Quantification of the Value of Structural Health Monitoring Information for Fatigue Deteriorating Structural Systems

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ABSTRACT: This paper addresses the quantification of the value of structural health monitoring (SHM) before its implementation for structural systems on the basis of its Value of Information (VoI). The value of SHM is calculated utilizing the Bayesian pre-posterior decision analysis modelling the structural life cycle performance, the integrity management and the structural risks. The relevance and precision of SHM information for the reduction of the structural system risks and the expected cost of the structural integrity management throughout the life cycle constitutes the value of SHM and is quantified with this framework. The approach is focused on fatigue deteriorating structural steel systems for which a continuous resistance deterioration formulation is introduced. In a case study, the value of SHM for load monitoring is calculated for a Daniels system subjected to fatigue deterioration. The influence of and the value of SHM in regard to the structural system risks and the integrity management is explicated and explained. The results are pointing to the importance of the consideration of the structural system risks for the quantification of the value of SHM.

1. INTRODUCTION
Structural systems constitute a part of almost any civil infrastructure and are thus of a high societal importance. Structural systems are subjected to deterioration and thus require a continuous structural integrity management throughout their life cycle. An efficient structural risk and integrity management is crucial considering limited resources and the large extend of aged infrastructure throughout the developed part of the world (see e.g. EUROCONSTRUCT (2007) and ASCE (2013)).

This paper addresses thus the optimization of the structural risk and integrity management in the perspective of structural health monitoring (SHM) by quantifying the value of the SHM information before implementation. The developed approach building upon Thöns and Faber (2013) and Faber and Thöns (2013) takes basis in the value of SHM as the relevance and precision of SHM information for the reduction of the structural system risks and the expected cost of the structural system integrity management throughout the life cycle. The quantification of the value of SHM facilitates thus the optimization of SHM strategies by the optimization of the structural system risk and integrity management.

The approach for the quantification of the value of SHM for structural systems is developed and described in section 2. A continuous deterioration formulation of a structural system subjected to fatigue is introduced. Consecutively, the modelling of SHM information in the context of a pre-posterior decision analysis and the structural integrity management model accounting for the consequences is described. In section 3, the approach is applied to a Daniels system and the utilized probabilistic models are described in detail. The value of SHM for the considered load monitoring strategy is quantified
and documented. The paper finishes with a conclusion in section 4.

2. QUANTIFICATION OF THE VALUE OF SHM

The quantification of the value of SHM takes basis in the value of Information theory and the Bayesian pre-posterior decision theory. As introduced in Thöns and Faber (2013) and Faber and Thöns (2013), the value of SHM can be calculated through the difference between the expected value of the life cycle benefits $B_i$ utilizing SHM and the expected value of the life cycle benefits $B_0$ without SHM (Equ. (1)).

$$V = B_i - B_0$$

The expected value of the life cycle benefit $B_0$ depends on the structural performance subjected to the uncertainties $Z$ consisting of epistemic and aleatory uncertainties $Z_E$ and $Z_A$, respectively and the decision rules $d$ for adaptive actions $a$ for the structural integrity management throughout the life cycle (Equ. (2)).

$$B_0 = \max_{a,d} E_{Z_i} \left[ E_{Z_i} \left[ B(\ldots) \right] \right]$$

$$B(\ldots) = B\left( d(a,Z_E,Z_A),Z_E,Z_A \right)$$

Utilizing SHM, the expected value of the life cycle benefit $B_i$ depends additionally on the SHM strategies $s$ which deliver the uncertain SHM information $X$. The decision rules and adaptive actions for the structural integrity management ($\bar{a}$ and $\bar{d}$) are now modified to account for the SHM information. Further, the uncertainties in regard to the life cycle performance may have changed due to the observations collected through SHM and are thus denoted as $\bar{Z}_E$ and $\bar{Z}_A$ (Equ. (4)).

$$B_i = \max_{s} E_{\bar{Z}_i} \left[ E_{\bar{Z}_i} \left[ \max_{a,d} E_{X_{s},Z_i} \left[ \bar{B}(\ldots) \right] \right] \right]$$

$$\bar{B}(\ldots) = \bar{B}\left( \bar{d}(\bar{a},X,\bar{Z}_E,\bar{Z}_A),s,X,\bar{Z}_E,\bar{Z}_A \right)$$

2.1. Structural system model and deterioration model

With the structural system model the performance throughout the life cycle is calculated taking into account the deterioration. For the calculation of the system performance, i.e. the structural system reliability, diverse approaches have been developed (see e.g. Ditlevsen and Bjerager (1986), Straub and Kiureghian (2011), Lee and Song (2011) and Naess, Leira et al. (2009)). The probability of system failure can be calculated by the integration of the joint probability density of the structural performance random variables $f_X(z)$ over the domain of system failure with the cited methods (Equ. (6)).

$$P(F_S) = \int_{\Omega_Z} f_X(z) dz$$

One of the most common deterioration mechanisms represents fatigue of structural steel which is considered in the following. Fatigue maybe modelled with a fracture mechanics (FM) model which is calibrated to an SN fatigue model. The SN limit state function $g_i^{SN}$ (Equ. (7)) for the component $i$, i.e. hot spot is formulated in dependency of fatigue capacity $\Delta$, the annual number of stress cycles $\nu$, the stress ranges $\Delta \sigma_i$, and the SN curve constants $m$ and $K$.

$$g_i^{SN} = \Delta - \nu t \frac{E[\Delta \sigma_i^m]}{K}$$

The expected value of the stress ranges $E[\Delta \sigma_i^m]$ (Equ. (8)) is calculated with the model uncertainty $M$, the cut-off stress range $s_0$ and the Weibull scale parameter $\lambda$ as well as the Weibull location parameter $k$.

$$E[\Delta \sigma_i^m] = \left( Mk \right)^m \Gamma \left( 1 + \frac{m}{\lambda}; \left( \frac{s_0}{k} \right)^{\lambda} \right)$$

The FM model is described with the limit state function $g_i^{FM}$ (Equ. (9)) containing the critical
crack depth \( a_{i,e} \) and the crack depth distribution \( a_{i}(t) \) at time \( t \) for the component \( i \).

\[
g_{i}^{FM} = a_{i,e} - a_{i}(t) \tag{9}
\]

The calibrated FM model allows for reliability based inspection planning. Further, with the quantification of the crack size distribution \( a_{i}(t) \), a continuous modelling of the deterioration state is facilitated. The crack depth at year \( t \) conditional on the inspection outcomes can be calculated with the approach recently proposed by Straub and Papaioannou (In press). The algorithm can be interpreted as an enhancement of the classical rejection sampling algorithm for Bayesian updating which can be based on subset simulation (Au and Beck (2001)).

2.1.1. Coupling of the structural system and the deterioration model

The structural system resistance consists of the individual component resistances \( R_i(t) \) which are continuously reduced by the development and growth of fatigue cracks over time. This continuous deterioration state can be described with the reduction initial component resistance \( R_{i,0} \) in dependency of a resistance reduction factor \( r_R \) multiplied with the crack size \( a_{i}(t) \) to wall thickness \( d_i \) ratio, see Equ. (10). The resistance reduction factor can be determined by the crack to thickness ratio induced lost cross sectional area.

\[
R_i(t) = R_{i,0} \left( 1 - r_R \frac{a_{i}(t)}{d_i} \right) \tag{10}
\]

2.2. SHM strategies

SHM concerns the loading, the structural and/or the structural response as well as the consequence characteristics. These characteristics can be represented with analytical, empirical or semi-empirical models which are subjected to model uncertainties. The model uncertainties may be determined by means of measurements (see e.g. JCSS (2006)) which implies that SHM data contain information about the model uncertainties. In this way, yet unknown SHM data can be modeled pre-posteriorly in the context of the Bayesian decision theory. This means that the expected stress ranges for fatigue are calculated in dependency of realizations of the model uncertainties \( \hat{M} \) (Equ. (11)) accounting for the SHM uncertainty \( U \).

\[
E[\Delta\sigma_i | \hat{M}] = (\hat{M} U \kappa)^m \Gamma \left( 1 + \frac{m}{\lambda}, \frac{s_0}{k} \right) \tag{11}
\]

In the context of structural systems, the SHM system information can also be utilized for the calculation of system failure probability by utilizing the realizations of a vector of system model uncertainties \( \hat{M} \) and accounting for the measurement uncertainty (Equ. (12)).

\[
P(F_S | \hat{M}) = \int_{\Omega_{S,U}} f_{Z,U}(z,u | \hat{M}) dzdu \tag{12}
\]

2.3. Service life integrity management and risk model

The service life integrity management model builds upon the reliability based inspection and repair planning decision rule (see Faber, Engelund et al. (2000), Straub (2004) and Schneider, Thöns et al. (2013)) with the adaptive actions inspection and repair. Additionally, the risks due to component fatigue failure and system failure are calculated.

The expected life cycle benefits \( B_0 \) without utilizing SHM are the sum of the expected costs (negative expected benefits) of the componential structural integrity management \( E[C_i] \), the risk of component fatigue failure \( R_{i,DR} \) and the risk for structural system failure \( R_{FS} \) (Equ. (13)). The expected costs of the componential structural integrity management consist of the expected value of the costs of inspection \( E[C_{i,Insp}] \) and the expected value of the costs of repair \( E[C_{i,Rep}] \).
(Equ. (14)) and are calculated following Straub (2004).

\[ B_0(d(a, Z), Z) = - \left( \sum_{i=1}^n \left( E[C_i] + R_{i,D} \right) + R_{F_5} \right) \]  

(13)

\[ E[C_i] = E[C_{i,\text{Insp}}] + E[C_{i,R}] \]  

(14)

The expected inspection costs are calculated with the probability of component survival given no repair at any previous inspections \(1-P(F_{i[0,t_{\text{insp}}]} | \bar{R}_{i[0,t_{\text{insp}}]-1}]\) and the inspection costs at time \(t_{\text{insp}}\) as the sum over all inspections \(n_i\), see Equ. (15). The inspection costs are discounted to the present value, i.e. to the value at the time of the decision, with the discount rate \(r\) (Equ. (15)). The repair event \(R\) is defined as a crack indication event and a crack sizing larger than 1 mm (see Straub (2004) for details).

\[ E[C_{i,\text{Insp}}] = \sum_{t_{\text{insp}}=1}^{t_{\text{insp}}-\text{max}} \left( 1-P(F_{i[0,t_{\text{insp}}]} | \bar{R}_{i[0,t_{\text{insp}}]-1}]\right) \frac{C_{i,\text{Insp}}}{(1+r)^{t_{\text{insp}}} } \]  

(15)

The expected repair costs are calculated as the sum of the product of the joint probability of repair at inspection time \(t_{\text{insp}}\) and component survival up to year \(t_{\text{insp}}\) given no repair at any previous inspections, i.e. \(P(R_{i,\text{insp}} | \bar{R}_{i[0,t_{\text{insp}}]-1}]\) and \(1-P(F_{i[0,t_{\text{insp}}]} | \bar{R}_{i[0,t_{\text{insp}}]-1}]\), respectively, and the repair costs \(C_{i,R}\) which are discounted over the inspection times (see Equ. (16) and (17)). It is assumed that a repaired hot spot behaves like a hot spot that has no indication at the inspection (see Straub (2004)).

\[ E[C_{i,R}] = \sum_{t_{\text{insp}}=1}^{t_{\text{insp}}-\text{max}} P_{i,R} \frac{C_{i,R}}{(1+r)^{t_{\text{insp}}} } \]  

(16)

\[ P_{i,R} = P(R_{i,\text{insp}} | \bar{R}_{i[0,t_{\text{insp}}]-1}] \left( 1-P(F_{i[0,t_{\text{insp}}]} | \bar{R}_{i[0,t_{\text{insp}}]-1}]\right) \]  

(17)

The risk of component fatigue failure \(R_{i,D}\) (Equ. (18)) is calculated as the sum of the yearly individual service life risks which itself are the product of the probability of failure in year \(t\) given no repair \(P(F_{i[t]} | \bar{R}_{i[0,t]-1}]\) and the costs of the component \(C_{i,D}\) which are discounted.

\[ R_{i,D} = \sum_{t=1}^{\text{insp}} P(F_{i[t]} | \bar{R}_{i[0,t]-1}] \frac{C_{i,D}}{(1+r)^{t}} \]  

(18)

The risk of system failure \(R_{F_5}\) is calculated utilizing the annual probability of system failure \(P(F_{S[t]}\), see Equ. (6) and the discounted consequences for system failure \(C_{F_5}\):

\[ R_{F_5} = \sum_{t=1}^{\text{insp}} P(F_{S[t]} \frac{C_{F_5}}{(1+r)^{t}} \]  

(19)

The expected value of the life cycle benefit utilizing SHM \(B_1(d(a, X, Z), s, X, Z)\) is calculated similarly with the expected value of the costs for the componential structural integrity management \(E[C_{i,\text{SHM}}]\), the risk of component fatigue failure \(R_{i,D}^{\text{SHM}}\) and the risk of system failure \(R_{S}^{\text{SHM}}\) which are changing due to the different probabilistic characteristics (see section 2.2) and the costs for SHM.

\[ B_1(d(a, X, Z), s, X, Z) = - \left( \sum_{i=1}^{n_i} \left( E[C_{i,\text{SHM}}] + R_{i,D}^{\text{SHM}} + R_{S}^{\text{SHM}} \right) \right) \]  

(20)

The expected value of the costs for the component structural integrity management include now additionally the expected value of the costs for the SHM system and operation \(E[C_{i,\text{SHM}}]\) comprising the SHM system investment \(C_{i,\text{inv}}\), the installation costs \(C_{i,\text{inst}}\) and the operation costs \(C_{i,\text{op}}\) (Equ. (21)). The expected SHM system operation costs are accumulated for each service life year and are based on the probability of component survival given the SHM information \(P(D_{i[t]} | \hat{M})\) (see section 2.2) are discounted (Equ. (22)).

\[ E[C_{i,\text{SHM}}] = E[C_{i,\text{Insp}}] + E[C_{i,R}] + E[C_{i,\text{SHM}}] \]  

(21)

\[ E[C_{i,\text{SHM}}] = C_{i,\text{inv}} + C_{i,\text{inst}} + \sum_{t=1}^{\text{insp}} P(D_{i[t]} | \hat{M}) \frac{C_{i,\text{op}}}{(1+r)^{t}} \]  

(22)
3. CASE STUDY

3.1. Structural system and deterioration model
A Daniels system model is chosen for the case study as it applies to widely used redundant structural systems and accounts for its mechanical behavior (see e.g. Gollwitzer and Rackwitz (1990)).

The Daniels system (Figure 1) consists of \( n = 5 \) hot spots which are designed with fatigue design factors of 2.0 (three hot spots) and 3.0 (two hot spots). The system loading \( S \) is resisted by the components with time dependent component resistances \( R_i(t) \). Both, the loading and resistance models are subject to model uncertainties \( SM \) and \( RM \), respectively. System failure \( FS \) is then described with \( \Omega_{FS} = \{g_{FS} \leq 0\} \) and Equ. (23) for ideal ductile behavior.

\[
P(g_{FS} \leq 0) = P\left(\sum_{i=1}^{n} M_{R,i} R_i(t) - M_S S \leq 0\right) \tag{23}
\]

The loading of the Daniels system and the resistance of the components are Log-Normal and Weibull distributed with a standard deviation of 0.1, see Table 1. The probabilistic models for the model uncertainties \( M_R \) and \( M_S \) are determined in accordance with JCSS (2006).

![Figure 1: Daniels System](image)

Table 1: Probabilistic structural system model

<table>
<thead>
<tr>
<th>Var.</th>
<th>Dim.</th>
<th>Dist.</th>
<th>Exp. value</th>
<th>Std. dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_R )</td>
<td>-</td>
<td>LN</td>
<td>1.0</td>
<td>0.05</td>
</tr>
<tr>
<td>( R_{0,i} )</td>
<td>-</td>
<td>LN</td>
<td>1.0</td>
<td>0.1</td>
</tr>
<tr>
<td>( M_S )</td>
<td>-</td>
<td>LN</td>
<td>1.0</td>
<td>0.1</td>
</tr>
<tr>
<td>( S )</td>
<td>1/y</td>
<td>WBL</td>
<td>3.5</td>
<td>0.1</td>
</tr>
<tr>
<td>( r_R )</td>
<td>-</td>
<td>Det.</td>
<td>0.6</td>
<td>-</td>
</tr>
</tbody>
</table>

LN: Lognormal, WBL: Weibull

The SN fatigue resistance \( \Delta \) and the model uncertainties \( M_L \) (load calculation), \( M_\sigma \) (nominal stress calculation), \( M_{HS} \) (hot spot stress calculation) and \( M_\phi \) (weld quality) are modeled following Folsø, Otto et al. (2002), see Table 2. The location parameter \( k \) of the long-term stress distribution is scaled so that the accumulated fatigue damage after \( t = FDF \cdot t_{SL} \) years equals one applying the characteristic value for \( K \).

Table 2: Probabilistic SN fatigue deterioration model

<table>
<thead>
<tr>
<th>Var.</th>
<th>Dim.</th>
<th>Dist.</th>
<th>Exp. value</th>
<th>Std. dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta )</td>
<td>-</td>
<td>LN</td>
<td>1.0</td>
<td>0.3</td>
</tr>
<tr>
<td>( \ln K )</td>
<td>-</td>
<td>N</td>
<td>28.995</td>
<td>0.572</td>
</tr>
<tr>
<td>( m )</td>
<td>-</td>
<td>Det.</td>
<td>3.0</td>
<td>-</td>
</tr>
<tr>
<td>( k )</td>
<td>MPa</td>
<td>LN</td>
<td>Dep. on</td>
<td>FDF</td>
</tr>
<tr>
<td>( 1/\lambda )</td>
<td>-</td>
<td>Det.</td>
<td>1.2</td>
<td>-</td>
</tr>
<tr>
<td>( s_\sigma )</td>
<td>MPa</td>
<td>Det.</td>
<td>0.0</td>
<td>-</td>
</tr>
<tr>
<td>( \nu )</td>
<td>yr(^{-1})</td>
<td>Det.</td>
<td>3.0x10(^6)</td>
<td>-</td>
</tr>
<tr>
<td>( t_{SL} )</td>
<td>yr</td>
<td>Det.</td>
<td>20.0</td>
<td>-</td>
</tr>
<tr>
<td>( M_L )</td>
<td>LN</td>
<td>0.89</td>
<td>0.27</td>
<td></td>
</tr>
<tr>
<td>( M_\sigma )</td>
<td>LN</td>
<td>1.01</td>
<td>0.12</td>
<td></td>
</tr>
<tr>
<td>( M_{HS} )</td>
<td>LN</td>
<td>1.02</td>
<td>0.20</td>
<td></td>
</tr>
<tr>
<td>( M_\phi )</td>
<td>LN</td>
<td>1.02</td>
<td>0.20</td>
<td></td>
</tr>
</tbody>
</table>

LN: Lognormal, N: Normal

The FM model is based on a 2D-FM-model and a single slope Paris’ law crack growth model, see BS 7910 (2005). For simplicity identical hot spots in terms of the wall thickness and the degree of bending are assumed (Table 3). The initial crack size is modelled exponentially distributed following Moan and Song (2000).

Table 3: Probabilistic FM model

<table>
<thead>
<tr>
<th>Var.</th>
<th>Dim.</th>
<th>Dist.</th>
<th>Exp. value</th>
<th>Std. dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d )</td>
<td>mm</td>
<td>Det.</td>
<td>16</td>
<td>-</td>
</tr>
<tr>
<td>( a_e )</td>
<td>Det.</td>
<td>16</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>( DoB )</td>
<td>Det.</td>
<td>0.5</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>( r_{as} )</td>
<td>Det.</td>
<td>0.2</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>( a_0 )</td>
<td>EX</td>
<td>0.11</td>
<td>0.11</td>
<td></td>
</tr>
<tr>
<td>( \ln C )</td>
<td>N and mm</td>
<td>N</td>
<td>Cal.</td>
<td>0.77</td>
</tr>
<tr>
<td>( M_{SFF} )</td>
<td>LN</td>
<td>Cal.</td>
<td>0.1</td>
<td>-</td>
</tr>
</tbody>
</table>

LN: Lognormal, N: Normal, EX: Exponential
The expected values of the crack growth parameter and of the stress intensity factor model uncertainty are calibrated to the SN model. A correlation of the fatigue deterioration of 0.6 is assumed following on Moan (1994). Further, the component resistances including their model uncertainties are assumed to be correlated with 0.5.

### 3.2. SHM strategy

The SHM strategy consists of monitoring the system loading and thus of hot spot stresses, i.e. hot spot loading. The probability of structural system failure utilizing SHM is calculated with the realizations of the system loading model uncertainty \( \hat{M}_S \) by:

\[
P(\hat{S}_S^{\text{SHM}} \leq 0) = P\left( \sum_{i=1}^{n} M_R R_i(t) - \hat{M}_S U_L S \leq 0 \right) \tag{24}
\]

The expected values of the stress ranges for the individual hot spots are modeled conditional on the realizations of the hot spot loading model uncertainties, i.e.:

\[
E[\Delta \sigma_i | \hat{M}_L] = \left( \hat{M}_L M_{\sigma} M_{\text{HS}} M_Q U \frac{k}{k} \right)^m \Gamma \left( 1 + \frac{m}{\lambda} ; \left( \frac{s_0}{k} \right) \right) \tag{25}
\]

In Equ. (24) and (25), the measurement uncertainty \( U_L \) is introduced to account for the uncertainties associated with the observations of the structural system and the hot spot loading. The probabilistic model builds upon the quantified measurement uncertainties in Thöns (2011), see Table 4. The costs of the considered 5 channels SHM system consisting of investment, installation and operation are chosen in accordance with Thöns, Faber et al. (2014).

### 3.3. Service life integrity management model

The service life integrity management model takes basis in the reliability based inspection and repair planning at component, i.e. hot spot, level. The inspection plans for the individual hot spots are determined such that a given maximum threshold for the annual probability of component fatigue failure \( \Delta p_0 \) for each of the hot spots is maintained throughout the service life of 20 years. The inspection strategy is magnetic particle inspections (MPI) which are modelled with the parameters \( \alpha \) and \( \beta \) following e.g. Straub (2004), see Equ. (26) and Table 5.

\[
\text{PoD} = \frac{\exp(\alpha + \beta \ln(\alpha))}{1 + \exp(\alpha + \beta \ln(\alpha))} \tag{26}
\]

### Table 5: Probabilistic inspection model

<table>
<thead>
<tr>
<th>Var.</th>
<th>Dim.</th>
<th>Dist.</th>
<th>Exp. value</th>
<th>Std. dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>-</td>
<td>Det.</td>
<td>0.63</td>
<td></td>
</tr>
<tr>
<td>( \beta )</td>
<td>-</td>
<td>Det.</td>
<td>1.16</td>
<td></td>
</tr>
</tbody>
</table>

The cost model for the service life integrity management and the calculation of risks builds upon generic normalized values for the adaptive actions inspection and repair and the consequences in case of component, i.e. hot spot, fatigue failure and structural system failure (see Straub (2004) and Baker, Schubert et al. (2008)). The discounting rate is assumed to be equal to 0.05.
3.4. Value of load monitoring

The value of load monitoring (Equ. (1)) is calculated by quantifying the service life benefits $B_1$ utilizing SHM (Equ. (20)) and $B_0$ without SHM (Equ. (13)) with the structural system model accounting for the fatigue deterioration throughout the service life (sections 2.1 and 3.1). The structural integrity management is performed with four different probability of component fatigue failure thresholds $\Delta p_D$, namely $1.0 \times 10^{-2}, 3.0 \times 10^{-3}, 1.0 \times 10^{-3}$ and $3.0 \times 10^{-4}$.

The value of SHM (Figure 2) varies between 19.6 and 4.9 in dependency of the threshold. It is observed that the expected value of the service life benefit is substantially increased by SHM for high probability of component fatigue failure thresholds. For the threshold $1.0 \times 10^{-3}$, the value of SHM has its minimum because the uncertainty reduction due to SHM is counteracted by higher annual component and thus system failure probabilities (below the component fatigue failure threshold) caused by less inspections.

\[
V_{\text{insp}} = \sum_{i=1}^{n} \left( E[C_{i,\text{insp}}] - E[C_{i,\text{insp}}^{\text{SHM}}] \right) \quad (27)
\]
\[
V_{\text{rep}} = \sum_{i=1}^{n} \left( E[C_{i,R}] - E[C_{i,R}^{\text{SHM}}] \right) \quad (28)
\]
\[
V_{\text{risk}} = \sum_{i=1}^{n} \left( R_{i,D} - R_{i,D}^{\text{SHM}} \right) \quad (29)
\]

It is observed that (1) the values of SHM for the accumulated component structural integrity management are significantly lower in comparison to the value of SHM caused by the significantly higher consequences for system failure (see Table 6) and (2) that the values of SHM are positive throughout the considered thresholds. For thresholds higher than $1.3 \times 10^{-3}$, the value of SHM is caused by component risk reduction and inspection cost reduction. For thresholds lower than $1.3 \times 10^{-3}$, the value of SHM is lower and it is caused by risk, inspection and repair cost reduction.

![Figure 2: Value of SHM, the service benefits utilizing no SHM and utilizing SHM for the system](image)

Figure 3 summarizes the value of SHM in regard to the accumulated component structural integrity management due to inspections and repair and the component risks of fatigue failure, i.e. $V_{\text{SIM}} = V_{\text{insp}} + V_{\text{rep}} + V_{\text{risk}}$ (Equ.(27) to (29)).

![Figure 3: Accumulated value of SHM for components and in regard to inspections, repair and the component risks of fatigue failure](image)

4. CONCLUSIONS

This paper addresses the quantification of the value of SHM before its implementation for deteriorating structural systems on the basis of its Value of Information. The quantification of the value of SHM facilitates the optimization of SHM strategies by the optimization of the structural system risk and integrity management.
The paper focusses on the value of SHM for fatigue deteriorating systems with the SHM strategy load monitoring which provides information about the structural system and the component loading. The structural system performance is modeled by a continuous structural resistance reduction due to fatigue deterioration.

The value of SHM for load monitoring is quantified with a generic structural system formulation utilizing a ductile Daniels System. Positive values of SHM due to structural system risk reduction and lower expected cost of the structural system integrity management are calculated in dependency of the service life probability of fatigue failure threshold.

The value of SHM for load monitoring is dominated by the system risk reduction as the consequences of system failure are significantly higher than the accumulated component structural integrity management costs and risks of fatigue failure. For high thresholds, high values of SHM are calculated. For lower thresholds, the value of SHM decreases.

5. REFERENCES
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