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NUMERICAL RIGID PLASTIC MODELLING OF SHEAR CAPACITY OF KEYED JOINTS

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Abstract

Keyed shear joints are currently designed using simple and conservative design formulas, yet these formulas do not take the local mechanisms in the concrete core of the joint into account. To investigate this phenomenon a rigid, perfectly plastic finite element model of keyed joints is used. The model is formulated for second-order conic optimisation as a lower bound problem, which yields a statically admissible stress field that satisfies the yield condition in every point. The dual solution to the problem can be interpreted as the collapse mode and will be used to analyse the properties of the different joints. This approach makes it possible to investigate both global and local failure mechanisms in keyed joints. The results of the model will be compared to experiments found in the literature as well as the current design rules of the Eurocode.

Keywords: Finite Element, Keyed Joints, Limit Analysis, Numerical Method, Precast Concrete Elements, Rigid Plasticity

1 Introduction

Construction using precast concrete elements has many advantages over conventional in-situ cast concrete buildings. The precast elements are cast and cured in a controlled environment and the construction phase is less labour intensive, however, the use of precast elements also poses new challenges because the elements have to be connected by in-situ cast joints. The joints are composed of two interfaces and the concrete core, which is often reinforced in both transverse direction (U-bars or similar) and the longitudinal direction (locking rebar). This study will focus on the joints found in shear walls, where it is assumed that only in-plane forces are present.

The shear walls are an essential part of the structural system and the horizontal loads, e.g. wind or seismic action, are transferred from the façades and deck slabs via the shear walls to the foundations. The shear capacity of the joints located in these shear walls are of the utmost importance to the load capacity of the entire structure. During the 70s and 80s several researchers conducted experiments to investigate the behaviour and mechanisms of such joints (see Cholewicki, 1971; Fauchart & Cortini, 1972; Bhatt, 1973; Bljuger, 1976; Hansen & Olesen, 1976; Chakrabarti, Bhise, & Sharma, 1979; Rizkalla & Serrette, 1988; Rizkalla, Serrette, Heuvel, & Attigbe, 1989). In present practice structural engineers rely on simple empirical – often very conservative – design formulas, where only sliding failure of the interface and complete crushing failure of the concrete core are taken into account. The experiments showed that local effects in the concrete core also may affect the shear capacity:
Two experiments by Hansen & Olesen (1976) with similar reinforcement degree yielded significantly different results due to the reinforcement layout – a phenomenon not accounted for in code design formulas such as the one in the Eurocode 2 (European Committee for Standardization, 2005) simply neglects. In the tests by Hansen & Olesen, the transverse loop reinforcement (U-bars) was placed with a mutual distance, which introduced local compression struts in the concrete core. This particular mechanism will be investigated in this paper.

In recent times the keyed joints as well as the local mechanisms of the concrete core have received some attention by researchers – mainly due to the increased use of in-situ cast joints in bridge decks (see Issa & Abdalla, 2007; Joergensen & Hoang, 2013). Joergensen & Hoang investigated the behaviour of joints loaded in tension, and the article presents experiments, which show local mechanisms, as well as an upper bound solution (based on yield lines) for the observed behaviour. Nielsen & Hoang (2010) also covers rigid, perfectly plastic solutions of joints and a lower bound solution to the shear capacity of keyed joint is presented. An enhanced version of this lower bound solution is presented by Ismaili (2014), where both compression struts and friction struts are taken into account. This lower bound solution provides a simple tool, superior to the design formulas of the Eurocode, but the local mechanisms of the core are not accounted for.

This paper presents a detailed rigid, perfectly plastic finite element model of keyed joints, where the local effects in the core caused by the transverse reinforcement are taken into account. The model is based on the lower bound theorem, which states that if a stress field satisfies equilibrium and does not violate the yield condition in any point, the stress field is safe and will not cause collapse. For the model an enhanced version of a triangular disk element with a linear variation of stresses originally presented by Poulsen & Damkilde (2000) and Sloan (1988) is used, and a bar element with quadratic stress distribution is used to model the reinforcement (see Poulsen & Damkilde, 2000). The primal solution to the problem gives the statically admissible stress field, while the dual solution, i.e. the corresponding upper bound problem, can be interpreted as the collapse mode.

The formulation of numerical methods for assessment of the load carrying capacity of structures of rigid plastic material is usually credited to Anderheggen & Knöpfel (1972). Manual methods for limit analysis have been available for more than 70 years. For instance the idea of yield line theory originates from the work of A. Ingerslev and K. W. Johansen in the 20s and 30s (see Johansen, 1962). The manual methods provide the structural engineer with a general tool for assessment of the ultimate limit state capacity of structures, but the results of these manual methods are very dependent on the intuition and skill of the individual structural engineer. As the structural systems become more complex the manual methods will often yield a result far from the actual capacity. The numerical methods are based on a discretisation known from the finite element method, but numerical limit analysis is formulated as an optimisation problem rather than a set of linear or non-linear equations. For a detailed description of the formulation of numerical limit analysis the reader is referred to Anderheggen & Knöpfel (1972), Sloan (1988, 1989), or Krenk, Damkilde, & Høyer (1994).

2 Problem formulation

The lower bound formulation of numerical limit analysis consists of a set of equilibrium equations, yield conditions, and an object function. The equilibrium equations can be stated as follows:

\[ \mathbf{H} \beta = R_0 + R \lambda \]  

(1)

where \( \mathbf{H} \) is the global equilibrium matrix, \( \beta \) is a vector containing the stress variables. The loading consists of a constant part \( R_0 \) and a scalar part \( R \) which is multiplied by the load factor \( \lambda \). The global equilibrium matrix contains contributions from each element, and the number of columns (i.e. number
of stress variables) will on a system level usually exceed the number of rows (i.e. number of equilibrium equations), thus, the structural system is statically indeterminate and plastic redistribution of stresses is possible. For the stress field to be statically admissible, the yield condition must be satisfied for every point:

\[ f(\beta_i) \leq 0, \quad i = 1, 2, \ldots, p \]  

(2)

where \( f \) is the yield function and \( p \) is the number of sets of stress variable, which depends on the chosen element and number of elements. The yield criterion is generally non-linear, but convex. Many researchers have linearised the yield condition to fit the format of linear optimization (also known as linear programming or LP), and the disk element used in this study was originally formulated with a yield condition consisting of several hyper planes. Second-order cone programming (SOCP), a generalisation of LP, allows for second-order terms in the inequalities, and many yield conditions can be formulated exactly without any approximations using SOCP. The modified Mohr-Coulomb yield condition for plane stress and plane strain, which is used in this study, can be formulated as second-order conic inequalities. For a detailed description of SOCP the reader is referred to Boyd & Vandenberghe (2004).

The goal of load optimisation in limit analysis is to maximise the load factor \( \lambda \). The objective function of the optimization problem can be written as:

\[ f_0(x) = c^T x = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \beta \\ \lambda \end{bmatrix} = \lambda \]  

(3)

Both LP and SOCP requires a linear object function, thus, (3) can be used for both types of optimisation. The complete optimisation problem can be written as:

\[
\begin{align*}
\text{maximize} & \quad \lambda \\
\text{subject to} & \quad H \beta - R \lambda = R_0 \\
& \quad f(\beta_i) \leq 0, \quad i = 1, 2, \ldots, p
\end{align*}
\]  

(4)

The problem (4) can be solved efficiently using primal dual interior point algorithms, a class of algorithms originally proposed by Karmarkar (1984) for linear programming, and later generalised to many types of convex optimisation. Andersen, Roos, & Terlaky (2000) describes the algorithm in details including many of the numerical tricks involved. The commercial solver MOSEK (2013) developed by E. D. Andersen and K. D. Andersen is used for this study.

3 Interface element

The in-situ cast joint consists of two interfaces and a concrete core as mentioned in the introduction. The core can be modelled using the triangular disk elements, while the interfaces require a new, one-dimensional element, which can be placed between disk elements to limit the stresses that can be transferred. In such interface two stress components (\( \sigma_n \) and \( \tau_{nt} \)) are present, both with a linear distribution. To reflect this, the element requires four stress variables, two at each end.
\( \sigma_{ni} \) is the normal stress associated with node \( i \) normal to the interface and \( \tau_i \) is the shear stress associated with node \( i \). All four stress variables seen on Fig. 1(a) are given in local coordinates, thus, no transformation is needed for the equilibrium equations and the generalised nodal forces \( q \), Fig. 1(b), can be written as:

\[
q = \begin{bmatrix}
q_{\sigma_1}^+ \\
q_{\tau_1}^+ \\
q_{\sigma_2}^+ \\
q_{\tau_2}^+ \\
q_{\sigma_1}^- \\
q_{\tau_1}^- \\
q_{\sigma_2}^- \\
q_{\tau_2}^-
\end{bmatrix}
= \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & -1 \\
-1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\sigma_{n1} \\
\tau_1 \\
\sigma_{n2} \\
\tau_1
\end{bmatrix}
= \mathbf{h} \beta_e
\]

(5)

\( \mathbf{h} \) is the local equilibrium matrix and \( \beta_e \) is a vector containing the stress variables of element \( e \). Since the stress distribution is linear it is only necessary to impose the yield criterion at the nodes to ensure a statically admissible stress distribution. A suitable yield condition for the interface of joints is discussed in Nielsen & Hoang (2010). The yield condition corresponds to a modified Mohr-Coulomb for plane stress with one free normal stress. In principal stresses the yield condition can be written as:

\[
\sigma_1 \leq f_A \\
k \sigma_1 - \sigma_2 \leq 2c/\sqrt{k}
\]

(6)

where \( \sigma_1 \) and \( \sigma_2 \) are the largest and smallest principal stress, respectively. \( f_A \) is the tensile strength of the interface, \( c \) is the cohesion, and \( k \) is a friction parameter defined as \( k = \left( \mu + \sqrt{1 + \mu^2} \right)^2 \), where \( \mu \) is the friction coefficient; usually \( \mu = 0.75 \) is used for monolithic concrete, corresponding to a friction angle of \( \varphi \approx 37^\circ \). The non-linear yield condition (6) can be formulated in terms of \( \sigma_n \), \( \sigma_c \), and \( \tau \) as a second-order conic constraint, suitable for second-order cone programming.

4 Model, results and discussion

This section presents the geometry of keyed joints, experimental results, and corresponding numerical models. The joints consist of evenly spaced keys with a spacing \( h_1 \) and a length \( h_2 \). The depth of
the keys is $d$, the core of the joint has a height $b$ and the U-bars have a overlap $o$, and finally $u$ is the mutual distance between a pair of U-bars. All measurements are shown in Fig. 2.

![Diagram](image_url)

**Fig. 2:** Typical geometry of keyed joints and notation: Low value of $u$ (a); larger value of $u$ (b).

Fig. 2 shows two joints with different values of $u$. Experiments by Hansen & Olesen (1976) suggest that the joint with the larger value of $u$, Fig. 2(b), will have a smaller shear capacity due to local mechanisms caused by the loop reinforcement.

The numerical model consists of three types of finite elements, namely the enhanced triangular disk element (Poulsen & Damkilde, 2000), the bar element (Poulsen & Damkilde, 2000), and the interface element. The enhanced disk element, originally formulated for linear programming, uses the exact formulation of the yield condition. The enhanced version consists of three subelements - each with a linear stress variation. This improves the dual solution significantly (Nielsen, 2014).

The yield condition of the joint concrete is given by the modified Mohr-Coulomb yield criteria for plane stress under the assumption of a tensile strength of zero. The tensile strength of the interfaces is also chosen as zero. Based on the experiments found in the literature, a friction coefficient of $\mu = 0.75$ and a cohesion between 0 and 0.5 MPa seems to give the best results. Nielsen & Hoang (2010) suggests a friction coefficient of 0.75 and a cohesion of $0.55 \sqrt{f_c}$, but the preliminary results of the numerical model indicates a lower value, as $c = 0.55 \sqrt{f_c}$ seems to be on the unsafe side for the experiments used in this study. The effectiveness factor $\nu$ is chosen as 1, i.e. $\nu f_c = f_c$. Finally, the yield condition of the bar elements for the reinforcement is described by the yield strength and cross section area. It is assumed that the reinforcement only carries tension.

![Diagram](image_url)

**Fig. 3:** Sketch of the numerical model including boundary conditions and loading.

The numerical model consists of two infinitely strong precast elements and a keyed joint, see Fig. 3. 19040 disk elements are used for the modelling of the experiments by Hansen & Olesen and 7520 disk elements for the experiments by Fauchart & Cortini. The precast elements are loaded by two normal forces each, which ensures that the moment at the center of the keyed joint is zero. The bottom boundary of the bottom precast element is fully supported.
The assumption of plane stress is of course not accurate close to the U-bars. The three-dimensional nature of the loop reinforcement will create a triaxial stress state, which may increase the capacity of the joint concrete due compression in three directions. Some of the results presented in the following subsections will be somewhat conservative as the local (two-dimensional) mechanisms are governing the shear capacity.

4.1 Comparison between experiments and numerical model

Hansen & Olesen (1976) presented 16 experiments, out of which 10 can be classified as deck joints where the usual transverse loop reinforcement is replaced by rebars located outside the joint, thus, no local mechanisms are be present. The remaining 6 specimens can be classified as wall joints with U-bars as transverse reinforcement and local effects can occur. Several different reinforcement layouts were tested for the wall joints, and Hansen & Olesen reported that specimens 24 and 26 (see Fig. 4) yielded lower shear capacities than specimens 23 and 25, despite almost identical material characteristics, and that the failure mode of specimens 24 and 26 included almost complete destruction of the concrete core of the joint.

Fig. 4: Specimens tested by Hansen & Olesen, measurements in millimetres: a) shows the deck joint and the transverse reinforcement located at outside the joint, b) to g) show the wall joints reinforced with U-bars. The mutual distance between the U-bars of a pair is 10 mm unless specified otherwise.

Fig. 4 shows the six different reinforcement layouts of the wall joints as well as the deck joints. All joints have a total of 14 keys (and two half keys at the ends) and a total length of 1200 mm. The keys have a depth of \( d = 6 \) mm and a length of \( h_2 = 40 \) mm. The spacing between the keys is \( h_1 = 40 \) mm, and the height of the joint is \( b = 50 \) mm. All joints and precast elements have a out-of-plane thickness of \( t = 50 \) mm.

The following figures presents the load capacity given as \( \tau/f_c \) as a function of the mechanical reinforcement degree, \( \Phi \), defined as:

\[
\Phi = \frac{\sum A_s f_y}{l_t f_c}
\]

(7)
where $A_k$ is the reinforcement area, $f_y$ is the reinforcement strength, and $l$ is the length. The shear stress discussed in this section is the average shear stress, i.e., the net shear force over the total area.

For a given reinforcement configuration, the strength of the reinforcement is varied to obtain a shear capacity curve. The capacity curves can generally be divided into three parts: For low values of $\Phi$, the curve is non-linear with a steep slope, and the failure mode involves shearing off the keys completely. As the reinforcement degree increases, the curve becomes linear with a lower slope and the failure mode is now a combination of sliding and local crushing in the keys. Finally, the curve will reach a horizontal plateau, and no further increase of the load carrying capacity is possible. The concrete core of the joint is now failing - either due to local mechanisms caused by the reinforcement, or the absolute maximum of $\tau = f_c/2$ has been reached.

The results of the testing of the deck joints appear to be somewhat scattered and three specimens with widely different reinforcement degree (between 0.043 and 0.188) yielded almost similar capacity, see Fig. 5(a). The choice of $\mu = 0.75$ and a low cohesion seems to give reasonable agreement, especially for lower values of $\Phi$. As the reinforcement is placed outside the joint, no local effects will occur, and in the plotted domain the plateau is not reached. From Fig. 5(a) it can be observed that even under the assumption of a cohesionless interface, a considerable *pseudo-cohesion* is present (approximately 0.06 $f_c$) due to the keys where local crushing of the corners occurs.

Fig. 5(b) shows the shear capacity curves for specimens 23 and 24. A clear difference between the two curves is observed and the model for specimen 24 yields a significantly smaller shear capacity. The stress distribution and collapse mode suggest that this is due to the local mechanisms caused by the mutual distance between the loop reinforcement (10 mm for 23 and 170/150 mm for 24, see Fig. 4).

![Graphs showing predicted and experimental shear capacity results for deck and wall specimens.](image)

**Fig. 5**: Predicted shear capacity by the rigid plastic finite element model and experimental results of the deck specimens (a) and wall specimens 23 and 24 (b) from Hansen & Olesen (1976): a) The model uses the average concrete strength for the joint concrete, and the yield condition of the interface is defined by $\mu = 0.75$ and $c = 0$ or 0.50 MPa. b) A friction coefficient of $\mu = 0.75$ and a cohesion of $c = 0.50$ MPa is used.

The collapse mechanism can be illustrated using the variables of the dual optimisation problem. These dual variables correspond to the upper bound problem and can be interpreted as deformations (Krenk et al., 1994). Fig. 6(a) shows that the predicted collapse mechanism of specimen 23 involves crushing of the keys and yielding of a small area between the loop reinforcement. For specimen 24, Fig. 6(b), large, skew yield lines throughout the entire concrete core is present, and the mechanism can be described as a *rigid block mechanism*. 

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*Note: The diagrams and graphs are placeholders for actual images.*
Fig. 6: Collapse mode of specimen 23 (a) and specimen 24 (b).

Fig. 7(a) shows that the two different reinforcement configurations only gives a minor difference in the shear capacity of the joints. It is also observed that this difference becomes more pronounced as the reinforcement ratio increases. Fig. 7(b) shows the shear capacity for two specimens where all U-bars are placed with a mutual distance of 10 mm. It is seen that specimen 27 reaches the horizontal plateau earlier as the 11 U-bars of specimen 27 will have a higher individual strength than the 15 U-bars of specimen 28 for the same reinforcement degree. For specimen 28 a significant difference between the experimental results and the numerical model is observed. In this context it is noted that the two-dimensional model cannot capture the actual three-dimensional behaviour, and the local mechanisms becomes the governing mechanism in the numerical model.

![Graph](image)

Fig. 7: Shear capacity curves predicted by the rigid plastic joint model using $\mu = 0.75$ and $c = 0.50$ MPa: a) Curves for specimens 25 and 26, b) curves for specimens 27 and 28.

Fig. 8 shows that the collapse modes of specimens 25 and 26 are similar to specimens 23 and 24, respectively. Specimen 25 fails by sliding in the interface and local crushing of the keys as well as local yielding between the U-bars. Specimen 26 displays a rigid block mechanism involving several blocks with the shape of a parallelogram.
Fig. 8: Collapse mode of specimen 25 (a) and specimen 26 (b).

The collapse mode of specimens 27 and 28, illustrated on Fig. 9, involves sliding of the interface and local crushing of the keys as well as local yielding between the U-bars. For specimen 27 the governing mechanism is the sliding and local crushing of the keys, while for specimen 28 the local effects are the critical mechanism. This can also be seen from the shear capacity curves (Fig. 7(b)), where specimen 28 has reached the horizontal plateau.

Fig. 9: Collapse mode of specimen 27 (a) and specimen 28 (b).

Fauchart & Cortini (1972) presented 10 experiments of keyed shear joints, all with loop reinforcement and a concrete compressive strength of 20 MPa. The specimens have 6 keys and a total length of 1500 mm. The keys have a depth of 20 mm, a length of 83 mm and the spacing between the keys are 167 mm. The joints were significantly wider than the ones used by Hansen & Olesen and have a width of $b = 145$ mm. The out-of-plane thickness is 90 mm. A friction coefficient of $\mu = 0.75$ and a cohesion of $c = 0$ for the interface are used in the numerical model.

Fig. 10 shows the shear capacity curve for varying mechanical reinforcement degree. The curve shows the three phases mentioned earlier: The first part of the curve has a steeper slope and the specimens fails when the keys are sheared off. The next part of the curve is almost linear and involves sliding in the interface and local crushing of the keys. The last part (seen around $\Phi = 0.25$) involves failure of the core of the joint, i.e. local yielding in-between the U-bar pairs. The curve captures the experimental tendency well and gives a good prediction of the shear capacity of such joints.

To sum up all the results presented in this section, all experimental results are plotted against the shear capacities predicted by the numerical model. Fig. 11 shows that most of the data points are close to the solid line ($\tau_{\text{model}} = \tau_{\text{test}}$). The majority of the points are slightly above the solid line, but a single experiment stands out (indicated by the circle). It is one of the deck specimens presented by Hansen & Olesen, and the only deck specimen with such high reinforcement degree. As mentioned previously, the results of the deck specimens appear to be somewhat scattered, and in the case of this
very high reinforcement degree, other parameters of the experimental set-up might have affected the results.

4.2 Comparison between the Eurocode and the numerical model

This subsection compares the current design rules of the Eurocode to the presented numerical model. The Eurocode approach is of course developed to yield a safe estimate of the shear capacity and it is expected that the numerical model will predict a larger shear capacity. A compressive concrete strength of 30 MPa is used and a tensile strength of zero is assumed in the numerical model. The cohesion used in the Eurocode approach is proportional to the concrete tensile strength, which is estimated to be $f_t = 0.21 f_c^{2/3} = 2.03$ MPa for this study. The Eurocode assumes that stresses only are transferred at the keys, and to get the average shear stress over the entire the following formula is
used:

\[
\tau = \min \left\{ c f_t A_{\text{key}} / A + \mu \Phi f_c, \frac{1}{2} \nu f_c A_{\text{key}} / A \right\}
\]

(8)

For a keyed joint the Eurocode uses a friction coefficient of \( \mu = 0.90 \) and a cohesion parameter of \( c = 0.50 \). \( \nu \) is the effectiveness factor and given as \( 0.7 - f_c/200 = 0.55 \). The numerical model uses a friction coefficient of \( \mu = 0.75 \) and a cohesion of 0.50 MPa based on the experimental data. A geometry similar to the one used by Hansen & Olesen (1976) is used for this comparison (where \( A_{\text{key}} / A = 1/2 \)). Two different cases are analysed: A joint with external reinforcement (like the deck specimens), where no local effects occur, and a joint with loop reinforcement, where local effects may limit the shear capacity depending on the reinforcement configuration.

Fig. 12: Shear capacities predicted using the numerical method and the design formulas of the Eurocode: a) External reinforcement and varying key depth, \( d \). b) Loop reinforcement and varying reinforcement configuration, \( u \).

Fig. 12(a) shows that the numerical model predicts a significantly larger shear capacity of the joint. For lower reinforcement degrees this difference is more pronounced and it is worth noting that the Eurocode formula does not capture the effect of deeper keys which increase the pseudo-cohesion. It is also seen that the horizontal cut-off of the Eurocode severely underestimates the limit of the shear capacity. Fig. 12(b) shows that the Eurocode formula generally yields lower capacity compared to the numerical model, even when the local effects become governing according to the numerical model. The Eurocode does not take these local phenomenons into account, yet the conservative nature of the approach will yield a safe estimate of the shear capacity. It is noted that the Eurocode underestimates the pseudo-cohesion of the joints and compensates by an increased friction parameter.

The Eurocode design formula have been used to predict the shear capacity of the experiments by Fauchart & Cortini (1972) and Hansen & Olesen (1976). Fig. 13(a) shows that the Eurocode generally underestimates the shear capacity, in some cases by more than 50%. Only a single experiment shows a lower capacity than predicted by the Eurocode (same experiment as the one indicated by the circle on Fig. 11). Fig. 13(b) compared the Eurocode design formula to the numerical model, an a pattern almost similar to Fig. 13(a) is observed; the Eurocode predicts a significantly lower shear capacity than the numerical model, and in no cases does the Eurocode predict a larger shear capacity than the numerical model.
Fig. 13: Shear capacities predicted using the Eurocode compared to the experimental testing (a) and the numerical model.

5 Conclusion

The mathematical framework for rigid, perfectly plastic limit analysis has been presented and a new interface element for modelling the interface between precast concrete elements and in-situ cast joints has been introduced. The model is based on the lower bound method, which yields a safe and statically admissible stress field. An enhanced version of a triangular disk element is used for modelling of the concrete, and a bar element is used to model the reinforcement. The interface and concrete is modelled using a modified Mohr-Coulomb yield condition, which can be formulated as a second-order cone and the resulting optimisation problem can be solved efficiently using second-order cone programming.

The numerical model is compared to several experiments found in the literature. The model predicts the shear capacity and failure mode reasonably well and it captures some of the local mechanisms found in the in-situ cast joint caused by the loop reinforcement. The model shows that the configuration of the loop reinforcement affects both the capacity of the joint as well as the failure mode. It is observed that the assumption of plane stress may be conservative and does not capture the actual triaxial stress state created by the loop reinforcement.

Finally the numerical model is compared to the current design formula of the Eurocode and it is concluded that the Eurocode provides safe, but often very conservative, estimates of the shear capacity of joints - especially for low reinforcement degrees. In the case of the local mechanisms caused by the U-bars, the Eurocode still yields a safe estimate compared to the numerical model due to the horizontal cut-off. It is also observed that the Eurocode severely underestimates the pseudo-cohesion caused by the keys.

The larger scope is to develop tools and methods for modelling entire structures of precast concrete elements within the presented framework. For this to be feasible, a generalisation of the observed behaviour of the joint is needed, i.e. a special joint finite element that can capture the general behaviour as well as the local mechanisms.
References


