) identify the AC power flowing through the feeder $i-j$, and constraints (3.17) further impose the capacity limits individually. Constraints (3.15) and (3.16) identify the angle bounds and voltage limits for the other buses. Constraints (3.18) describe the capacity bounds for each compensator. Specifically for the potential load-shedding bus, the amount of load curtailment invoked by the PDISCO is capped with the constraints (3.19). Constraints (3.20) express the limits of the elastic reactive power for each demand. Constraints (3.21) and (3.22) keep the power factor in constant, if the corresponding demand is involved in load-shedding or GEP-generating. Constraints (3.23) and (3.24) state consumption control over the whole timespan (e.g., 24 hours per day) with the bounds $\zeta_{\text{min}}/\zeta_{\text{max}}$. For instance, when $\zeta_{\text{min}}=\zeta_{\text{max}}=1$, these constraints guarantee the consumption of each demand across the whole timespan should be equal to the same amount purchased by the PDISCO from the day-ahead market. That means, these constraints ensure the load shifting between the hours while maintaining the total consumption at a certain level.

As indicated in (3.25), the objective of the lower-level problem is to minimize the minus-profit of each RA $k$, i.e. the cost of purchasing GEPs from contractual demands minus the revenue of selling the aggregated quantities to the PDISCO, correspondingly.

Observe that the PDISCO offering price $\lambda_{kt}^{\text{Agg}}$ is an upper-level decision variable treated as a parameter within the lower-level problem. This means that once the RAs rationally submit their bidding prices $\lambda_{kt}^{\text{Agg}}$ with kW quantities $P_{kt}^{\text{Agg}}$, the PDISCO figures out the preferable offers through the upper-lever problem. Constraints (3.26) and (3.27) illustrate the valid GEPs (active power) with the essential reactive power $Q_{it}^{\text{DE}}$ assembled by each RA, while the output limits on individuals are imposed by constraints (3.28) and (3.29). Constraints (3.30) preserve the GEPs of each demand ought to be less than the amount $P_{it}^{\text{DSE}}$ purchased from the day-ahead market.

Finally, constraints (3.31) classify the positive variables and free variables for this bi-level model.
3.2.3 MPEC

In this subsection, the bi-level model for the procurement decision-making of the PDISCO with RAs is transformed into a single-level optimization problem. The upper-level PDISCO problem is non-linear and non-convex, while the lower-level RAs' problems are linear and thus convex. Therefore, the RAs' problems can be replaced by their first-order optimality conditions, which renders an MPEC.

Since each lower-level problem is linear and thus convex, its KKT optimality conditions are necessary and sufficient. However, the complementarity derived from KKT conditions is numerically difficult to handle especially when the upper-level PDISCO problem is already non-linear and non-convex. Therefore, the KKT approach is not appropriate to solve this bi-level problem tractably.

To avoid complementarity conditions, the Primal-dual approach is applied in this chapter. It is relevant to note that an MPPDC is generally easier to be solved than its associated MPCC. Note that the lower-level problems are generally formulated using their canonical form, so that the dual problem can be straightforwardly written. The complementarity conditions obtained by the first approach (KKT conditions) are equivalent to the corresponding strong duality equality and the set of primal-dual constraints. Finally, the bi-level problem results in a non-linear problem without complementarity constraints.

Constraints (3.32)-(3.34) are dual constraints of lower-level RAs problems. The constraint (3.35) is the associated strong duality equality, which ensures the equality of the primal and dual objective function values, one per RA $k$.

**Dual constraints:**

\[
\lambda_{kt}^{AggPro} - \lambda_{kt}^{Agg} + \eta_{kt} + \rho_{kt}^+ - \rho_{kt}^- = 0, \forall k, t \tag{3.32}
\]

\[
\mu_{kt} + \sigma_{kt}^+ - \sigma_{kt}^- = 0, \forall k, t \tag{3.33}
\]

\[-\eta_{kt} P_{lt}^{DSE} + \phi_{lt}^+ - \phi_{lt}^- = 0, \forall l : (l, k) \in \mathcal{M}_{Agg}, t \tag{3.34}
\]

**Strong duality equality:**

\[
\sum_t \left( \lambda_{kt}^{AggPro} P_{kt}^{Agg} - \lambda_{kt}^{Agg} P_{kt}^{Agg} \right) + \sum_t \left( P_{kt}^{Agg_{max}} \rho_{kt}^+ - P_{kt}^{Agg_{min}} \rho_{kt}^- \right) + \sum_{t, l : (l, k) \in \mathcal{M}_{Agg}} \left( Q_{lt}^{DE} \right) + \sum_{t, l : (l, k) \in \mathcal{M}_{Agg}} \left( P_{kt}^{Agg_{max}} \sigma_{kt}^+ - Q_{kt}^{Agg_{max}} \sigma_{kt}^- \right) + \sum_{t, l : (l, k) \in \mathcal{M}_{Agg}} \phi_{lt}^+ = 0, \forall k \tag{3.35}
\]
3.3 33-bus Distribution Network

Substituting lower-level RAs problems with MPPDC, the final single-level model’s structure is to minimize the PDISCO’s objective, subject to PDISCO’s constraints, RAs’ MPPDC constraints, and in the end declarations of positive and free variables, as shown in (3.36). The final non-linear model without complementarity can be solved by the commercial off-the-shell large-scale non-linear optimization solver CONOPT3 [37].

\[
\text{Minimize} \quad \Xi_{\text{Agg}} \cup \Xi_{\text{PDISCO}} \cup \Xi_{\text{Dual}} \quad (3.1) \\
\text{s.t.} \\
\text{PDISCO’s problem constraints:} \\
(3.2) - (3.24) \\
\text{RAs’ problems MPPDC constraints:} \\
(3.26) - (3.30) \text{ and } (3.32) - (3.34) \\
\text{Variables Declarations: } (3.31)
\]

3.3 33-bus Distribution Network

To illustrate the feasibility and effectiveness of the approach proposed in the Section 3.1-3.2, numerical analyses are presented in this section to identify a PDISCO’s optimal procurement.

A 33-bus distribution network [38] is modified to validate the model’s effectiveness as proposed in Section 3.2.2. The PDISCO procurement decisions, RA trading behaviors and demand performances are exhibited explicitly. Meanwhile, the impacts of sensitive parameters, i.e. demand elasticity limit \( \Gamma \), inelasticity control factor \( \varepsilon \), consumption control factor \( \zeta_{\text{min/max}} \), RA procurement price \( \lambda_{\text{AggPro}} \), and RA integration number \( k \), are defined and analyzed in detail. In addition, comparison results of differing DR, network congestion, and prices are fully elaborated.

All cases are solved by CONOPT3 with GAMS 24.4.1 [37] on a 3.6 GHz Intel Core i7 processor carried out on 16 GB of RAM and 64-bit Windows 7 system.

3.3.1 Data

The 33-bus distribution network presented in [38] is considered to be owned and operated by the PDISCO, the topology, feeder capacity, and impedance parameters are shown in this section. The base parameters of this 33-bus distribution
network can be found in Appendix A. Capacity of each feeder $S_{ij}^{Flow}$ is set to 10 MVA, main substation capacity limit $\bar{S}$ to 20 MVA. As the reference bus, the voltage is 1 p.u. with the voltage angle of 0. The tap ratio $\tau_i$ per transformer is fixed to 1. For the other buses, the bounds of voltage ($V_{i}^{min/max}$) are 0.9 and 1.1 p.u. Each compensator’s output is enforced in the range of 0-200 kVar. To address the inelastic portion per demand, an inelasticity control factor $\varepsilon$ is adopted to specify this proportion, i.e. $P_{it}^{DSI} = \varepsilon P_{it}^{S}$, $P_{it}^{DSE} = (1 - \varepsilon)P_{it}^{S}$. As shown in Fig. 3.4, three RAs are concerned to be hourly involved in the real-time trading with the PDISCO, i.e. 24 times per day. The individual zoning is initialised as, RA1 includes \{2, 3, 4, 5, 6, 7, 19, 20, 21, 23, 26\}, RA2 contains \{10, 11, 12, 13, 14, 15, 16, 17, 18, 24, 25\}, and RA3 consists of \{8, 9, 22, 27, 28, 29, 30, 31, 32, 33\}. Referring to the NordPool prices, the day-ahead market price $\lambda_{it}^{S}$ and real-time market price $\lambda_{it}^{RT}$ can be estimated and given in Table 3.1, which also enumerates the day-ahead market volume $P_{it}^{S}$, RA-demand contract price $\lambda_{kt}^{AggPro}$, and load-shedding price $\lambda_{it}^{LS}$.
Table 3.1: Essential Input Parameters for PDISCO Procurement Model (33-bus Network)

<table>
<thead>
<tr>
<th>Time ( t ) [Hour]</th>
<th>Real-time market price ( \lambda_{t}^{RT} ) [€/kW]</th>
<th>Day-ahead market price ( \lambda_{t}^{S} ) [€/kW]</th>
<th>Day-ahead market transaction ( P_{t}^{S} ) [kW]</th>
<th>RA1-demand contract price ( \lambda_{1t}^{AggPro} ) [€/kW]</th>
<th>RA2-demand contract price ( \lambda_{2t}^{AggPro} ) [€/kW]</th>
<th>RA3-demand contract price ( \lambda_{3t}^{AggPro} ) [€/kW]</th>
<th>Load-shedding price ( \lambda_{t}^{LS} ) [€/kW]</th>
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<td>8.00</td>
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</table>
For simplicity, the profit guarantee factor $\delta_{kt}$ held by each RA $k$ is identical and equal to 1.1. Other system-wide parameters are imposed as: the demand purchase price $\lambda^{D} = 0.6$ €/kW, elasticity limit $\Gamma = 1.2$, and daily consumption control factor $\zeta^{min} = \zeta^{max} = 1$.

### 3.3.2 PDISCO Procurement Strategies

The results of optimal PDISCO procurement are shown in Fig. 3.5 and in Table 3.2 (a)-(b). As expected, during high-price hours (10-14, 18-22) in the real-time market, the PDISCO performs as an active producer with the exchanging volume 3715.74 kW, 3753.65 kW, 2083.71 kW, 480.25 kW, 1066.61 kW, and 3330.55 kW, respectively.

At each hour $t$, for PDISCO-RA trading, the procurement volume $P_{Agg}^{kt}$ for each RA $k$ is settled at the individual offer price $\lambda_{Agg}^{kt}$. The main trading hours of RAs focus on hours 10-14, 18-22. In the other time slots, PDISCO acquires very little from RAs, who executes demand shifting according to shifting schedules passed by PDISCO. Therefore, the demand consumes more in the low-price periods, and results in a more rational load curve. Furthermore, to avoid an extremely high penalty price, load-shedding is not used at any bus.

Regarding the PDISCO profit $O_{PP}^{t}$, observe that the maximum income is not placed during hours 10-14 or 18-22, but spread out in other time periods. That is because PDISCO-RA trading shaves high-price peak load and shifts it to the other hours, while the daily consumption remains unchanged and the electricity purchase price $\lambda^{D}$ by the demand side is fixed. Thus, the PDISCO daily profit can reach 17958.97 €.

### 3.3.3 RA Real-time Trading

For each hour $t$, the profit results of RAs are shown in Fig. 3.5 (b). Each RA rationally bids at the equilibrium price and quantity, and dispatches individual contractual demands. Consequently, the numerical results of the lower-level problem are obtained (33 bus × 24 hour). For illustration purposes, this section takes RA1’s optimal decisions for instance to show the RA’s virtual generation in Table 3.3.

Except high-price periods in the real-time market, RA1 calls for very little virtual generation, and provides nearly no output in PDISCO-RA trading. Same as in subsection 3.3.2, the RA1’s profits $O_{AP}^{t}$ are dramatically increased during
3.3 33-bus Distribution Network

Figure 3.5: Results of optimal PDISCO procurement.
**Table 3.2:** PDISCO Procurement and Prots in Real-time Trading (33-bus Network)

<table>
<thead>
<tr>
<th>Time</th>
<th>Load Real-time</th>
<th>Real-time Pro</th>
<th>Real-time Shed</th>
<th>Total Real-time</th>
<th>Total Real-time</th>
<th>Total Real-time</th>
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**Note:** The table details are truncated for brevity. Full data can be provided upon request.
**Table 3.3:** Contractual Demands’ GEPs, Revenues and RA1 Profits in Real-time Trading

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<thead>
<tr>
<th>Time</th>
<th>( GEP \beta_{lt}P^D_{lt}^{DE} )</th>
<th>Profit ( O^A_{lt} )</th>
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hours 10-13 and 20-21. The highest appears at hour $t=21$ with 19.60 €, and daily profit is about 72.59 €. Meanwhile, the revenue of GEP for individual demands can be evaluated by $\lambda_{kt}^{AggPro} \beta_{lt} P_{DSE}^t$, $\forall l \in M_{Agg}$, e.g., for demand $l=2$, $\sum_t \lambda_{kt}^{AggPro} \beta_{2t} P_{DSE}^t=78.53$ €. Subsequently, the daily profit for RA2 is 164.83 €, while for RA3 it is 112.45 €, and the daily revenue of the total demand attains 3498.68 €.

### 3.3.4 Impact of Elasticity Limit

As indicated above, the elasticity limit $\Gamma$ enforces the the upper bound of GEP/CEP produced/consumed by each demand $l$ via constraint (2). Thus, $\Gamma$ is the key factor impacting the real-time demand consumption, PDISCO procurement decision, RA bidding, and demand revenue, simultaneously.

Keeping the other parameters unchanged, given the increment of $\Gamma$ with 0.1 per interval, as shown in Fig. 3.6 (a), the PDISCO’s daily profit, RAs’ profit and demand revenue keep increasing rapidly, but tend to be flat when $\Gamma$ goes beyond 1.4. That implies, on basis of the system operation constraints, the PDISCO has to find a trade-off among exchanging with the real-time market, procuring RAs’ virtual generation, and maintaining daily consumption at a critical level. On the other side, extra availability of GEPs causes a more competitive market, some RAs (e.g., RA1 and RA2) may reduce their interests in trading with PDISCO.

### 3.3.5 Impact of Inelasticity Control Factor

The inelasticity control factor $\varepsilon$ directly reflects the consumption characteristic, and indirectly affects the GEP per demand, as illustrated in Section 3.1. With the other parameters unchanged, Fig. 3.6 (b) depicts the relations of daily PDISCO profit, RA profit and demand revenue under differing $\varepsilon$ values. Observe that, with the $\varepsilon$ increasing, the diminishing elasticity results in limited trading possibilities among the participants.

### 3.3.6 Impact of Consumption Control Factor

To embody the sensitivity of daily consumption adopted in this proposed bi-level model, the section categorizes the control factor $\zeta^{\min/\max}$ into three cases. With the other parameters unchanged, the results are shown in Fig. 3.6 (c).
1) Case 1: $\zeta_{\text{min}} \geq 1$, $\zeta_{\text{max}} \geq 1$. Compared with $\zeta$ allocated in the range of [1-1.1], each market participant obtains slightly more profit if the daily consumption is strictly controlled at $\zeta_{\text{min}} = \zeta_{\text{max}} = 1$. This indicates the PDISCO intends to evade demand-side risks and hope consumption at each bus can be realized as the same amount as purchased from the day-ahead market.

2) Case 2: $\zeta_{\text{min}} < 1$, $\zeta_{\text{max}} > 1$. In this situation, this proposed trading setup allows moderate arbitrage to have both PDISCO and RAs pursue higher profit. Compared with Case 1, this section sets $\zeta$ in [0.9-1.1] and [0.9-1], both of the results show that, a proper bound of consumption control can increase around 15% of the daily profit for PDISCO, and critically prompt 75%-87% of profit for individual RAs. Thus, the demand obtains 81% extra revenues.

3) Case 3: $\zeta_{\text{min}} < 1$, $\zeta_{\text{max}} < 1$. With the tendency of exacerbating arbitrage behavior, i.e. $\zeta$ arranged in [0.8-0.9] and [0.7-0.8], the RAs’ profits persist with a rising trend. On the contrary, the PDISCO’s daily profit drops rapidly compared to Case 2, and a reduction of 23% from Case 1.

Therefore, the moderate arbitrage strategy may be more preferable for PDISCO for profit maximization, and for RAs to trade with PDISCO rationally.

### 3.3.7 Impact of Profit Guarantee Factor

Aiming to be a profitable participant gaming with PDISCO, the profit guarantee factor $\delta_{kt}$ represents the RAs’ expected price margin. As shown in Fig. 3.6 (d), increasing $\delta_{kt}$ to 1.5 with 0.1 increments, the RAs’ profits rise up sharply, while the PDISCO’s daily profit decreases progressively and the demand revenue stays at a certain level. On one hand, this validates the PDISCO is willing to continuously procure RAs’ virtual generation under the premise that the bidding prices $\lambda_{kt}^{\text{Agg}}$ are comparable to the real-time price $\lambda_{kt}^{\text{RT}}$. On the other hand, to maximize RA’s own profit, the procurement price $\lambda_{kt}^{\text{AggPro}}$ should be optimized with the contractual GEP providers.

### 3.3.8 Impact of RA Procurement Price

To capture the characteristics of RA procurement price $\lambda_{kt}^{\text{AggPro}}$, this section assumes the data in Section 3.3.1 represents a base case and impose a $\lambda_{kt}^{\text{AggPro}}$ multiplier from [0.5-2.5]. Fig. 3.6 (e) indicates the PDISCO’s daily profit declines gradually, while each RA has varied performance. For RA1 and RA3,
Figure 3.6: Impacts of individual sensitive parameters.
their profits are maximized at multiplier=2 and go down immediately afterwards, since their bidding prices are approaching the price ceiling $\lambda^{RT}_{kt}$. Prior to that point, a similar observation can be obtained from RA2. Accordingly, the PDISCO’s profit has been reduced significantly at each time when a RA quits real-time trading. When multiplier=2.5, RA1 is left as the single player to trade with PDISCO. In this case, the PDISCO-RA trading turns into a monopolistic structure.

3.3.9 Impact of RA Aggregation

In the proposed approach, associated with a certain number of RAs, PDISCO makes optimal decisions on electricity procurement. To test the PDISCO’s profitability through multiple RA combinations, $\kappa$ is used to distinguish the number of RAs, and the results are shown in Fig. 3.7.

1) $\kappa = 1$: As a base case, if no RA exists in this PDISCO-RA trading model, the daily PDISCO profit is 12938.88 €. Particularly, RA1, RA2 and RA3 respectively represent the participants with the low, high, and medium prices involved in the PDISCO-RA trading process. As mentioned above, a single RA interacting with the PDISCO is in a monopolistic setting. We can observe that the RA’s bidding price goes higher while the PDISCO’s profit gets lower, e.g., RA1 bids with $\lambda_{1t}^{AggPro}$ and the PDISCO gains 21651.20 € daily profit, in contrast, RA2 bids with $\lambda_{2t}^{AggPro}$ and the PDISCO’s profit reduces to 19930.40 €.

2) $\kappa = 2$: The observation mentioned above can also be obtained in the two-RA case, which denotes a moderately competitive market. The PDISCO’s profit shows a reverse trend with the RA’s bidding prices, and the lowest profit (18543.44 €) appears in the existence of RA2 and RA3.

3) $\kappa = 3$: In a more competitive market case, the PDISCO claims more RA virtual generation, and three rival RAs compete with each other on bidding prices and quantities to maximize their own profit. Contrarily, the PDISCO’s profit further declines to 17958.97 €, but still shows 39% higher than the non-RA case.

Among these cases, the demand revenue grows gradually along with the increasing number $\kappa$ of RAs, and presents an opposite trend against the PDISCO’s profit.
3.3.10 DR Comparison \((\alpha, \beta)\) vs. \(\alpha\)

In general, the proposed approach is on basis of a novel DR definition, where the RAs are eligible to assemble the dispersed GEPs to trade with PDISCO. Note that \(\beta\) represents the GEP virtual generation capability of each demand, further constrains the LL problem, and finally impacts the system-wide PDISCO’s decisions. Thus, the particular \(\beta\) contribution of the proposed approach needs to be verified. To illustrate the merits of this \((\alpha, \beta)\) based DR formulation, two different cases are stimulated to compare the cases with \((\alpha)\) only DR and without DR. The results are summarized in Table 3.4 and described by Fig. 3.8.

![Figure 3.7: Impact of integrated RA number \(k\).](image)

<table>
<thead>
<tr>
<th>Case (DR type)</th>
<th>Daily exchanging power [kW]</th>
<th>Daily PDISCO profit [€]</th>
</tr>
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<tbody>
<tr>
<td>1 ((\alpha, \beta))</td>
<td>-8002.15</td>
<td>17958.97</td>
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<tr>
<td>2 ((\alpha)) only</td>
<td>2413.13</td>
<td>14677.45</td>
</tr>
<tr>
<td>3 (without DR)</td>
<td>4334.66</td>
<td>12938.88</td>
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</table>
In case 1, PDISCO acts as an active producer during high real-time market price periods, i.e. hours 10-14 and 18-22, with a generation volume of 14430.50 kW. In the other hours, PDISCO procures 6428.36 kW to meet the daily demand consumption control (as discussed in Section 3.3.6). Thus, the daily exchanging power in the real-time market is -8002.15 kW. For case 2, the PDISCO also intends to be proactive during hours 10-13 and 20-22, the DR load shifting function can be directly enabled by PDISCO. However, due to lack of $\beta$, the sales and purchases volume in the real-time market become 6927.97 kW and 9341.10 kW, and the daily balance is 2413.13 kW. For these two cases, the load shifting function is accomplished, but with varying load shaving performance, shown in Fig. 3.8 (a). The reason is that PDISCO has to find the trade-off among procuring RA production, exchanging in the real-time market and supplying electricity to demand. Compared with case 1 and 2, case 3 indicates that a large amount of power (4334.66 kW) needs to be acquired to meet the daily consumption.

Fig. 3.8 (b) exhibits the respective profits at each hour $t$, and the major variation occurs across hours 10-14 and 18-22. Due to case 1 equipped with $(\alpha, \beta)$ based DR, RA-PDISCO trading can help PDISCO increase the profit by 22% and 39% compared with cases 2 and 3. Consequently, over the daily operation, the revenue brought by $\beta$ can be seen as $17958.97 \in - 14677.45 \in = 3281.52 \in$, which accounts for 18% of the PDISCO’s daily profit in case 1.

### 3.3.11 Network Congestion

In the previous sections, the capacity limit of each feeder is assumed to be 10 MVA. Note that, in the competitive market environment, no congestion occurs in the physical network, since the bidding prices and virtual generation of RAs have to satisfy the network constraints in advance. However, the congestion may occur in real-time dispatching, due to the peak load occasionally exceeding the capacity limit.

To estimate the possible impacts of congestion, three scenarios are selected as follows. For scenario 1, the capacity limit of feeder 3-23 (within RA1) is narrowed to 1 MVA. In order to create a worse case, additional feeders (13-14 (RA2), 28-29 (RA3)) are enforced to experience congestions in scenario 2, and the capacity limits are adjusted to 0.5 MVA and 1 MVA, respectively. In addition, the competition outcomes of each RA can be observed under this multiple congestion situation. To demonstrate the system-wide congestion, scenario 3 limits the main path (1-2) capacity to 5 MVA.

The comparison results are analyzed between with and without $(\alpha, \beta)$ based DR.
Figure 3.8: Comparison of (α, β) DR, (α) only DR and without DR.
### Table 3.5: Scenarios of Network Congestion

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Feeder from-to</th>
<th>Capacity limit [MVA]</th>
<th>Load-shedding cost with $(\alpha, \beta)$ DR [€]</th>
<th>Load-shedding cost without DR [€]</th>
<th>PDISCO profit without DR [€]</th>
<th>DISCO profit without DR [€]</th>
<th>RA1 profit [€]</th>
<th>RA2 profit [€]</th>
<th>RA3 profit [€]</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>3-23 (RA1)</td>
<td>1</td>
<td>18220.72</td>
<td>20440.91</td>
<td>-2758.87</td>
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<td>66.28</td>
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<td>112.45</td>
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<td>2</td>
<td>13-14 (RA2) 28-29 (RA3)</td>
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<td>33390.40</td>
<td>36391.64</td>
<td>-23335.10</td>
<td>-29115.95</td>
<td>65.92</td>
<td>51.43</td>
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<td>1795.88</td>
<td>-61785.90</td>
<td>69.97</td>
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in this network, and outlined in Table 3.5. Observing that the load-shedding occurs in both scenarios 1 and 2, the RA-based DR relieves partial congestion and avoids a certain amount of profit loss for PDISCO, i.e., 68% and 20%, individually. In scenario 3, without DR, a sizable loss for PDISCO appears. On the contrary, DR can thoroughly mitigate the congestion system-wide and retain the profitability of PDISCO and RAs. Meanwhile, we can also infer the RAs’ profits in existence of a single, mixed, or system-wide congestion. The observation shows that RAs can obtain higher profit if they can alleviate more congestion.

In other words, to maximize the profit from a long-term perspective, PDISCO can also make extensive use of RA-based DR to postpone network expansion or reinforcement.

### 3.4 69-bus Distribution Network

In this section, the well-established 69-bus distribution network [10] is used to verify the scalability of the proposed approach. The base parameters of this 69-bus distribution network can be found in Appendix B. The network initial conditions can be referred to in 2.3.1.1. As shown in Fig. 3.9 three RAs are imposed as RA1 \{4, 47, 48, 49, 50\}, RA2 \{5, 6, 7, 8, 51, 52\}, and RA3 \{9, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65\}. In view of the supportive outcomes from Section 3.3, to maximize the PDISCO’s profit, the sensitive parameters are assumed as \(\Gamma = 1.4\), \(\zeta_{\text{min}} = 0.9\), \(\zeta_{\text{max}} = 1.1\), \(\varepsilon = 0.5\), \(\delta_{k1} = 1.3\), while RA-demand contract prices and day-ahead market transaction volume are adjusted as shown in Table 3.6. The other input parameters remain the same as in Section 3.3.1. The 69-bus test results are presented in Table 3.7, each RA’s profit and related demand revenue are summarized in Table 3.8. This case study is also carried out on the same benchmark as described in Section 3.3.

As expected, load-shedding is not actuated by PDISCO in daily operation. We can observe that, in the real-time market, a large volume of power exchanging happens in hours 10-14 and 18-21, which leads to a high profit due to PDISCO behaving as an active producer. The PDISCO daily profit reaches 30338.20 €. As a DR provider, the real-time demand tracks the virtual generation amounts to benefit from PDISCO-RA trading. Since the demand consumption is also a large amount, GEPs could be abundant on the demand side. RAs seize every possible opportunity to take part in PDISCO-RA trading, even in the lower price periods, e.g., RA3 persists in selling a certain amount of generation at hours 9 and 17. This indicates that, in a DR adequate situation, the zoning scheme plays a decisive role for RAs to increase income, i.e., higher DR capacity.
Figure 3.9: Modified 33-bus distribution network and RAs' zoning.
Table 3.6: Essential Input Parameters for PDISCO Procurement Model (69-bus Network)

<table>
<thead>
<tr>
<th>Time $t$ [Hour]</th>
<th>Day-ahead market transaction $P_t^S$ [kW]</th>
<th>RA1-demand contract price $\lambda_{\text{AggPro}}^{1t}$ [€/kW]</th>
<th>RA2-demand contract price $\lambda_{\text{AggPro}}^{2t}$ [€/kW]</th>
<th>RA3-demand contract price $\lambda_{\text{AggPro}}^{3t}$ [€/kW]</th>
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Table 3.7: PDISCO Procurements and Profits in Real-time Trading (69-bus Network)

<table>
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<tr>
<th>Time ( t ) [Hour]</th>
<th>Real-time market ( p^{RT}_{t} ) [( \epsilon ) / kW]</th>
<th>RA price ( \lambda^{Agg}_{1} ) [( \epsilon ) / kW]</th>
<th>RA quantity ( q^{Agg}_{1} ) [kW]</th>
<th>RA2 price ( \lambda^{Agg}_{2} ) [( \epsilon ) / kW]</th>
<th>RA2 quantity ( q^{Agg}_{2} ) [kW]</th>
<th>RA3 price ( \lambda^{Agg}_{3} ) [( \epsilon ) / kW]</th>
<th>RA3 quantity ( q^{Agg}_{3} ) [kW]</th>
<th>Load-shedding ( \sum_{t} \sum_{l} p^{LS}_{lt} ) [kW]</th>
<th>Real-time demand ( \sum_{t} p^{d}_{t} ) [kW]</th>
<th>Profit ( O^{P}_{l} ) [( \epsilon )]</th>
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leads to higher profit. The daily profit of each RA is 514.05 €, 119.29 €, and 741.01 €, respectively. The hourly demand revenue is consistent with the RAs’ profits, and daily revenue reaches up to 4581.16 €.

Table 3.8: RAs’ Profits and Demand Revenue in Real-time Trading

<table>
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<tr>
<th>Time $t$ [Hour]</th>
<th>RA1 profit $O_{1t}^{AP}$ [10^{-2}€]</th>
<th>RA2 profit $O_{2t}^{AP}$ [10^{-2}€]</th>
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<th>Demand revenue [€]</th>
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3.5 Conclusion

This chapter presents a bi-level programming model for optimal electricity procurement for a PDISCO with DERs, typically indicating by the aggregator-based DR resources in the real-time trading framework.

The PDISCO and RAs’ decisions are modeled through upper-level and lower-level problems. Combined with commercial and physical constraints, this model
3.5 Conclusion

can explicitly address the decisions of PDISCO’s procurements about real-time market exchanging, RAs’ GEP-generating and load shedding. This proposed bi-level model is finally transformed into an MPEC for computational efficiency and tractability.

To validate the proposed methodology, the numerical results and impact analysis are fully conducted in the case studies, illustrating the integration of PDISCO-RA trading can result in a reciprocal situation where both PDISCO and RAs are satisfied with profit maximization. Consistently following the RAs’ trading schemes, the contractual demand can benefit more as a GEP provider, with both DR shaving and shifting fulfilled.

The interests of PDISCO, RAs and demands are all captured by the proposed bi-level game structure. Based on the numerical analysis, other conclusions are:

1) For PDISCO, \((\alpha, \beta)\) based DR is preferable, \(\zeta\) should be flexible in a certain range, RAs are helpful for eliminating congestions and increasing profits in various situations.

2) For a RA, a lower purchase price combined with higher \(\delta\) is a superior option and \(\varepsilon\) ought to be lower, while \(\Gamma\) is desired to be bounded. Also, higher DR capacity leads to higher competitiveness.
This chapter proposes a methodology to address the trading strategies of a PDISCO engaged in the transmission-level markets by procuring distribution-level resources. On the basis of the market framework presented in Section 2.1, assuming the PDISCO-RA procurement is proceeded prior to the PDISCO trading in the transmission-level markets, the procurement prices (per time $t$) for the individual DERs can be seen as available parameters. To characterize the impacts of stochastic DERs physically dispersed in the distribution-level network, the virtual RA’s participation is omitted from the distribution-level market framework. Then a one-leader multi-follower bi-level model can be presented to focus on formulating the game between the PDISCO and the transmission-level markets.

The lower-level problems include the transmission-level day-ahead market and scenario-based real-time markets, respectively with the objectives of maximizing social welfare and minimizing operation cost. The upper-level problem is to maximize the PDISCO’s profit crossing these markets. The PDISCO’s strategic offers/bids interactively influence the outcomes of each market. Since the lower-level problems are linear and convex, while the upper-level problem is non-linear and non-convex, an equivalent primal-dual approach is used to reformulate this bi-level model to a solvable MPEC. The effectiveness of the proposed model is

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2The content of this chapter is mainly taken from [41]
verified by the case studies.

Nomenclature

Sets and Indices

\[ i,j,B_{DS}^{DS} \]
Index and set of the distribution-level and transmission-level buses, respectively.

\[ n,m,B_{TS}^{TS} \]
Index and set of the distribution-level feeders and transmission-level lines, respectively.

\[ l,L_{DS}^{DS} \]
Index and set of the distribution-level and transmission-level demands, respectively.

\[ g,G_{TS}^{TS} \]
Index and set of the transmission-level generators.

\[ k,K_{DS}^{DS} \]
Index and set of the distribution-level DERs.

\[ t,T_{DS}^{TS} \]
Index and set of the time periods (e.g., hours per day).

\[ \omega,\Omega_{DS}^{TS} \]
Index and set of the scenarios.

\[ M_{L},M_{D}^{DS} \]
Mapping of the set of distribution-level/transmission-level demands onto the set of distribution-level/transmission-level buses, respectively.

\[ M_{G}^{TS} \]
Mapping of the set of the transmission-level conventional generations onto the set of transmission-level buses.

\[ M_{K}^{DS} \]
Mapping of the set of DERs onto the set of distribution-level buses.

Variables

\[ P_{tg}^{G} \]
Day-ahead offer of generator \( g \) at time \( t \).

\[ R_{tg}^{UP},R_{tg}^{DN} \]
Day-ahead up and down regulation reserve capacities of generator \( g \) at time \( t \).

\[ r_{tg\omega}^{UP},r_{tg\omega}^{DN} \]
Real-time up and down regulation power of generator \( g \) at time \( t \) for scenario \( \omega \).

\[ \lambda_{t}^{DDA} \]
Day-ahead offering/bidding price of the PDISCO at time \( t \).
\[ P_{t}^{DDA}, Q_{t}^{DDA} \]

Day-ahead offering/bidding quantity of the PDISCO at time \( t \) (non-negative is offer, negative is bid).

\[ \lambda_{t}^{DRT} \]

Real-time offering/bidding price of the PDISCO at time \( t \) for scenario \( \omega \).

\[ P_{t}^{DRT}, Q_{t}^{DRT} \]

Real-time offering/bidding quantity of the PDISCO at time \( t \) for scenario \( \omega \) (non-negative is offer, negative is bid).

\[ P_{t}^{ST} \]

Transmission-level load-shedding of demand \( d \) at time \( t \) for scenario \( \omega \).

\[ \theta_{tn}, \theta_{tn\omega} \]

Voltage angles of bus \( n \) at day-ahead time \( t \), and at real-time time \( t \) for scenario \( \omega \).

\[ \lambda_{tn}^{DA}, \lambda_{tn\omega}^{RT} \]

LMP at transmission-level bus \( n \) at day-ahead time \( t \), and at real-time time \( t \) for scenario \( \omega \).

\[ P_{tk}^{DER}, Q_{tk}^{DER0} \]

Active and reactive power procured from DER \( k \) at day-ahead time \( t \).

\[ P_{tl\omega}^{SD}, Q_{tl\omega}^{SD} \]

Active and reactive power of distribution-level load-shedding for demand \( l \) at time \( t \) for scenario \( \omega \).

\[ Q_{ti}^{C0}, Q_{ti\omega}^{C} \]

Reactive power from distribution-level shunt compensator at bus \( i \) at day-ahead time \( t \), and at real-time time \( t \) for scenario \( \omega \).

\[ P_{t,ij}^{FD0}, Q_{t,ij}^{FD0} \]

Day-ahead active and reactive power flows through distribution-level feeder \( i-j \) at time \( t \).

\[ P_{t,ij\omega}^{FD}, Q_{t,ij\omega}^{FD} \]

Real-time active and reactive power flows through distribution-level feeder \( i-j \) at time \( t \) for scenario \( \omega \).

\[ \delta_{ti}, \delta_{ti\omega} \]

Voltage angles of bus \( i \) at day-ahead time \( t \), and at real-time time \( t \) for scenario \( \omega \).

\[ V_{ti}, V_{ti\omega} \]

Voltage magnitudes of distribution-level bus \( i \) at day-ahead time \( t \), and at real-time time \( t \) for scenario \( \omega \).

**Parameters**

\[ P_{tk\omega}^{DER}, Q_{tk\omega}^{DER} \]

Active and reactive power generation realization of DER \( k \) at time \( t \) for scenario \( \omega \).

\[ P_{g}^{G}, R_{g}^{UP}, R_{g}^{DN} \]

Maximum production, maximum up and down regulation reserve capacities of generator \( g \).
\( C^G_g, C^U_p, C^D_p \) \( g \) Day-ahead generation cost, up and down regulation reserve costs of generator \( g \).

\( c^U_p, c^D_p \) Real-time up and down regulation cost of generator \( g \).

\( C^D_t \) Operation cost of the PDISCO at time \( t \).

\( \lambda^TSD_{td}, P^TSD_{td} \) Day-ahead bidding price and consumption of transmission-level demand \( d \) at time \( t \).

\( \lambda^DER_t \) DER procurement price of the PDISCO at time \( t \).

\( \lambda^DSD_t \) Distribution-level sale price at time \( t \).

\( P^P_{tl}, Q^P_{tl} \) Consumption of distribution-level demand \( l \) at time \( t \).

\( P^PS \) Active power injection limit for the PDISCO.

\( \lambda^ST_t, \lambda^SD_t \) Transmission-level/distribution-level load-shedding price at time \( t \).

\( P^TS_{nm} \) Capacity limit of each transmission-level line \( nm \).

\( S, S_{ij} \) Capacity limits of the distribution-level main substation and each distribution-level feeder \( ij \).

\( S_k \) Capacity limit of each DER \( k \).

\( Q^C_i, Q^C_i \) Reactive power limits of the distribution-level shunt compensator at bus \( i \).

\( V_i, \overline{V}_i \) Limits of voltage magnitude at distribution-level bus \( i \).

\( \tau_i \) Transformer tap ratio at distribution-level bus \( i \).

\( B_{nm} \) Susceptance of the transmission-level line \( nm \).

\( G_{ij}, B_{ij}, b_{ij} \) Conductance, susceptance and charging susceptance of the distribution-level feeder \( ij \).

### 4.1 Market Design

As shown in Fig. 4.1 to trade in the transmission-level markets, the PDISCO has two optional strategies to take part in each market, i.e. offer or bid in the day-ahead market and real-time market per scenario \( \omega \). For two-stage market operation, the day-ahead market is cleared prior to the real-time markets. At each time \( t \), the PDISCO’s strategic day-ahead offer/bid \((\lambda^DDA_t, P^DDA_t)\) goes
4.2 Problem Formulation

Participating in the transmission-level markets, the PDISCO’s strategic trading problem can be formulated as a bi-level one-leader multi-follower game-theoretic model [36]. One follower is characterized as a lower-level day-ahead market with the day-ahead market aiming to meet the transmission-level demands by generators’ offers, while each real-time market balances the PDISCO’s strategic real-time offer/bid \((\lambda_{DDA}^t, P_{DRT}^{t\omega})\) with the transmission-level regulation power and/or load-shedding.

Crossing the transmission-level markets, to maximize profit, the PDISCO’s power quantity \(P_{DDA}^t\) is scheduled by rationally procuring the DERs’ portfolios (a set of \(P_{DER}^{tk}\)) against physical distribution-level constraints at the day-ahead stage, and the scenario-based power quantity \(P_{DRT}^{t\omega}\) is determined as the variation pertaining to total DERs’ outputs \(\sum_k P_{DER}^{tk\omega}\) versus network constraints at the real-time stage. At each time \(t\): If \(P_{DDA}^t/P_{DRT}^{t\omega}\) is non-negative, the PDISCO behaves as an active producer with a lower offering price \(\lambda_{DDA}^t/\lambda_{DRT}^{t\omega}\); If this quantity is negative, the PDISCO acts as an active consumer with a higher bidding price. For the stochastic DERs, the PDISCO’s procurement is settled with the price \(\lambda_{DER}^t\), and the scenario-based method [11] can be used to represent the individual DERs’ uncertainties.
problem with the goal to maximize the transmission-level social welfare. Other followers are scenario-based lower-level real-time market problems. A lower-level problem per scenario $\omega$ seeks to achieve the cost minimization during the real-time transmission-level balancing. In particular, to obtain the trade-offs between the markets, the offering/bidding strategies of the PDISCO are conducted by the upper-level problem with the purpose of minus-profit minimization.

As indicated in Fig. [4.2] the upper-level constraints consist of two aspects, i.e. the day-ahead and real-time aspects. The former includes the bounds of the PDISCO's day-ahead offering/bidding quantity, individual DERs' production limits, and physical network constraints at each bus. The latter covers the bounds of the PDISCO's real-time offering/bidding quantity, real-time physical network constraints at each bus, and power factor control of each DER and possible load-shedding. The lower-level problems are considered to represent a set of market clearing issues, constraining the day-ahead and real-time bounds of the PDISCO's offers/bids, power balance at the PDISCO located bus and others, capacity limits of each transmission-level line, reserve/regulation capacity of individual transmission-level generators, and the bounds of the transmission-level voltage value and voltage angle.

### 4.2.1 Assumptions

The assumptions of the proposed bi-level model are as follows:

1) The day-ahead and real-time offering/bidding strategies of a single PDISCO are considered to be involved in the corresponding transmission-level markets.

2) The distribution-level network is assumed to be operated and owned by the PDISCO, and interconnected to the transmission-level network via only one main substation.

3) Only the active power is considered to be traded in markets. At each time $t$, the reactive power exchanging between distribution and transmission levels is assumed to be balanced at the distribution-level main substation.

4) The PDISCO can explicitly anticipate the impacts of its strategic offers/bids, versus the markets' outcomes. One offer/bid per time $t$ for each market.
4.2 Problem Formulation

**Upper-level problem**

**Minimize** Minus-profit of the PDISCO

**s.t.**

1) For the distribution-level day-ahead aspect, at time $t$:
   1.1) Bounds of day-ahead offering/bidding quantity
   1.2) DER production limits
   1.3) Distribution-level physical network constraints for the main substation
       - Bus: AC power balance
       - Compensator: capacity bounds
       - Transformer: capacity limits
       - Voltage angle and voltage value: reference
   1.4) Distribution-level physical network constraints for the other buses
       - Buses: AC power balance
       - Feeders: capacity limits
       - Compensator: capacity bounds
       - Voltage angle and voltage value: bounds

2) For the distribution-level real-time aspect, at time $t$ per scenario $\omega$:
   2.1) Bounds of real-time offering/bidding quantity
   2.2) Distribution-level physical network constraints for the main substation
       - Bus: AC power balance
       - Compensator: capacity bounds
       - Transformer: capacity limits
       - Voltage angle and voltage value: reference
   2.3) Distribution-level physical network constraints for the other buses
       - Buses: AC power balance
       - Feeders: capacity limits
       - Compensator: capacity bounds
       - Voltage angle and voltage value: bounds
   2.4) Power factor control of distribution-level DERs and load-shedding

**Lower-level problems**

3) Transmission-level day-ahead market clearing

**Minimize** Minus social welfare

**s.t.**

- 3.1) PDISCO’s day-ahead offering/bidding bounds
- 3.2) DC power balance at PDISCO located bus
- 3.3) DC power balance at other transmission-level buses
- 3.4) Capacity limits of transmission-level lines
- 3.5) Reserve capacity limits of transmission-level generators
- 3.6) Bounds of voltage angle and voltage value

4) Transmission-level real-time market clearing per scenario $\omega$

**Minimize** Operation cost

**s.t.**

- 4.1) PDISCO’s real-time offering/bidding bounds
- 4.2) DC power balance at PDISCO located bus
- 4.3) DC power balance at other transmission-level buses
- 4.4) Capacity limits of transmission-level lines
- 4.5) Regulation capacity limits of transmission-level generators
- 4.6) Bounds of voltage angle and voltage value

**Figure 4.2:** Bi-level structure of the PDISCO’s trading model in markets.
4.2.2 Markets

Note that the outcomes of the lower-level markets’ problems can directly affect the PDISCO’s offering/bidding strategies in the upper-level problem. Thus, the corresponding lower-level problem formulations are given below.

1) Lower-level day-ahead market problem:

\[
\begin{align*}
\text{Minimize } & \sum_{t,g} \left(C_G^g P_G^g + C_{UP}^g R_{UP}^g + C_{DN}^g R_{DN}^g\right) \\
+ & \sum_t \lambda_I^{DDA} P_{t}^{DDA} - \sum_{t,d} \lambda_{td}^{TSD} P_{td}^{TSD} \\
\text{s.t.} & \sum_{(g,I) \in M_G} P_G^{tg} + P_{t}^{DDA} = \sum_{Im \in \Lambda^{TS}} B_{Im} (\theta_{tI}^0 - \theta_{tm}^0) : \lambda_{tI}^{DA}, \forall t \\
- & P_{DS}^0 \leq P_{t}^{DDA} \leq P_{DS}^0 : \beta_t^{0-}, \beta_t^{0+}, \forall t \\
\text{For PDISCO located bus } I: & \sum_{(g,n) \in M_G} P_G^{tg} - P_{td}^{TSD} = \sum_{nm \in \Lambda^{TS}} B_{nm} (\theta_{tn}^0 - \theta_{tm}^0) : \lambda_{tn}^{DA}, \forall t,n \\
0 \leq & P_{t}^{G} + R_{t}^{UP} \leq P_{t}^{G} : \gamma_t^g, \gamma_t^g, \forall t,g \\
0 \leq & P_{t}^{G} - R_{t}^{DN} \leq P_{t}^{G} : \gamma_t^g, \forall t,g \\
0 \leq & R_{t}^{UP} \leq R_{t}^{DN} : \phi_t^0, \phi_t^0, \forall t,g \\
- & P_{nm}^{TS} \leq B_{nm} (\theta_{tn}^0 - \theta_{tm}^0) \leq P_{nm}^{TS} : \mu_{t,nm}, \mu_{t,nm}, \forall t,nm \\
\forall t,nm \in \Lambda^{TS} & \\
- & \pi \leq \theta_{tn}^0 \leq \pi : \sigma_{tn}^0, \sigma_{tn}^0, \forall t,n \\
\theta_{t1}^0 = & 0 : \zeta_t^0, \forall t,n = 1 \\
\end{align*}
\]

where \(\Xi^{DAT} = \{P_{tg}, R_{tg}^{UP}, R_{tg}^{DN}, P_{t}^{DDA}, \theta_{tn}^0\}\) is the variable set of the lower-level day-ahead problem.

The day-ahead market clearing is modeled within the lower-level problem (4.1). The objective function (4.1.1) is to minimize the minus (maximize) transmission-level social welfare, i.e. the generation and reserve cost of individual generators plus the PDISCO’s strategic offers/bids (non-negative \(P_{t}^{DDA}\) is offer, while the
negative is bid, $\forall t$), and minus the revenue of sales to the other transmission-level demands. This chapter assumes all the generators are dispatchable. DC power flow is adopted to formulate the transmission-level operation conditions.

At each time $t$:

Constraints (4.1.2) and (4.1.4) represent the power balance at the PDISCO’s location bus $I$ and the other transmission-level buses, respectively. Constraints (4.1.3) impose the bounds of the PDISCO’s offering/bidding quantity $P_t^{DDA}$. Constraints (4.1.5) (4.1.6) (4.1.7) and (4.1.8) specify the capacity limits of the up/down reserve $R_{tg}^{UP}/R_{tg}^{DN}$ for generator $g$. Constraints (4.1.9) indicate the capacity limits of the transmission-level line $n-m$. Constraints (4.1.10) express the range of the transmission-level voltage angles, and constraints (4.1.11) set transmission-level bus 1 as the reference bus. Correspondingly, the dual variables for each group of constraints are indicated at the right side of a colon.

2) Lower-level real-time market problems:

$$\left\{ \begin{array}{l}
\text{Minimize} \sum_{t,g} \left( c_{t,g}^{UP} r_{tg}^{UP} - c_{t,g}^{DN} r_{tg}^{DN} \right) \\
+ \sum_t \lambda_t^{DRT} P_{t}^{DRT} + \sum_{t,d} \lambda_t^{ST} P_{td}^{ST}
\end{array} \right. (4.2.1)$$

s.t.

For PDISCO located bus $I$:

$$\sum_{(g:I) \in M_G} (r_{tg}^{UP} - r_{tg}^{DN}) + P_{t}^{DRT} = \sum_{l,m \in \Lambda_{TS}} B_{lm}$$

$$\left( \theta_{tI} - \theta_{tI}^0 - \theta_{tm} + \theta_{tm}^0 \right) : \lambda_{tI}^{RT}, \forall t \tag{4.2.2}$$

$$-P_{t}^{DS} \leq P_{t}^{DRT} \leq P_{t}^{DS} : \beta_{tI}^-, \beta_{tI}^+, \forall t \tag{4.2.3}$$

For other buses:

$$\sum_{(g:I) \in M_G} (r_{tg}^{UP} - r_{tg}^{DN}) + P_{t}^{ST} = \sum_{nm \in \Lambda_{TS}} B_{nm}$$

$$\left( \theta_{tn} - \theta_{tn}^0 - \theta_{tm} + \theta_{tm}^0 \right) : \lambda_{tn}^{RT}, \forall t, n_{(d;n)} \in M_D \tag{4.2.4}$$

$$0 \leq r_{tg}^{UP} \leq R_{tg}^{UP} : \phi_{tg}^-, \phi_{tg}^+, \forall t, g \tag{4.2.5}$$

$$0 \leq r_{tg}^{DN} \leq R_{tg}^{DN} : \psi_{tg}^- \psi_{tg}^+, \forall t, g \tag{4.2.6}$$

$$0 \leq P_{td}^{ST} \leq P_{td}^{TSD} : \nu_{td}^-, \nu_{td}^+, \forall t, d \tag{4.2.7}$$

$$-P_{nm}^{TS} \leq B_{nm} (\theta_{tn} - \theta_{tm}) \leq P_{nm}^{TS} : \mu_{t, nm}^-, \mu_{t, nm}^+, \forall t, n, m \in \Lambda_{TS} \tag{4.2.8}$$

$$-\pi \leq \theta_{tn} \leq \pi : \sigma_{tn}^-, \sigma_{tn}^+, \forall t, n \tag{4.2.9}$$
where $\mathcal{Z}^{RTT} = \{r_{t g, w}^{UP}, r_{t g, w}^{DN}, P_{t w}^{DRT}, P_{t d, w}^{ST}, \theta_{t n, w}\}$ is the variable set of each lower-level real-time problem per scenario $\omega$. At this stage, the day-ahead market variables $R_{t g}^{UP}$, $R_{t g}^{DN}$ and $\theta_{t n}^{0}$ are recognized as parameters.

The real-time market is cleared by (4.2). For each scenario $\omega$, the objective (4.2.1) indicates the transmission-level operation cost minimization, which comprises the regulation cost of each generator $g$, the strategic offers/bids of the PDISCO, and the cost of load-shedding invocations.

On basis of DC power flow, at each time $t$ per scenario $\omega$:

Constraints (4.2.2) guarantee the power balance at the PDISCO bus $I$ to deal with its strategic offering/bidding quantity $P_{t w}^{DRT}$, which is further limited in (4.2.3). For other buses, constraints (4.2.4) ensure the power balance at each bus when involuntary load-shedding occurs. Constraints (4.2.5) and (4.2.6) enforce the up and down regulation $(r_{t g, w}^{UP}, r_{t g, w}^{DN})$ of each generator should not exceed its up and down reserve capacity $(R_{t g}^{UP}, R_{t g}^{DN})$, respectively. Constraints (4.2.7) indicate the transmission-level load-shedding limits. Constraints (4.2.8) identify the transmission capacity bounds. Constraints (4.2.9) and (4.2.10) limit the real-time transmission-level voltage angle at bus $n$. The related dual variables are also separated by a colon.

### 4.2.3 PDISCO

Interrelated with the lower-level problems, the offers/bids determined by the upper-level PDISCO’s problem inevitably influence the markets’ outcomes in the lower-level problems. Therefore, the upper-level problem covers the PDISCO’s offering/bidding strategies in individual markets, distribution-level network constraints, and markets’ arguments regarding the LMPs and production/consumption quantities.

\[
\text{Minimize } \mathcal{Z}^{DRT} \cup \mathcal{Z}^{RTT} \cup \mathcal{Z}^{PDISCO} \cup \mathcal{Z}^{Dual} \\
\quad - \sum_t \left( \lambda_{t I}^{DA} - C_{t}^{DS} \right) P_{t DDA} \\
\quad + \sum_{t,k} \lambda_{t k}^{DER} P_{t k}^{DER0} - \sum_{t,l} \lambda_{t DSD}^{DS} P_{t l}^{DSD} + \\
\quad E \left[ - \sum_t \left( \lambda_{t I, w}^{RT} - C_{t}^{DS} \right) P_{t I, w}^{DRT} + \sum_{t,l} \left( \lambda_{t}^{SD} + \lambda_{t}^{DSD} \right) P_{t l, w}^{SD} \right]
\]
4.2 Problem Formulation

\[-\sum_{t,k} \left( \lambda_{t,k}^{RT} - \lambda_{t,k}^{DER} \right) \left( P_{tk}^{DER} - P_{tk}^{DER0} \right) \] (4.3.1)

s.t.

PDISCO day-ahead constraints (4.3.2)-(4.3.16):

\[ P_{tDDA}^k \leq \sum_k P_{tk}^{DER0} - \sum_l P_{tl}^{DSD}, \forall t \] (4.3.2)

For the main substation (PDISCO’s reference bus 1):

\[-P_{tDDA}^1 + \sum_{(k:1)\in M_K} P_{tk}^{DER0} - P_{t}^{DSD} = \sum_{1j\in \Lambda_{DS}} P_{t1j}^{FD0}, \forall t \] (4.3.3)

\[ V_{t1}^0 = 1, \forall t \] (4.3.4)

\[ \delta_{t1}^0 = 0, \forall t \] (4.3.5)

\[(P_{tDDA}^1)^2 + (Q_{tDDA}^1)^2 \leq S, \forall t \] (4.3.6)

For the other buses:

\[ \sum_{(k:i)\in M_K} P_{tk}^{DER0} - P_{ti}^{DSD} = \sum_{ij\in \Lambda_{DS}} P_{tij}^{FD0}, \forall t, i_{(1;i)} \in M_L \] (4.3.7)

\[ V_{ti}^0 \leq V_i, \forall t, i \] (4.3.8)

\[ -\pi \leq \delta_{ti}^0 \leq \pi, \forall t, i \] (4.3.9)

\[ -\pi \leq V_{ti}^0 \leq V_i, \forall t, i \] (4.3.10)

PDISCO real-time constraints (4.3.17)-(4.3.33):
\[ P_{t\omega}^{DRT} \leq \sum_{k} (P_{tk\omega}^{DER} - P_{tk}^{DER0}) + \sum_{l} P_{tl\omega}^{SD}, \forall t, \omega \quad (4.3.17) \]

For the main substation (PDISCO’s reference bus 1):
\[ -P_{t\omega}^{DRT} + \sum_{(k:1) \in M_K} (P_{tk\omega}^{DER} - P_{tk}^{DER0}) + P_{t1\omega}^{SD}, \forall t, \omega \quad (4.3.18) \]
\[ -Q_{t\omega}^{DRT} + \sum_{(k:1) \in M_K} (Q_{tk\omega}^{DER} - Q_{tk}^{DER0}) + Q_{t1\omega}^{SD} + Q_{t1\omega}^{C}, \forall t, \omega \quad (4.3.19) \]
\[ V_{t1\omega} = 1, \forall t, \omega \quad (4.3.20) \]
\[ \delta_{t1\omega} = 0, \forall t, \omega \quad (4.3.21) \]
\[ (P_{t}^{DDA} + P_{t\omega}^{DRT})^2 + (Q_{t}^{DDA} + Q_{t\omega}^{DRT})^2 \leq \bar{S}^2, \forall t, \omega \quad (4.3.22) \]

For the other buses:
\[ \sum_{(k:i) \in M_K} (P_{tk\omega}^{DER} - P_{tk}^{DER0}) + P_{ti\omega}^{SD}, \forall t, i, \omega \quad (4.3.23) \]
\[ \sum_{(k:i) \in M_K} (Q_{tk\omega}^{DER} - Q_{tk}^{DER0}) + Q_{ti\omega}^{SD} + Q_{ti\omega}^{C} - Q_{ti}^{C0}, \forall t, i, \omega \quad (4.3.24) \]
\[ P_{t,ij,\omega}^{FD} = -\tau_i V_{ti\omega}^2 G_{ij} + V_{ti\omega} V_{tj\omega} |G_{ij}| \cos (\delta_{ti\omega} - \delta_{tj\omega}) + B_{ij} \sin (\delta_{ti\omega} - \delta_{tj\omega}), \forall t, i, j \in \Lambda_{DS}, \omega \quad (4.3.25) \]
\[ Q_{t,ij,\omega}^{FD} = \tau_i V_{ti\omega}^2 B_{ij} - 0.5 b_{ij} + V_{ti\omega} V_{tj\omega} |G_{ij}| \sin (\delta_{ti\omega} - \delta_{tj\omega}) - B_{ij} \cos (\delta_{ti\omega} - \delta_{tj\omega}), \forall t, i, j \in \Lambda_{DS}, \omega \quad (4.3.26) \]
\[ (P_{t,ij,\omega}^{FD})^2 + (Q_{t,ij,\omega}^{FD})^2 \leq (S_{ij})^2, \forall t, i, j \in \Lambda_{DS}, \omega \quad (4.3.27) \]
\[ 0 \leq P_{tl\omega}^{SD} \leq P_{tl}^{DS}, \forall t, l, \omega \quad (4.3.28) \]
\[ 0 \leq Q_{tl\omega}^{SD} \leq Q_{tl}^{DS}, \forall t, l, \omega \quad (4.3.29) \]
\[ P_{tl}^{DS} Q_{tl\omega}^{SD} - P_{tl\omega}^{SD} Q_{tl}^{DS} = 0, \forall t, l, \omega \quad (4.3.30) \]
\[ Q_{ti}^{C} \leq Q_{ti\omega}^{C}, \forall t, i, \omega \quad (4.3.31) \]
\[ V_{i} \leq V_{ti\omega} \leq V_{i}, \forall t, i, \omega \quad (4.3.32) \]
4.2 Problem Formulation

\[-\pi \leq \delta_{ti\omega} \leq \pi, \forall t, i, \omega\]  
\[\lambda_{tI}^{DA}, P_{tI}^{DDA} \in \arg(1)^{DA}\]  
\[\lambda_{tI\omega}^{RT}, P_{tI\omega}^{DRRT} \in \arg(2)^{RT}\]  

where \(\Xi^{PDISCO} = \{\lambda_{tI}^{DA}, \lambda_{tI\omega}^{DRRT}, P_{tk}^{DER0}, Q_{tk}^{DER0}, P_{tl\omega}^{SD}, Q_{tl\omega}^{SD}, Q_{C}^{C0}, P_{t,ij}^{PFD0}, Q_{t,ij}^{PFD0}, \delta_{ti}, V_{ti}, Q_{tI}^C, P_{t,ij,\omega}^{PFD}, Q_{t,ij,\omega}^{PFD}, \delta_{tI\omega}, V_{tI\omega}\}\) is the variable set of the upper-level PDISCO-O problem (4.3).

\(\Xi^{DUAL}\) is the set of the dual variables.

To minimize the PDISCO’s minus-profit (maximize profit), the objective (4.3.1) of the upper-level problem is made up of day-ahead and real-time aspects. The day-ahead aspect include the revenue/cost from the PDISCO’s strategic offers/bids in the day-ahead market, the procurements from the individual DERs, and the revenue of electricity sales to the distribution-level demands. The real-time aspect is the expected minus-profit according to the revenue/cost from the PDISCO’s strategic offers/bids in the real-time markets, the penalty of possible distribution-level load-shedding, and the revenue/cost from the DERs’ production deviations. The PDISCO’s main substation located at the transmission-level bus \(I\) is seen as the distribution-level reference bus 1. AC power flow is used to formulate the PDISCO’s operation model.

For the day-ahead aspect, at each time \(t\):

Constraints (4.3.2) illustrate the upper bound of the PDISCO’s day-ahead offering/bidding quantity. Constraints (4.3.3) and (4.3.4) enforce the AC power balance at the distribution-level reference bus, which keeps the voltage magnitude and voltage angle as constant values with constraints (4.3.5) and (4.3.6). Constraints (4.3.7) impose the capacity limits of the main substation. Constraints (4.3.8) and (4.3.9) represent the AC power balance at the other buses. Constraints (4.3.10) and (4.3.11) depict the AC power flow through feeder \(i-j\), which is restricted by constraints (4.3.12). Constraints (4.3.13) specify the production limits of DER \(k\). The capacity of each compensator is bounded in constraints (4.3.14). Constraints (4.3.15) and (4.3.16) limit the voltage magnitude and angle at each distribution-level bus.

For the real-time aspect, at each time \(t\) per scenario \(\omega\):

Constraints (4.3.17) indicate the upper bound of the PDISCO’s real-time offering/bidding quantity. Constraints (4.3.18) and (4.3.19) guarantee the AC power balance at the distribution-level reference bus. Constraints (4.3.20), (4.3.21), (4.3.22) and (4.3.33) enforce the value of voltage magnitude and angle at each bus \(n\). The capacity limits of the main substation, each feeder and each compensator are constrained by (4.3.22), (4.3.27) and (4.3.31), respectively. Constraints (4.3.23) and (4.3.24) realize the AC power balance for
the other distribution-level buses. The AC power flow is expressed in constraints (4.3.25) and (4.3.26). Constraints (4.3.28)-(4.3.30) retain the demand power factor in constant if the distribution-level load-shedding occurs. Note that the PDISCO’s offering/bidding prices $\lambda_{DA}^{DDA}$ and $\lambda_{DRT}^{DRT}$ are upper-level decision variables treated as parameters in the lower-level problems. Thus, constraints (4.3.34) and (4.3.35) represent the interactive impacts between the PDSICO’s trading strategies (offering/bidding price and quantity) and markets’ outcomes.

### 4.2.4 MPEC

Note that the proposed bi-level model is put forward by the non-linear upper-level problem and the linear lower-level problems. Thus, this bi-level model can be translated into a single-level model by replacing the lower-level markets’ problems with their first-order optimality conditions, which renders an MPEC. As discussed in Section 3.2.3, since the primal-dual approach is more tractable and efficient than the KKT conditions for off-the-shell branch-and-cut software [12], the former approach is applied in this chapter.

The primal-dual approach renders a MPPDC, which is carried out on the transformation of each lower-level problem. The MPPDCs for the day-ahead market problem and real-time market problems are indicated in (4.4) and (4.5), respectively. Constraints (4.4.1)-(4.4.5) are the dual constraints of the primal constraints (4.1.2)-(4.1.11). Constraint (4.4.6) is the associated strong duality equality of problem (4.1). For each scenario $\omega$: Constraints (4.5.1)-(4.5.5) are the dual constraints of the primal constraints (4.2.2)-(4.2.11). Constraint (4.5.6) is the associated strong duality equality of problem (4.2). The strong duality constraint ensures the equality of the primal and dual objective values, one per lower-level market problem.

For the lower-level day-ahead market problem:

\[
\begin{align*}
C_g^G + \lambda_{DA}^{DA} + \lambda_{DA}^{0+} - \lambda_{DA}^{0-} - \eta_{g}^{0} &= 0, \forall t, g, (g, n) \in \mathcal{M}_G \quad (4.4.1) \\
C_g^{UP} + \gamma_{tg}^{0+} - \gamma_{tg}^{0-} + \phi_{tg}^{0+} - \phi_{tg}^{0-} &= 0, \forall t, g \quad (4.4.2) \\
C_g^{DN} + \eta_{tg}^{0+} + \psi_{tg}^{0+} - \psi_{tg}^{0-} &= 0, \forall t, g \\
\lambda_t^{DDA} + (\lambda_{tI}^{DA})_{n=I} + \beta_t^{0+} - \beta_t^{0-} &= 0, \forall t \quad (4.4.4) \\
\sum_{nm \in \Lambda^{PS}} B_{nm} \left( \lambda_{tm}^{DA} - \lambda_{tn}^{DA} \right) + \sigma_{tn}^{0+} - \sigma_{tn}^{0-} &= 0, \forall t \\
+ \sum_{nm \in \Lambda^{PS}} B_{nm} \left( \mu_{t,mn}^{0+} - \mu_{t,mn}^{0+} \right) + \left( s_t \right)_{n=1} &= 0
\end{align*}
\]
4.2 Problem Formulation

\[- \sum_{nm \in \Lambda_{TS}} B_{nm} (\mu_{t,nm}^{0-} - \mu_{t,mn}^{0-}) = 0, \forall t, n \quad (4.4.5)\]

\[\sum_{t,g} (C_G^P t_g + C_g^U P t_{tg} + C_g^D N R_{tg}^D N) + \sum_{t} \lambda_t^D A P_t^D A + \sum_{t,g} \psi_{tg}^{0+} R_g^D N + \sum_{t,n} \pi (\sigma_{tn}^{0+} + \sigma_{tn}^{0-}) = 0 \quad (4.4.6)\]

For the lower-level real-time market problems:

\[c_g^{U P} + \lambda_{t \omega}^{R T} + \psi_{t \omega}^{0+} - \phi_{t \omega}^{0-} = 0, \forall t, g_{(g;n) \in \mathcal{M}_G}, \omega \quad (4.5.1)\]
\[c_g^{D N} - \lambda_{t \omega}^{R T} + \psi_{t \omega}^{0+} - \phi_{t \omega}^{0-} = 0, \forall t, g_{(g;n) \in \mathcal{M}_G}, \omega \quad (4.5.2)\]
\[\lambda_t^{D R T} + (\lambda_{t \omega}^{R T})_{n=1} + \beta_t^{+} - \beta_t^{0-} = 0, \forall t, \omega \quad (4.5.3)\]
\[\lambda_t^{S T} + \lambda_{t(n:d) \in \mathcal{M}_D \omega}^{R T} + \nu_{t \omega}^{0+} - \nu_{t \omega}^{0-} = 0, \forall t, d, \omega \quad (4.5.4)\]

\[- \sum_{nm \in \Lambda_{TS}} B_{nm} (\lambda_{t \omega}^{R T} - \lambda_{t \omega}^{R T}) + \sigma_{tn}^{0+} - \sigma_{tn}^{0-} \quad (4.5.5)\]

\[\sum_{t,g} (C_G^P t_g + C_g^U P t_{tg} + C_g^D N R_{tg}^D N) + \sum_{t} \lambda_t^{D R T} P_t^{D R T} + \sum_{t} \psi_{tg}^{0+} R_g^D N + \sum_{t,n} \pi (\sigma_{tn}^{0+} + \sigma_{tn}^{0-}) \quad (4.5.6)\]

Replacing each lower-level market problem with the corresponding MPPDC, the proposed bi-level model is transformed into a single-level optimization problem, as expressed in (4.6). Thus, the commercial off-the-shell large-scale non-linear
optimization solver CONOPT3 [37] can be employed to solve this reformulated non-linear model.

\[
\text{Minimize} \quad (4.3.1) \\
\text{s.t.} \\
\text{PDISCO's problem constraints: (4.3.2) - (4.3.35)}; \\
\text{Day-ahead market problem MPPDC constraints:} \\
(4.1.2) - (4.1.11) \text{ and (4.4)}; \\
\text{Real-time market problems MPPDC constraints:} \\
(4.2.2) - (4.2.10) \text{ and (4.5).}
\]

4.3 14-bus Distribution-level Network with 9-bus Transmission-level Network

In this section, a modified 14-bus distribution-level network [42] interconnected to a 9-bus transmission-level network [43] is used to validate the effectiveness of the proposed bi-level model for deriving the PDISCO’s strategic offers/bids and markets’ performances, as shown in Fig. 4.3. To simulate individual DERs’ uncertainties, the scenario handling approach [11] is adopted to create 1000 scenarios and reduce to 15 scenarios for the case studies. This case study is also carried out on the same benchmark as described in Section 3.3.

4.3.1 Data

For the PDISCO-owned 14-bus network:

The base parameters of this 14-bus distribution network can be found in Appendix C. The capacity of the main substation $S$ and each feeder $S_{ij}$ are set to 200 MVA and 100 MVA, respectively. The voltage keeps 1 p.u. at the reference bus 1, and limits the other buses from 0.9 to 1.1 p.u.. The transformer tap ratio $\tau_i$ is imposed to 1. Each compensator is with 0-80 MVar. The wind turbines (WTs) and photovoltaic systems (PVs) are selected to represent the stochastic DERs. The mappings of the DERs and buses are WT1:11, WT2:13, WT3:9, PV1:2, PV2:7, and PV3:12. For simplicity, the capacity limit of each DER is set to 18 MVA, and the related power factor is given as 0.90/WT and 0.95/PV. The PDISCO rationally procures the DERs' portfolios and strategically trades in individual markets with 24 times per stage. The purchase price
4.3 14-bus Distribution-level Network with 9-bus Transmission-level Network

Figure 4.3: Modified 14-bus distribution-level network with 9-bus transmission-level network.

\( \lambda_{t}^{DER} \) for each DER’s generation is assumed to be unique, as shown in Table 4.1, which also includes the sale price \( \lambda_{t}^{DSD} \), and network operation cost \( C_{t}^{DS} \). The distribution-level multiplier \( \alpha_{t} \) is used to obtain the demand \( P_{DSD}^{t}, Q_{DSD}^{t} \).

For the market-operated 9-bus network:

The base parameters of this 9-bus transmission network can be found in Appendix D. The PDISCO’s network is assumed to be interconnected at transmission-level bus 4 with a power injection limit 100 MW. The line capacity is set to 500 MW. Table 4.2 shows the capacity, reserve limit and corresponding cost of each generator. The real-time regulation cost \( c_{g}^{UP} \) and \( c_{g}^{DN} \) can be regarded as the generation cost \( C_{g}^{G} \). The transmission-level multiplier \( \beta_{t} \) shown in Table 4.1 is used to get the day-ahead bidding price \( \lambda_{id}^{TSD} \) relying on the base value 45 €/MW and consumption \( P_{id}^{TSD} \) according to the base in [43]. In addition, the distribution-level and transmission-level load-shedding prices (\( \lambda_{t}^{SD}, \lambda_{t}^{ST} \)) are
considered as 200 times as $\lambda_t^{DDA}$ and $\lambda_t^{DRT}$, respectively.

Other distribution-level and transmission-level parameters can be found in [42][43]. All the price parameters mentioned above are estimated by the NordPool [39] prices.

**Table 4.1:** Essential Input Parameters for the PDISCO Trading in Markets

<table>
<thead>
<tr>
<th>$t$ [Hour]</th>
<th>$\lambda_t^{DER}$ $[€]$</th>
<th>$\lambda_t^{DSD}$ $[€]$</th>
<th>$C_t^{DS}$ $[€]$</th>
<th>$\alpha_t$</th>
<th>$\beta_t$</th>
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<td>2.4</td>
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</table>

### 4.3.2 PDISCO Trading Strategies

The PDISCO’s strategic offers and bids make up continuous curves for trading in the two-stage market, as shown in Fig. 4.4. As expected, in the day-ahead market, the PDISCO behaves as an active producer to provide productions in hours 1-9, 14-17, and 22-24, with lower offering prices (around 28 €/MW). In the rest hours, the PDISCO performs as an active consumer to acquire power with higher prices (about 32 €/MW or more).
For illustrative purposes, in a scenario-based ($\omega=1$) real-time market, sequences of the PDISCO’s strategies are also presented in Fig. 4.4. Lower prices (approx. 14 €/MW) are consistent with the power offers as in hours 1, 3, 8-9, 14-17, and 23-24. Power bids appear along with the higher prices (45-49 €/MW) at hours 2, 4-6, 10-13, 18-20, and 22. However, low prices (14 €/MW) are also conducted at hours 7 and 21 during the PDISCO’s real-time bidding process. The reason for this special phenomenon is that, the PDISCO’s trading strategies are made across the two-stages markets, implying the PDISCO’s arbitrage behavior, i.e. the PDISCO submits a strategic offer/bid to raise the system-wide upward reserve beyond the actual required and thus cleared in the day-ahead market. Subsequently, in the real-time market, when the PDISCO bids more but has not reached the artificially high system-wide upward reserve, the cleared price falls to a much lower point. Taking this price, the PDISCO can evade a certain comparatively high prices in real-time markets and thus gain more profit. Specifically, the PDISCO hands over a proper offer and a crucial bid at day-ahead hour 7 and 21, individually, to avoid the minus-profit at the corresponding real-time hours.

Coordinating the trading strategies, the PDISCO simultaneously makes rational acquisitions on the DER portfolios at the day-ahead stage. The details of the PDISCO’s procurements are shown in Fig. 4.5. Large amounts of DERs are motivated in the peak periods, i.e. hours 10-13 and 18-21. Throughout the day, WT renders relatively high outputs, since the PVs’ generations declined significantly during hours 1-4 and 23-24. Consequently, the PDISCO’s profits by trading in markets are obtained and shown in Fig. 4.6. Scenario-based ($\omega=1$) real-time profits are account for comparison with the day-ahead profits. Observe that, the day-ahead profit peaks occur consistently with the load peaks. In contrast, the real-time profits have few volatilities, which are basically recognized as minus-profits to cover the deviations of DERs’ generations, such as hours 2, 4-6, 13, 18-20, and 22. This leads to a 10% reduction in the PDISCO’s total profit. Note that hours 7 and 21 are the PDISCO’s bidding periods, while the related profits are retained at around non-negative values. These results demonstrate that the arbitrage schemes mentioned above are functional, i.e. a moderate arbitrage involved at hour 7, and a heavy arbitrage realized at hour 21.
Figure 4.4: The PDISCO’s strategic offers and bids in two-stage markets.
Figure 4.5: The PDISCO’s power procurements from individual DERs.
Figure 4.6: The PDISCO’s day-ahead profit, real-time profit and total profit.
4.3.3 Generators

The PDISCO’s offers/bids can vary the markets’ outcomes by interrelating with the transmission-level generators’ offers. For brevity, the cleared power quantities of generator 1 are taken as an illustrative example to address the PDISCO’s impacts on the markets, as shown in Fig. 4.7. The comparative situation is carried out on the transmission-level network without the PDISCO’s trading strategies and executed high real-time deviation (±10%) on each distribution-level demand.

The generation offered and reserves committed (upward and downward) at the day-ahead stage are exhibited in Fig. 4.7(a), and the real-time up and down regulation power are outlined in Fig. 4.7(b). Observe that, the PDISCO’s involvement results in a dramatic generation drop for generator 1 on day-ahead offers, in total 19% per day. In order to mitigate the unpredictable oscillations of the PDISCO’s offers/bids, the notable power increment occurs in both reserve and regulation circumstances, i.e. 192% (71%) and 553% (56%) for daily upward (downward) reserve and up (down) regulation, respectively. The peaks of the upward reserve and up regulation capacity are coincidentally launched at hour 21, corresponding to the PDISCO’s heavy arbitrage scheme indicated above as for Fig. 4.4 and 4.6. The effect of the PDISCO’s moderate arbitrage at hour 7 is not explicitly expressed, since the upward reserve and up regulation power of the generator 1 are already merged in the system-wide reserve and regulation capacity, respectively.

4.3.4 Cases

In addition, the comparison results also include the day-ahead social welfare and real-time operation cost of the markets, as shown in Fig. 4.8. The social welfare continuously declines by considering the PDISCO’s participation, daily reducing 12%. On the contrary, the cost increases 962% for the real-time market operation. In other words, the PDISCO’s strategic trading negatively impacts the markets’ objectives, which are worse than its existence as a highly fluctuating demand.

Concerning the PDISCO’s trading strategies essentially depend on the DERs’ capacity, this section further focuses on the DER’s impacts for the PDISCO’s decisions by resetting the DERs’ availability in additional cases. Thus, the discussions above are seen as Case 1. Case 2 increases the individual DERs’ capacities to two-times as higher than Case 1, while these are reduced to half in Case 3. The results of PDISCO’s profits and markets’ objectives are summarized
Figure 4.7: Cleared power quantities from markets for generator 1.
4.3 14-bus Distribution-level Network with 9-bus Transmission-level Network

Figure 4.8: Day-ahead social welfare and real-time operation cost in individual markets.
in Table 4.3. The obtained social welfare is almost the same in Case 1 and 3, but falls by 13% for Case 2. This means higher DERs’ penetration yield more PDISCO’s offers and lower social welfare in the day-ahead market. The operation cost of the real-time market increases significantly with the DERs’ capacity growth in Case 1-3. Observe that the higher DERs cannot ensure a higher profit for the PDISCO, while lower DERs definitely bring lower profit, and even render minus-profit (in Case 3). To improve the profitability for the PDISCO, appropriate portfolio procurement is the best option.

<table>
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<tr>
<th>No.</th>
<th>PDISCO profit [€]</th>
<th>Day-ahead market social welfare [€]</th>
<th>Real-time market cost (ω=1) [€]</th>
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<td>10642.9</td>
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<td>Case 2</td>
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<td>Case 3</td>
<td>-54138.2</td>
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<td>237.0</td>
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4.4 Conclusion

This chapter presents a bi-level model for a PDISCO strategically trading within the transmission-level two-stage markets, considering rational procurements of DERs’ portfolios. Crossing the markets, the continuous offers and bids are optimized to maximize the PDISCO’s profit, accompanying moderate and heavy arbitrages.

Interacted with the PDISCO’s strategies, the social welfare is maximized in the day-ahead market, and the operation cost minimization of the real-time market is achieved in each scenario ω. The proposed model is translated to a single-level solvable MPEC with the primal-dual approach. The results of the case studies reveal that the PDISCO’s strategic offers and bids are effective, and the portfolio procurements from DERs need to be appropriate.
To achieve the cost-effective use of DERs, this thesis covers the distribution-level market design for the PDISCO procurement and strategic trading in the transmission-level markets. Focusing on the short-term transactions, optimization tools are presented to exhibit the PDISCO’s market value.

5.1 Conclusion Overview

In order to address the PDISCO procurement strategies, a bi-level game-theoretic model is proposed for the PDISCO trading with DERs. Specifically, the aggregator-based DR is modeled as a resource to represent a characterized DER. The detailed numerical analyses show that the proposed methodology for the PDISCO’s procurements between real-time market exchanging and RAs’ virtual generation is effective. It should be notice that, the proposed distribution-level market framework and clearing methodology can also be extended to include the participants of various types of DGs. For instance, a market structure presented in [44] has discussed the real-time trading strategies for the PDISCO to deal with heterogeneous DG owners.
Furthermore, the investigation of the PDISCO trading strategies in the wholesale markets is launched on a two-stage framework with a bi-level structure. Crossing the day-ahead and real-time markets, the proposed model continuously renders strategic offers and bids to achieve the PDISCO’s profit maximization, according to the markets’ outcomes. Case studies also show the effectiveness of the presented approach for the PDISCO trading in the transmission-level markets. In particular, the PDISCO can also participate in individual single-stage transmission-level markets, e.g., a strategy making approach is addressed in [45] to maximize the PDISCO’s profit with demand response by trading in real-time markets.

Therefore, liberating the distribution-level market is a strong complement to the existing market architecture. PDISCO and aggregators are high-efficiency market players to stimulate individual DERs to trade across the markets. With the proposed methodologies, the interrelated trading between the PDISCO, aggregators and wholesale markets can result in a bidirectional transaction fashion.

5.2 Research Perspectives

The works presented in this thesis also feature an interesting field for the future research.

To depict the interrelations regarding the hierarchical market structure, a tri-level model is also suitable for presenting the gaming structure between the PDISCO, aggregators, and wholesale markets. However, according to the literature, current methods are not capable to formulate a solvable tri-level model, which needs the new modeling and solving approaches to get involved. Furthermore, a bi-level programming is also possible to realize the competition between aggregators and individual DERs, implying the negotiation behavior could be modeled as a vital market performance. In particular, risk aversion needs to be considered in these hierarchical programming, e.g., modeling with the conditional value at risk (CVaR).

In this thesis, one of the key assumptions is that the PDISCO is able to own and operate the distribution grid, while it may not be eligible for the European framework. However, accessing to the transmission buses, each proactive distribution grid can also be seen as a centralized renewable energy (e.g., large-scale wind or PV farms), which can be considered to integrate in the transmission-level markets. This may facilitate the proposed distribution-level market to interact with the transmission-level markets in real life. Besides, the distribution-level market between aggregators and the PDISCO can be extended to multiple
mechanisms with various clearing procedures, such as ancillary market, reserve market, and flexibility market, which could enhance the energy efficiency and power systems reliability. In addition, the proposed models for the PDISCO to interface the distribution-level and transmission-level markets are all assumed by interconnecting to the transmission-level network with only one main substation. Thus, the future developments of these models could take multiple interconnection points into account to demonstrate a more complex situation.

Although the reactive power is already concerned via AC power flow in the proposed models, the corresponding results only exhibit the impacts on the correlated active power. Since no uniform reactive power market has been acknowledged, the potential benefits of the reactive power exchanging/trading between PDISCO, DERs, and markets are not discussed in detail. To this end, the market issue regarding reactive power involvement should be investigated in the future.
The base parameters of 33-bus distribution network are shown in Table A.1.
### Table A.1: 33-bus Distribution Network

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<th>Bus $i$</th>
<th>Bus $j$</th>
<th>Resistance $R$ (Ω)</th>
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<th>Bus $j$ $P$ (kW)</th>
<th>Bus $j$ $Q$ (kVar)</th>
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Appendix B

69-bus Distribution Network: Base Parameters

The base parameters of 69-bus distribution network are shown in Table B.1-3.
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The base parameters of 14-bus distribution network are shown in Table C.1.
Table C.1: 14-bus Distribution Network

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