Numerical Modeling of Shoreline Undulations

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Publication date: 2011

Document Version
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Numerical Modeling of Shoreline Undulations

PhD Thesis

Kasper Kærgaard
September 2011
Preface

This thesis is submitted as part of the requirement for obtaining a Ph.D. degree from the Danish Technical University. The study has been conducted at the Danish Technical University, Department of Mechanical Engineering, Section for Coastal, Maritime and Structural Engineering under supervision of Prof. Jørgen Fredsøe. The work has been financed partly by the Danish Technical University and partly by DHI, who have also supplied their software package Mike21FM free of charge for use in the numerical modeling. Both of these institutions are thanked for their support.

Two research visits have been conducted during the work, one visit to the Danish Coastal Authorities (in danish Kystdirektoratet) who were helpful in both providing an office space, field data and relevant technical discussions. Regard is given to Søren B. Knudsen who especially helped with obtaining data and Per Sørensen whose office I intruded while visiting.

The second research visit was to Prof. A.B. Murray at Duke University, NC. in the United States. Prof. Murray is thanked for valuable theoretical discussions and for providing an office space during the visit.

Special regard is given to my supervisor Prof. Fredsøe. His distinctive style of supervising, where one is guided in the right direction without being told the details in what to do, has made work on the thesis a joy most of the time and has surely increased my own benefit from working on the thesis. My co-supervisor Ph.D. Rolf Deigaard is thanked for pointers at a few meetings we had in the beginning of the work on the thesis.

I have for the past 7 years been privileged to work closely together with Ph.D. Niels G. Jacobsen. We have had many hours of technical and theoretical discussions, and the present thesis would have been worse without these discussions.

My beautiful, fiancée Anne-Sophie also deserves a big thank you on these pages. This is for letting me off the hook at home in the past months so I could focus more of my time on writing the present thesis, and for listening to all my blablarings about misbehaving shoreline models during the last three years.

Lastly I want to thank both my parents for always supporting me and helping me, and especially my dad for proof reading the thesis.
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Abstract

The present thesis considers undulations on sandy shorelines. The aim of the study is to determine the physical mechanisms which govern the morphologic evolution of shoreline undulations, and thereby to be able to predict their shape, dimensions and evolution in time. In order to do so a numerical model has been developed which describes the longshore sediment transport along arbitrarily shaped shorelines. The numerical model is based on a spectral wave model, a depth integrated flow model, a wave-phase resolving sediment transport description and a one-line shoreline model.

First the theoretical length of the shoreline undulations is determined in the linear regime using a shoreline stability analysis based on the numerical model. The analysis shows that the length of the undulations in the linear regime depends on the incoming wave conditions and on the coastal profile. For larger waves and flatter profiles the length of the undulations increases.

Secondly the evolution of the shoreline undulations from the linear regime to the fully non-linear regime is described using the numerical shoreline evolution model. In the fully non-linear regime down drift spits and migrating shoreline undulations are described by the model. The shoreline evolution is considered for both constant and varying wave forcing and both periodic model domains with a single undulation as well as periodic model domains with multiple undulation are considered. Three different shoreline shapes are found depending on the wave conditions and the coastal profile: undulations with no spits, undulations with flying spits and undulations with reconnecting spits. It is further shown that the evolution of the shoreline undulations is governed mainly by the angle between the shoreline and the incoming waves and the curvature of the shoreline.

Thirdly the shoreline evolution model is tuned to two naturally occurring shorelines. On one of the shorelines, the west coast of Namibia, the shoreline model is able to describe the observed shoreline features in both a qualitative and quantitative way. The model over-predicts the scale of the feature and under predicts the migration speeds of the features. On the second shoreline, the shoreline model predicts undulations lengths which are longer than the observed undulations.

Lastly the thesis considers field measurements of undulations of the bottom bathymetry along an otherwise straight coast at the Danish West Coast.
Two bathymetric datasets and two time series of wave measurements are used in order to determine the following properties: The offshore extent of shoreline undulations, the amount of sediment transported alongshore in the shoreline undulations, the relationship between the shoreline undulations and longshore bars and the relationship between the morphology and the hydrodynamics. In one of the data sets the shoreline undulations are well correlated with undulations on the depth contours between -5 m and +2 m relative to mean sea level. An analysis of the wave climate shows that this shoreline is right at the limit between a stable and an unstable shoreline.
Dansk Resumé


Til sidst analyseres et datasæt fra den danske vestkyst hvori der observeres undulationer på både kystlinien og i kystprofilet på en ellers lige kystlinie.
To bathymetriske datasæt samt to tidsserier af bølgemålinger benyttes til at bestemme følgende egenskaber: Udstrækningen af kystundulationerne i kystprofilet, volumenet af sand som kystundulationerne transporterer på langs af kysten, forholdet mellem kystundulationerne og de langsgående revler samt forholdet mellem kystmorfologien og hydrodynamikken. På den ene kyststrækning kan kystundulationerne ses i kystprofilet mellem +2 og -5 m vanddybde i forhold til middelvandsspejlet. En sammenligning mellem observationerne og en stabilitetsteori for kystlinien viser at vi er lige på grænsen mellem en stabil og en ustabil kyst.
## List of Symbols

### Greek Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>Angle between the incoming wave direction and the normal to the shoreline</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Direction in which the front is advanced in the mesh generation routine.</td>
</tr>
<tr>
<td>$\alpha_p$</td>
<td>Phase between undulation at off-shore contour ($\Delta y$ from the shoreline) and undulation on shoreline.</td>
</tr>
<tr>
<td>$\alpha_s$</td>
<td>Wave steepness.</td>
</tr>
<tr>
<td>$\alpha_{crest}$</td>
<td>Migration direction of the crest of the undulation.</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Angle between the actual migration direction of the spit and the theoretical migration direction of the spit when the shoreline curvature is ignored.</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Slope of beach profile.</td>
</tr>
<tr>
<td>$\Delta \alpha$</td>
<td>Direction interval</td>
</tr>
<tr>
<td>$\Delta h$</td>
<td>Distance normal to the shoreline.</td>
</tr>
<tr>
<td>$\Delta n$</td>
<td>Movement of shoreline edge.</td>
</tr>
<tr>
<td>$\Delta s$</td>
<td>Length of shoreline edge.</td>
</tr>
<tr>
<td>$\Delta t$</td>
<td>Time step size.</td>
</tr>
<tr>
<td>$\Delta T_{MWD}$</td>
<td>Time step determining how often a new mean wave direction is determined when running with a variable wave climate.</td>
</tr>
<tr>
<td>$\Delta x$</td>
<td>Length of edge projected onto $x$-direction.</td>
</tr>
<tr>
<td>$\Delta y$</td>
<td>Distance from shoreline to off-shore contour.</td>
</tr>
</tbody>
</table>
CONTENTS

\( \Delta y \) Length of edge projected onto \( y \)-direction.
\( \epsilon \) Surf scaling parameter, used to characterize beach.
\( \eta \) Surface elevation.
\( \gamma \) Diffusivity in Laplace equation for mesh movement.
\( \gamma \) Ratio of wave height and water depth when waves are breaking, \( \gamma = H_b/D \).
\( \kappa \) Curvature of shoreline.
\( \nabla \) Gradient operator, \( \nabla = \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y} \right) \).
\( \nu \) Kinematic viscosity of water.
\( \Omega \) Dean parameter, used to characterize beach state.
\( \omega \) Wave frequency.
\( \rho \) Density of water.
\( \rho_a \) Density of air.
\( \sigma \) Growth rate of undulation.
\( \sigma \) Sediment grain grading coefficient.
\( \sigma \) Standard deviation of the wave height.
\( \tau_b \) Bottom shear stress.
\( \theta \) Angle between vectors in torsion spring system
\( \theta \) Direction coordinate.
\( \theta_c \) Critical Shield parameter.
\( \theta_i \) Phase of the \( i \)th sinusoidal component.

**Roman Symbols**

\( A \) Area.
\( A \) Fraction of incoming waves coming from either left or right.
\( A \) Steepness parameter in Dean profile.
\( a \) Amplitude of undulation.
Distance the advancing front is projected towards the interior of the domain. Determines the cross-shore discretization of the mesh.

Breaker amplitude.

Polygon

Amplitude of the i’th sinusoidal component.

Courant number.

Alongshore migration speed.

Sediment grain drag coefficient.

Drag coefficient due to bottom friction.

Water depth.

Diameter of sediment.

Node with bisectional line $L_i$, in torsion spring system.

Median sediment grain diameter.

Closure depth, the largest depth of the active part of the coastal profile.

Reference grain diameter.

Area of i’th element.

Area of i’th element projected onto the vertical plane which is parallel to the shoreline edge.

Vector representing shoreline node movement.

Shortest distance to shoreline.

Vector representing the correction to the node position.

Vector representing shoreline edge movement.

Directional Spreading Index, represents the direction spreading of the waves, a large value mean a low directional spreading.

Element in mesh.

Energy of torsion spring system.

East coordinate in UTM system.
\( e_r \) Volume error.
\( f \) Unknown function.
\( f^* \) Filtered value of unknown function 
\( g \) Gravitational acceleration.
\( H_b \) Breaking wave height.
\( H_s \) Significant wave height.
\( H_{sm} \) Average significant wave height.
\( k \) Spring constant in torsion spring system.
\( k_s \) Bed roughness length.
\( L \) Length of undulation.
\( L_0 \) Off-shore wave length.
\( L_i \) Bisectional line of internal angle at node \( D_i \), in torsion spring system.
\( M \) Manning number.
\( m \) Parameter in the Dean profile, in this work equal to \( 2/3 \).
\( MSL \) Mean sea level.
\( MW D \) Mean wave direction.
\( MW D_m \) Average mean wave direction.
\( n \) North coordinate in UTM system
\( n \) Number of nodes on shoreline.
\( n \) Number of nodes surrounding node being smoothed in the torsion spring system.
\( n \) Shoreline normal coordinate.
\( N' \) New position of node in torsion spring system.
\( O \) Order of magnitude.
\( p \) Porosity of sediment, in this work equal to 0.4.
\( q \) Longshore sediment transport.
\( q_a \) Aeolian sediment transport.
 CONTENTS 

$q_f$ Sediment flux across edge in mesh.

$q_l$ Approximate longshore sediment transport.

$q_{max}$ Maximum longshore sediment transport.

$R$ Sediment grain Reynolds number.

$R^2$ Goodness of fit.

$S$ Slope of linear fit.

$s$ Relative density of sediment, in the present work equal to 2.65.

$s$ Shoreline parallel coordinate.

$s$ Steepness of waves.

$s$ Sum of the distances between the node $N$ and the bisectional lines $L_i$.

$S_{xx}$ Radiation stress component due to wave breaking.

$S_{xy}$ Radiation stress component due to wave breaking.

$T$ Wave period.

$t$ Time.

$T_p$ Peak wave period.

$T_{pm}$ Average peak wave period.

$U$ Fraction of waves coming from the unstable wave regime.

$u$ Depth integrated horizontal velocity in $x$-direction.

$u$ Variable being solved for in mesh movement Laplace equation.

$u_b$ Velocity above the bed.

$U_f$ Friction velocity.

$u_z$ Wind speed at height $z$.

$V$ Volume.

$v$ Depth integrated horizontal velocity in $y$-direction.

$V_{dep}$ Deposited sediment volume.

$vol$ Deposited volume of sediment per time step.
Shoreline position.

Width of undulation.

Average shoreline position.

Undulating part of shoreline position.

Width of active coastal profile.

Fall velocity of sediment.

Width of the spit, i.e. cross-shore distance from crest of undulation to tip of the spit.

Width of undulation, i.e. ignoring shoreline in embayment.

Maximum width of undulation.

Total width of the spit, i.e. width of spit at the crest of undulation.

Total width of undulation, i.e. the width of the entire undulation including embayment behind the spit.

Volume per alongshore unit length.

Horizontal coordinate.

Horizontal coordinate.

Position of off-shore contour line.

Undulation part of off-shore contour line position.

Vertical coordinate.

Apparent roughness height.

Subscripts

Downstream.

Index in summation.

Stable.

Unstable.

Upstream.
Chapter 1

Introduction

Everyone knows that sand very rarely forms a flat surface. In the desert there are sand dunes, in rivers there are sand bars, but sand features are probably most abundant at the boundary between the land and the sea: the near shore area. Well known rhythmic features made of sand in the near shore area include dunes, ripples and longshore bars. Many more are described in the literature; these include beach cusps, mega cusps, oblique bars, bar rips, longshore shoreline undulations, barrier islands, evolving spits and capes.

Sand features in the near shore area are present on a wide range of scales, from centimeters (e.g. wave ripples, see Fredsoe and Deigaard (1992) p. 301) to hundreds of kilometers, (e.g. the Carolina Capes, see McNinch and Wells (1999)). Many of the features are mobile, some migrate in a specific direction, one example being the pro-grading spit (see Petersen et al. (2008)), and some migrate back and forth depending the hydrodynamic forcing, sediment supply or other factors; an example is the movement of a longshore breaker bar which moves onshore in fair weather and offshore during stormy weather (see Aagaard and Masselink (1999)). Most of the sand features exhibit rhythmic or quasi rhythmic behavior, either as a rhythmic shape, e.g. the dunes in the desert, or as a rhythmic migration pattern, e.g. the off-shore migration of a breaker bar and the subsequent formation of a new breaker bar closer to the shore which then migrates offshore (see Ruessink and Terwindt (2000)). The rhythmic behavior of the sand features is reflected in many of the measurement made in the near shore area.

The evolution and migration of the larger of the sand features has an important impact on the shoreline morphology. This is discussed by Stive et al. (2002) who summarizes studies around the world where the variability of the shoreline evolution has been studied. They identify four different scales at which the variability occurs, see table 1.1. For most engineering projects the very long scale can be disregarded, but the other three scales should be considered. For large scale coastal engineering projects an impor-
### Table 1.1: Natural causes for shoreline variability on different scales, after Stive et al. (2002) table 1a.

<table>
<thead>
<tr>
<th>Scale</th>
<th>Natural causes/factors</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Very long:</strong></td>
<td>sediment availability&lt;br&gt;relative sea-level changes&lt;br&gt;differential bottom changes&lt;br&gt;geologic setting&lt;br&gt;long-term climate changes&lt;br&gt;inherited morphology</td>
</tr>
<tr>
<td>Time scale: centuries to millenia)</td>
<td>relative sea-level changes&lt;br&gt;differential bottom changes&lt;br&gt;geologic setting&lt;br&gt;long-term climate changes&lt;br&gt;inherited morphology</td>
</tr>
<tr>
<td>Space scale: 100km and more</td>
<td>relative sea-level changes&lt;br&gt;differential bottom changes&lt;br&gt;geologic setting&lt;br&gt;long-term climate changes&lt;br&gt;inherited morphology</td>
</tr>
<tr>
<td><strong>Long term:</strong></td>
<td>relative sea-level changes&lt;br&gt;regional climate variations&lt;br&gt;coastal inlet cycles&lt;br&gt;sand waves&lt;br&gt;extreme events</td>
</tr>
<tr>
<td>Time scale: decades to centuries</td>
<td>relative sea-level changes&lt;br&gt;regional climate variations&lt;br&gt;coastal inlet cycles&lt;br&gt;sand waves&lt;br&gt;extreme events</td>
</tr>
<tr>
<td>Space scale: 10-100km</td>
<td>relative sea-level changes&lt;br&gt;regional climate variations&lt;br&gt;coastal inlet cycles&lt;br&gt;sand waves&lt;br&gt;extreme events</td>
</tr>
<tr>
<td><strong>Middle term:</strong></td>
<td>wave climate variations&lt;br&gt;surf zone bar cycles&lt;br&gt;extreme events</td>
</tr>
<tr>
<td>Time scale: years to decades</td>
<td>wave climate variations&lt;br&gt;surf zone bar cycles&lt;br&gt;extreme events</td>
</tr>
<tr>
<td>Space scale: 1-5 km</td>
<td>wave climate variations&lt;br&gt;surf zone bar cycles&lt;br&gt;extreme events</td>
</tr>
<tr>
<td><strong>Short term:</strong></td>
<td>wave, tide and surge conditions&lt;br&gt;seasonal climate variations</td>
</tr>
<tr>
<td>Time scale: hours to years</td>
<td>wave, tide and surge conditions&lt;br&gt;seasonal climate variations</td>
</tr>
<tr>
<td>Space scale: 10m-1km</td>
<td>wave, tide and surge conditions&lt;br&gt;seasonal climate variations</td>
</tr>
</tbody>
</table>

tant factor to be aware of is the sand waves, located under the Long term scale in the table. These sand waves are longshore shoreline undulations which migrate alongshore. They are the focus of the present work.
1.1. EXAMPLES OF LONGSHORE SHORELINE UNDULATIONS

The longshore shoreline undulations are longshore periodic features with lengths ranging from hundreds to thousands of meters and amplitudes from tens of meters to hundreds of meters or kilometers. A definition sketch of a longshore shoreline undulation is shown in figure 1.1, where the width and length of the feature is defined.

Longshore shoreline undulations are found on many coasts around the world, some examples are shown in figures 1.2 to 1.4. Figure 1.2 shows one shoreline undulation on Long Point in Lake Erie. A spit is seen on the downstream end of the shoreline undulation, the spit is seen to migrate parallel to the shoreline. The length is seen to be around 2 km.

Figure 1.3 shows the longshore undulations on the shoreline at Srd. Holmslands Tange on the West Coast of Denmark. The shoreline undulation are seen to be much less pronounced and no spits are formed on the downstream end. The length of the undulation is seen to be around 5 km.

On the West Coast of Namibia some very large shoreline undulation are seen in figure 1.4. The length of the undulations is 60 km (there are three undulation on the stretch of shoreline which is \( \approx 180 \) km long), and the width are between 8 and 12 km. As in Lake Erie spits are forming on the down drift side of some of the undulations, the direction in which the spits migrate is more or less parallel to the overall shoreline orientation.

1.2 The scope of the thesis

The focus of the present thesis is to study the longshore shoreline undulations in order to determine the mechanisms which govern the dimensions and the shape of the undulations. To achieve this a numerical model has been developed which can describe the evolution of shorelines subject to waves. This model is described in chapter 2 together with a stability analysis determining the most unstable length of the shoreline undulations. Theoretical results from the model are presented in chapter 3 and 4. In chapter 5 re-
Figure 1.2: Example of a longshore shoreline undulation in Lake Erie, Canada.
1.2. THE SCOPE OF THE THESIS

Figure 1.3: Example of a longshore shoreline undulation on Danish West Coast, Denmark.
Figure 1.4: *Example of longshore shoreline undulations on the west coast of Namibia*
sults from the model are compared with observations of naturally occurring shoreline undulations. Chapter 6 presents a data analysis of shoreline data collected by the Danish Coastal Authorities on the West Coast of Denmark. In chapter 7 various ideas which have not been fully developed are presented. Finally chapter 8 contains the overall discussion and conclusion of the thesis.
Chapter 2

Numerical Modeling of Shoreline Undulations: Model Set-up and Stability Analysis

2.1 Introduction

Field observations of longshore shoreline undulations were first reported by Bruun (1954a). He looked for periodic longshore features on the shoreline of the Danish North Sea Coast. He found evidence of large shoreline features with lengths from 0.5-2 km and widths of 60-80 m, traveling up to about one length per year. He speculated that there may be a connection between breaches in the longshore bar (rips) and the undulations, since the trough of the undulation was often found behind a breach and the crest just ahead. This mechanism is perhaps due to the increased wave energy which can reach the shoreline behind a rip as sketched in figure 2.1.

An explanation for the shoreline undulations connecting these to the rips in the longshore bars has also been proposed by Kystdirektoratet (2005).

Bruun also found evidence of sand waves traveling in the direction of the littoral drift at the 6-m depth contour and the 9-meter depth contour. He called these sand waves sand humps, and they had lengths from 1.5-3 km and amplitudes of 1-2 m. The connection, if any, between the sand waves and the shoreline undulations was not established. Bruun further speculated whether sudden increases in bottom shoaling of navigation channels without a marked change in the weather or wave climate could be due to the arrival of either sand humps or shoreline undulations to the place in question. In this way the first economic interest in longshore shoreline undulations was established.

From the paper it is evident that the described "ord" is in fact the narrow part of a longshore shoreline undulation (i.e. the trough). The average length of the "ord" over 6 years was around 1200 m and the average migration rate was 500 m per year. Pringle was mostly interested in the "ord" because of its relation to cliff erosion, i.e. the cliff was eroded much more where the beach was narrow than where the beach was wide. The economic aspect of this is evident if there are land owners on top of the cliff. Pringle found that the "ord" deepened and migrated most during northerly onshore winds and their associated water waves. On the Holderness coast northerly wind and waves arrive at the coast with an oblique angle as the coast is oriented NW.

Shoreline undulations along the Dutch coast has been studied by Verhaegen (1989). Although he calls the features sand waves it is clear the paper is concerned with longshore shoreline undulations. He finds longshore shoreline undulations with migration velocities around 50-200 m/yr, periods of 50-150 years and amplitudes of 30-500 meters. On one part of the Dutch shoreline groins were constructed; the longshore shoreline undulation were not affected by the groins, i.e. the celerity and amplitude of the longshore undulation were the same before and after the construction of the groin field. This is related to the large scale of the undulations compared with the scale of the groins.

The undulations on the position of the dune foot along the Dutch shoreline has been studied by Ruessink and Jeuker (2002), where a clear connection between undulations on the position of the foot of the dune and undulations of the position of the shoreline was observed.

In Hicks and Inman (1987) the morphology of an ephemeral river delta is studied through field observations. The river delta is located on the wave dominated Central Californian coast. It is found that the sand discharged
by the river to the river delta is dispersed down drift on the shoreline as a low amplitude longshore shoreline undulation. The large variation in the sediment supply responsible for the creation of the undulation is related to the fact that the sediment discharged from the river is discharged primarily during relatively rare flooding events.

The longshore shoreline undulations on Long Point in Lake Erie, Canada have been described in Stewart and Davidson-Arnott (1988) and Davidson-Arnott and Heyningen (2003). The lengths of the longshore undulations at Long Point are found to be 750 to 2050 m and the widths are found to be 40 to 100 m. The incidence angle of the waves at Long Point is very oblique due to the geometry of Lake Erie and the prevailing wind climate. The migration rates are between 100 and 300 m/year, however the maximum migration rates were much larger. The initial formation of the longshore undulations is related to the onshore migration of and subsequent welding to the beach of a near shore bar. After the initial formation the undulations grow in both length and amplitude as they migrate in the down drift direction. A maximum length of 1.5-2 km is observed, beach waves longer than this tend to break up into two shorter undulations.

Longshore shoreline undulations at Southampton Beach, New York are observed and described in Thevenot and Kraus (1995). Eleven longshore undulations are observed on this beach which is 15 km long. The average length of of the undulations is 0.75 km and the average amplitude is 40 m. The undulations are created by the periodic opening of a small inlet and subsequent welding of the ebb shoal to the beach. The undulations migrate around 0.35 km/year and do not disperse significantly during the migration. The foot of the dunes is not affected by the undulations. The offshore morphology is appearing as oblique finger shoals extending as far as 500m offshore and pointing in the down drift direction.

In a more recent paper Medellin et al. (2008) studied a shoreline undulation on a low energy beach at El Puntal in Spain. The El Puntal spit is located near Santander, Spain. The spit was monitored using the Argus video monitoring system and from this a shoreline could be extracted. Therefore the formation of a shoreline undulation can be followed in time in this data set. During the monitoring period longshore shoreline undulations accrete and erode during two different events. The lengths of the features is around 100 m, the maximum amplitude is 15 m and the time scale is weeks. No longshore migration of the undulations was observed in any of the two events.

The formation and evolution of a longshore shoreline undulation on an estuarine beach is studied in Vila-Concejo et al. (2009) on the Northern shoreline of Port Stephens estuary (NSW, Australia). On this beach a single longshore shoreline undulation is formed and migrates in the down drift direction. The length of the feature is approximately 1km and the width 200-300m, the average migration rate is found as 70 m/year.
2.2 What causes the rhythmic patterns of undulations?

The mechanism behind the formation of the shoreline undulations is not well understood. In the papers mentioned above, the formation of the undulations is at best only tentatively explained. At Southampton Beach the shoreline undulations were related to the opening of a small inlet and subsequent welding of the shoal to the beach. Other possible explanations include variations in sediment supply from rivers, as observed on the Central Californian coast, and the onshore migration and welding to the beach of near shore breaker bars, as observed at Long Point in Lake Erie.

A possible explanation for the existence of the longshore undulations is an instability mechanism of an otherwise straight and uniform coastline under very oblique wave incidence. The instability is explained in the following way.

Along a straight and uniform shoreline, the longshore sediment transport is formed by the incoming waves in combination with the longshore current. The longshore current is partly wave driven but can also have contributions from tide and wind, only the wave induced current is considered in the present work. A sketch of the longshore current and longshore sediment transport is shown in figure 2.2.

The longshore sediment transport profile can be integrated across the surf zone to give the total longshore sediment transport, $q$. The total longshore sediment transport depends among other things on the angle between
2.2. CAUSES OF THE UNDULATIONS

Incoming waves
Beach

Figure 2.3: Sketch showing the instability. $\alpha_u$ is the angle between the normal to the shoreline and the angle of the incoming waves at the up-drift side of the undulation and $\alpha_d$ is the angle between the normal to the shoreline and the angle of the incoming waves on the down-drift side of the undulation. $q$ is the longshore sediment transport.

the propagation direction of the incoming waves and the normal to the shoreline, $\alpha$ in figure 2.2. The dependency is such that a maximum for $q$ exists around $\alpha = 45^\circ$, this is the case in both the energetic approach by Komar and Inman (1970) (the CERC formulation) and the deterministic model by Deigaard et al. (1986). In the present work a model equivalent of the one used by Deigaard et al. (1986) is used.

If the angle of the incoming waves is larger than the critical angle, a reduction in the wave angle will lead to an increase in the longshore sediment transport. Therefore any small undulation will have a larger longshore sediment transport on the upstream side ($q_u$) than on the downstream side ($q_d$) as seen in figure 2.3. This means that any small undulation will grow in size, rendering the shoreline unstable under oblique wave incidence, waves from these angles are said to be in the unstable wave regime in the following. From the figure it is further seen than if the incoming wave direction is below the critical angle, undulations formed by any of the processes named above (welding of shoals, welding of near shore bars or variations in sediment supply from rivers) will disperse since the longshore sediment transport on the down-drift side ($q_d$) will then be larger than the longshore sediment transport on the up-drift side ($q_u$).

The instability was first outlined by Grijm (1960). He used an approximate mathematical analysis to show that if the incoming waves arrive at the shoreline with an angle equal to the one giving maximum longshore transport the resulting shoreline shape is either straight or what he terms a cusps. His assumptions were somewhat crude, among others the sea bed was assumed to be flat, that is wave refraction was not accounted for. LeMéhauté and Soldate (1978) determined under which conditions a straight shoreline is unstable taking refraction into account through the use of Snell’s law. The main interest in these early works was to determine under which conditions a shoreline is stable, because the shoreline models at that time all assumed
a stable shoreline; this is still the case for many shoreline evolution models used today, e.g. litline by DHI (2011a).

Ashton et al. (2001) were the first to try to model the evolution of shorelines subject to the instability. In this work the longshore littoral drift was determined using a CERC formulation approach for the description of the longshore transport. Waves were transformed from off-shore to breaking using linear theory for refraction and shoaling and assuming parallel depth contours (again applying Snell’s law). Their model simulates the non-linear evolution of small perturbations resulting in a variety of large scale shoreline morphologies. They speculate that the instability is responsible for the formation of the capes seen on the shoreline of North Carolina, on the east coast of the United States, as some of their model results resemble these features.

The effect of non-parallel depth contours on wave refraction in conjunction with the instability was first included in an analysis by Falqués and Calvete (2005). They looked at the diffusivity and instability of sandy coastlines. Their work is a linear stability analysis of a uniform shoreline subject to very oblique wave incidence described using a one-line model approach. The longshore sediment transport is determined using a CERC formulation approach, which means that inertia effects in the longshore current are ignored. The main new contribution was the determination of the most unstable undulation length. This length scales with breaker zone width (with a scaling factor of 50-100) but also scales with other parameters. Furthermore they determined that for a shoreline undulation to be unstable, the shoreline undulation must be felt a certain distance offshore.

Uguccioni et al. (2006) included inertia terms in the longshore current in their linear stability analysis of the most unstable undulation length. They found the most unstable length to increase when including the inertia terms and confirmed that the length of the most unstable undulation scales with break zone width.

Falqués (2006) analyzed the Dutch coast using the linear stability analysis and found the coast to be stable for the wave climate and shoreline orientation observed along the entire coast. Only by introducing offshore shoals did the shoreline become unstable. This is somewhat in contrast to the observations by Verhagen (1989) who found shoreline undulation along the entire Dutch shoreline. A possible explanation is that the offshore shoals are formed at tidal inlets, which then create the shoreline undulations.

In van der Vegt et al. (2007), the possibility that the undulations along the Dutch coast are caused by the tides or the combined effect of waves and tides, is studied. It is found that the tides can render a straight shoreline unstable, however with the tides alone the length scales that emerge from the model are larger than the shelf width. When adding the effect of waves to the model is is found that for the Dutch coast the waves damp the instability mechanism due to the tides, i.e. the shoreline becomes stable. Only by
2.3. NUMERICAL METHODS

including an offshore shoal, as in the analysis in Falqués (2006) is it possible to find unstable length scales.

The linear stability analysis is more successful in predicting the length scale of the shoreline undulation at the El Puntal spit, this is done in Medellín et al. (2009). The stability analysis of Falqués and Calvete (2005) is adopted to the local conditions at the El Puntal spit, and the result is a remarkable fit between the length scales predicted by the stability analysis and the length scales of the observed shoreline undulations. The predicted growth rate for the undulation is also in agreement with the observations, however the stability analysis predicts an alongshore migration of the shoreline undulations which is not seen in the observations.

Scope of present study.

The present chapter continues where Ashton, Falques and Uguccioni left. It first describes a non-linear stability analysis including the inertia terms in the longshore transport. This analysis is used to determine the most unstable length in the linear regime. Further the analysis is extended to study the full non-linear case of a sinusoidal undulation to estimate at which width the undulation will stop its growth. To finalize the study the model is extended to include the growth and decay of undulations in the non-linear regime, results from this extension are presented in the following chapters.

To describe this a numerical procedure has been developed which can calculate the longshore sediment transport along a coastline with arbitrary shape, so the full development can be studied. The cross-shore morphology is not solved, but a specified coastal profile perpendicular to the shoreline is always assumed.

2.3 Numerical Methods

2.3.1 Numerical Model

The computer model used in the present work is the Mike21FM framework created by DHI. The model consist of a spectral wave model, a hydrodynamic model and a sand transport model. The spectral wave model solves the wave action conservation equation which has been parametrized using the zero’th and first order moments of the wave action spectrum in the frequency domain, see Holthuijsen et al. (1989). The wave model computes the wave field taking into account linear wave refraction, linear wave shoaling and wave breaking using the model of Battjes and Janssen (1978) with a constant $\gamma$ in the domain, where $\gamma = \frac{H_b}{D}$, $H_b$ is the wave height when wave breaking occurs and $D$ is the water depth. Wind forcing and diffraction can be included in the model, but have been ignored in the present work. The spectral wave model hence discretizes the wave energy in two
horizontal coordinates \((x, y)\) and in the wave direction domain \((\theta)\) resulting in a three dimensional model. Thus the wave energy is computed in a domain described by \((x, y, \theta)\) for each time step. The domain and the governing equations are discretized using the finite volume method. The model is quasi-stationary using a Runge-Kutta iterative procedure to compute the stationary wave field at each time step, this is practical due to the extremely small time steps that are required if the solution is sought by time stepping the wave action equations in the domain. See DHI (2009c) for a detailed description of the spectral wave model.

The wave field computed using the spectral wave model is used as forcing for the hydrodynamic model. The hydrodynamic model solves the depth integrated Navier Stokes equations, i.e. the non-linear shallow water equations. The model includes contributions from bottom friction, included using a Manning formulation, and turbulence, included using a Smagorinsky approach. The domain and the governing equations are discretized using the finite volume method. The convective fluxes are computed using an approximate Riemann solver known as Roe’s scheme. See DHI (2009a) for a detailed description of the hydrodynamic model.

The computed flow field and the computed wave field is used as input for the sediment transport model, which computes the sediment transport field. The sediment transport is taken to be purely local, meaning that only the local wave and flow parameters determine the sediment transport rate. This transport rate is found by interpolation in a sediment transport table which is computed before the main model is run. The computation of the sediment transport rates in the sediment table are determined using an intra wave force balance description where the time evolution of the wave boundary layer is solved using the integrated momentum approach by Fredsoe (1984). The force balance includes forces stemming from near bed wave orbital motion, wave breaking and the sloping water surface. See DHI (2009b) for a detailed description of the sediment transport model.

### 2.3.2 Mesh Generation

The computational domain is a periodic domain in the east and west directions. The south boundary is the shoreline where zero flux of water and sediment is assumed, therefore boundary conditions are only needed on the northern boundary. The mesh is an unstructured finite volume mesh containing mainly quadrilaterals but which can contain some triangles as well. Such a mesh consists of nodes, edges and cells. Each node is a point \((x,y)\) in the computational domain. Each edge consists of two nodes and each cell consists of either three (triangle) or four (quadrilateral) edges.

The mesh is created using an advancing front technique described below. The method has been inspired by Owen et al. (1999). In the advancing front technique the mesh is generated by advancing a front of edges in the
computational domain. As the front is advanced new cells are generated in the mesh, this method is also known as paving (Blacker and Stephenson (1991)). The front is advanced by projecting the edges towards the interior of the domain, this is repeated until the entire domain is filled with cells.

In the present case the starting front is the shoreline described by a number of edges. First the direction in which new edges are projected must be determined. Based on the starting front, \((\Delta x, \Delta y)\) is computed using a central difference scheme for each point in the front, from \((\Delta x, \Delta y)\) the direction can be found as:

\[
\alpha = \tan^{-1}\left(\frac{\Delta x}{\Delta y}\right)
\]  

(2.1)

The directions are then filtered using a standard dissipative filter, i.e. \(f^*_i = 0.25f_{i-1} + 0.5f_i + 0.25f_{i+1}\) where \(f\) is the function to be filtered and \(f^*\) is the filtered function, \(i\) is the node number. The smoothing can lead to directions which do not point towards the interior of the domain, in those cases the unfiltered directions are used. See figure 2.4.

The nodes in the new front are then computed using:

\[
x_n = x + a \sin(\alpha)
\]

\[
y_n = y + a \cos(\alpha)
\]

(2.2)

(2.3)

where \(x_n\) is the \(x\)-coordinate of the new node and \(y_n\) is the \(y\)-coordinate of the new node. \(a\) is a constant that can vary for each new front.

The new front is checked for folding. Folding happens when the new front intersects itself somewhere. This is remedied by creating triangles in the folded area using Triangle, see Shewchuk (1996), and removing the folded section from the new front, see figure 2.5. After the folding check the quadrilateral cells created by the new front are made. This is followed by a check of the angle between adjacent edges in the new front, if this angle is smaller than \(\pi/10\), the element with the shortest edge is made into a triangle by projecting the shortest edge onto the long edge and removing the short edge, see figure 2.6 left. Next, nodes in the mesh that are closer
than a certain distance are merged. Triangles containing a merged node are removed and quadrilaterals containing a merged node are made into triangles, see figure 2.7 left. The angle between adjacent nodes is checked again after the merge but in case of a violation at this point, a triangle is created as shown in figure 2.6 right. The reason that this solution is not employed in the first check, is that during the first check the angle can risk being negative, in this case creating a triangle does not solve the problem.

Non-convex quadrilaterals are found next. In order to remove them, first a smoothing approach is tried. This means that the node responsible for the non-convex element (see figure 2.8) is found and smoothed using the method described in section 2.3.3. If the smoothing was unsuccessful in making the element convex, it is made into triangles by dividing along the shortest diagonal, see figure 2.8.

Next the front is filtered using the standard $f_i^* = \gamma f_{i-1} + (1-2\gamma)f_i + \gamma f_{i+1}$ where $\gamma$ can vary from case to case depending on the ratio between the length of edges in the front and the distance $a$ the front is projected. In case the filtering causes any cells to get negative areas or become non-convex, $\gamma$ is changed by $\gamma = 0.8\gamma$ and the original front is filtered again.

The last thing to be checked is the length of the edges in the new front: if the length of an edge is larger than 2 times the maximum edge length in the coastline, the edge is split into two by insertion of a triangle as seen in
2.3. NUMERICAL METHODS

Figure 2.7: Left: Sketch of a situation where the distance between two adjacent nodes is too small. Right: Sketch of a situation where the length of an edge in the front is too large.

Figure 2.8: Left: Node responsible for non-convex element. Right: Shortest diagonal in non-convex element.
The advancing of the front continues until a certain $y$-coordinate has been reached by all nodes in the front, thereby the minimum width of the domain is controlled. However, the outer boundary is not known before the mesh has been created, this is not a problem in the present situation due to the way the bathymetry is set, see section 2.3.4. Once the advancing of the front has been done, the positions of the nodes in the interior of the mesh are smoothed using the torsion smoothing technique explained in the next section. In figure 2.9 is shown an example of a mesh generated using the technique described above, before the smoothing is applied.
2.3. Smoothing the Mesh

Mesh smoothing is an area of research in itself. The most simple type of mesh smoothing is Laplacian smoothing where each internal node is simply moved to the average location of its connected nodes. This can in many cases lead to inverted cells in the mesh, which is highly undesirable (the solvers used in the present work do not permit inverted cells). Therefore a different approach has been used in the present case, namely the torsion spring system approach described below.

The basic idea behind the torsion spring system smoothing technique explained in the following was first presented by Zhou and Shimada (2000). Their algorithm smooths a mesh by moving each internal node in the mesh to a better location by modeling the edges that connects nodes as a spring torsion system. The energy of the torsion spring system surrounding one node is given as:

\[ E = \sum_{i=0}^{2n} 0.5k\theta_i^2 \]  

(2.4)

where \( k \) is the spring constant, \( n \) is the number of nodes connected to the internal node, \( \theta_i \) is the angle between a polygon edge and the edge from the internal node to the \( i\)-th connected node, see figure 2.11. Each edge from the internal node has two angles, therefore there are \( 2n \) angles to sum over in equation (2.4).

In Zhou and Shimada (2000) the energy of equation (2.4) is minimized by finding \( n \) positions of the internal node. For each connected node the optimum angle was determined and the optimum position of the internal node was found by rotating the vector \( \vec{v}_i \) around the connected node so the angles \( \theta_i \) and \( \theta_{i+1} \) are equal in size. The new position of the internal node was then found as the average of the \( n \) locations. Further details can be found in Zhou and Shimada (2000). This approach was tried but turned out to produce inverted cells in the mesh. This was not the case in the article by Zhou and Shimada (2000) and this appears to be due to the many stretched cells which exist in the meshes used in the present work.

In Xu and Newman (2005) the torsion spring system is viewed in a different way. Instead of minimizing the energy of equation (2.4) it is claimed that: "the energy of a torsion spring system model on the edges that connect nodes is provably minimal when the angle at each vertex is divided by a bisecting line". The problem is that the bi-sectional lines for a polygon about an internal node do not intersect in the same point, however a new objective function can be formulated as:

\[ s = \sum_{i=0}^{n-1} [\text{distance}(D, L_i)]^2 \]  

(2.5)
CHAPTER 2. MODEL SET-UP AND STABILITY ANALYSIS

Figure 2.11: Sketch of the torsion spring system modeled by (2.4). The dotted lines are the edges in the mesh, the full lines show the polygon made of the connected nodes. $N$ is the internal node, $\bar{v}_i$ is the vector from the connected node to the internal node.

Figure 2.12: Sketch of the torsion spring system modeled by (2.5). The dotted lines are the edges in the mesh, the full lines show the polygon made of the connected nodes. $N'$ is the new node position, $L_i$ is the bi-sectional line of angle $\theta_i$, $N$ is internal node and $\text{distance}(N', L_i)$ is the distance from $L_i$ to $N'$. 
2.3. NUMERICAL METHODS

where $L_i$ is the bi sectional line of the internal angle at node $D_i$. Xu and Newman (2005) goes a step further to formulate a way in which all node position can be optimized in an iterative way. In the present case $s$ is mini-
imized at each internal node at a time. As a first approach the $n$ intersections between $L_i$ and $L_{i+1}$ is determined, and next the new node position ($N'$) is 
found as the average of these $n$ positions. If this new node position is located 
outside the polygon ABCD (see figure 2.12) an optimization function (Mat-
labs fminsearch) is used to determine the optimal position, in the latter case the objective function (2.5) is used with the additional constraint of setting $s = 10^{12}$ if $N'$ is outside ABCD. It has been found that this formulation does not yield any inverted cells on the meshes which have been smoothed 
so far. The mesh shown in figure 2.9 has been smoothed 20 times with this 
formulation, the result is shown in figure 2.10.

2.3.4 The Bathymetry

The beach profile is a Dean beach profile (see Bruun (1954b)), this beach 
profile is given by:

$$D = Ay^m$$

where $D$ is the water depth, $y$ is the shore-perpendicular coordinate, $m = 2/3$ and $A$ is a constant which determines the steepness of the profile, it depends on the sediment size (see Dean (1991)).

The Dean profile is only used for water depths smaller than a certain depth, namely the closure depth. In the present model the closure depth is a measure for how far out in the coastal profile shoreline undulations can be observed; this is in slight contrast to the original definition by Hallermeier (1981), where the closure depth was the depth where no significant morphological changes were observed. He proposed the relationship:

$$D_{cld} = 2H_s + 11\sigma$$

where $D_{cld}$ is the closure depth, $H_s$ is the significant wave height and $\sigma$ is the standard deviation of the wave height.

The closure depth in the present model has been set based on measurement of how far out in the coastal profile shoreline undulations are felt. In chapter 6 (on Srd. Holmslands Tange), this depth is found to be at least 5 meters, where the yearly mean significant wave height was 1.3 m, thus $D_{cld} \approx 4H_s$ (with a deviation of 4%). This relationship is used for the theoretical sections of the present work.

It was chosen to fix the closure depth to the incoming wave height because Falqués and Calvete (2005) found that the offshore extent of shoreline undulations affects the stability of the shoreline. Therefore when investigating the stability and evolution of a shoreline with incoming waves of e.g. 1 m, it does not seem reasonable to investigate shoreline undulations that extend further offshore than these incoming waves can move the sediment.
The cross-shore coordinate in equation (2.6) is not self-explanatory since the coastline is not a straight line. It is chosen to take the cross-coordinate as the shortest distance to any point on the shoreline. The reason this is a reasonable choice is that short perturbations on the shoreline do not impact the bathymetry further offshore, as shown in figure 2.13.

This behavior is highly desirable. First of all because it make sense in a physical way that e.g. a perturbation on the shoreline with a length of 1m should not be felt on the bathymetry 100 m offshore. Second off all because with this formulation the model smooths out short undulations should they appear due to numerical errors.

### 2.3.5 Hydrodynamic Parameters

The hydrodynamic parameters, which are wave height, wave period, wave direction and directional spreading; control the forcing to the system and are therefore very important in the present work. In nature, these parameters are variable on several different time scales, i.e. the individual waves vary within one wave group, the significant wave height varies during one storm and the intensities of different storms also vary.

In contrast to this natural variability of the hydrodynamic parameters the theoretical work presented here mainly focuses on the behavior of shorelines under constant wave conditions. The constant wave conditions should represent some time averaged conditions. These time averaged conditions should be the conditions which give the same morphological evolution as if the full time varying conditions were used.

In coastal engineering and coastal geomorphology the wave climate is rarely specified as only an average wave height, average wave period and average wave direction. This is because information crucial to the understanding of the geomorphological processes is lost when using simple averag-
ing. In stead it is common to specify the wave climate as a wave rose where the percentage of waves of a certain height coming from a certain direction can be seen. Thereby different morphological features can be related to the amount of wave energy coming from different directions, i.e. the morphological features can be understood and explained by looking at the wave rose for the wave climate.

To understand the evolution of the large scale shoreline features in the present work, the degree of detail needed in the wave forcing is limited, therefore in the present work wave climates are included by following the approach by Ashton and Murray (2006b) where two parameters $A$ and $U$ are specified. $U$ is the fraction of waves coming from the unstable wave regime meaning directions larger than $45^\circ$ and smaller than $-45^\circ$, i.e. from direction intervals $\Delta \alpha_1$ and $\Delta \alpha_4$ in figure 2.14. $A$ is the fraction of waves coming from the left, meaning the fraction of waves from directions smaller than $0^\circ$, i.e. from direction intervals $\Delta \alpha_1$ and $\Delta \alpha_2$ in figure 2.14.

2.3.6 Stability Analysis

In the stability analysis, an initially straight shoreline is given a small sinusoidal perturbation, the change in the longshore sediment transport is then studied to evaluate if the small perturbation will grow or decay; if it grows the shoreline is unstable, if it decays the shoreline is stable. This analysis is done for different wave climates, hydrodynamic parameters and sediment parameters to study under which conditions it can be expected that undulations will form on an initially straight shoreline. A similar analysis has been made before by Falqués and Calvete (2005) and Uguccioni et al. (2006) in the linear regime as explained in section 2.1. The present analysis is different in the way that all non-linear terms in the hydrodynamics are included in the model, that the wave directional spreading is included in the wave model and that the sediment transport description is an intra wave model as explained in section 2.3.1. The first point allows the present model to be used for much larger perturbations than when linearizing the equations, thus allowing to extend the analysis to the case of a fully non-linear sinusoidal perturbation. In the following the different steps in the stability analysis are explained.

First the perturbed shoreline is created; the perturbed shoreline is de-
Figure 2.15: Sketch of a sinusoidal undulation with amplitude $a$ and length $L$.

scribed by:

$$y = a \cos \left( \frac{x}{L} \pi \right)$$

(2.8)

where $a$ is the amplitude set equal to 10m, $L$ is the length of the perturbation and $(x, y)$ are the two horizontal coordinates of the shoreline, see figure 2.15.

Based on this initial shoreline the mesh is created as explained in section 2.3.2 and the water depth set everywhere in the domain according to the chosen Dean beach profile, equation (2.6). The constant $A$ in equation (2.6) was set according to Dean (1991) as $A = 0.67 w_s^{0.44}$ where $w_s$ is the sediment fall velocity which is found as:

$$w_s = \sqrt{\frac{4(s-1)\mu g d}{\pi C_D}}$$

(2.9)

$$C_D = 1.4 + 36/R$$

(2.10)

$$R = \frac{w_s d}{\nu}$$

(2.11)

here $s$ is the relative density of the sediment (relative to the density of water), $d$ is the grain diameter, $\nu$ is the kinematic viscosity of water, $g$ is the gravitational acceleration, $R$ is the sediment grain Reynolds number and $C_D$ is the sediment grain drag coefficient (see Fredsoe and Deigaard (1992) page 199). As can be seen $A$ depends only on the sediment grain diameter since everything else is constant for a given sediment type.

Next the boundary conditions for the northern boundary of the spectral wave model are chosen. It is chosen to vary the significant wave height, $H_s$ and the mean wave direction, $MW D$, while keeping the off shore wave steepness constant. The wave steepness is defined as $\alpha_s = \frac{H_s}{L_0}$ where $L_0$ is the offshore wave length which is found from linear wave theory as $L_0 = 1.56T^2$ where $T$ is the wave period. Therefore the wave period is:

$$T_p = \sqrt{\frac{H_s}{\alpha_s}/1.56}.$$  

(2.12)

In the hydrodynamic model the resistance due to the shear stress between the flow and the bed, $\tau_b$, is represented by the Manning number. The bottom
shear stress is related to the Manning number by:
\[
\frac{\tau_b}{\rho} = c_f u_b |u_b| \tag{2.13}
\]
\[
c_f = \frac{g}{M D^{1/6}} \tag{2.14}
\]
where \(\tau_b\) is the bottom shear stress, \(\rho\) is the density of water, \(c_f\) is the drag coefficient and \(u_b\) is the velocity above the bed; since the flow model is depth integrated \(u_b\) is the depth integrated velocity. The Manning number can also be related to the bed roughness length, as:
\[
M = \frac{25.4}{k_s} \tag{2.15}
\]
where \(k_s\) is the bed roughness length.

Three different Manning numbers are used (20, 32 and 45). The Smagorinsky coefficient in the eddy viscosity model is kept constant at 0.28. For the sediment, two grain diameters are used, namely \(d_{50} = 0.2\) mm and \(d_{50} = 1\) mm. The remaining sediment parameters are kept constant. Table 2.1 contains an overview of the parameters in use. For the parameters being used to generate the sediment table see appendix A.

With all parameters set, the sediment transport field is computed using Mike21FM. The computed sediment transport field, which is 2-dimensional is then collapsed onto the shoreline by integration, thereby the longshore sediment transport along the shoreline is found. This longshore transport is then used to evaluate whether the initial perturbation on the shoreline will grow or decay. The integration of the sediment transport field is done by integrating the transport in the cells extending from the shoreline to the offshore. On figure 2.16 the sediment transport in \(E_{i-\frac{1}{2}}\) is determined by integrating the transport over the yellow cells, the transport in \(E_{i+\frac{1}{2}}\) is found by integrating over the green cells and so on.

The stability of the shoreline is evaluated next, by comparing the longshore sediment transport at the two points where the perturbed shoreline crosses the original shoreline, namely at \(x = L/4\) and \(x = 3L/4\) where \(L\) is the perturbation length, see figure 2.17.

Assuming the sinusoidal undulation keeps its sinusoidal shape and the coastal profile also retains its shape, the growth rate, \(\sigma\) of the undulation is:
\[
\sigma = \frac{q_{L/4} - q_{3L/4}}{LD_{cld}} \pi \tag{2.16}
\]
For each set of parameters in table 2.1 the most unstable perturbation length is determined as the length with the largest growth rate.

From the difference between the longshore transport at the crest \((x = L/2)\) and in the trough \((x = L)\) the migration speed of an undulation migrating with unchanging form is:
\[
c = \frac{q_{L/2} - q_L}{2aD_{cld}} \tag{2.17}
\]
Table 2.1: Model parameters used for the stability analysis. Values in parenthesis refers to equation numbers.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Wave parameters:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Significant Wave height</td>
<td>$H_s$</td>
<td>1 to 3 m</td>
</tr>
<tr>
<td>Mean Wave Direction</td>
<td>$MWD$</td>
<td>0 to 90 °</td>
</tr>
<tr>
<td>Directional Spreading Index</td>
<td>$DSI$</td>
<td>5 to 100</td>
</tr>
<tr>
<td>Peak wave period</td>
<td>$T_p$</td>
<td>(2.12)</td>
</tr>
<tr>
<td>Wave steepness</td>
<td>$\alpha_s$</td>
<td>1/39</td>
</tr>
<tr>
<td>Breaking wave parameter</td>
<td>$\gamma$</td>
<td>0.8</td>
</tr>
</tbody>
</table>

| **Hydraulic parameters:**      |        |            |
| Manning number                 | $M$    | 20 32 45 $m^{1/3}/s$ |
| Smagorinsky coef.              | -      | 0.28       |

| **Sediment parameters:**       |        |            |
| Sediment porosity              | $p$    | 0.4        |
| Sediment grain diameter        | $d_{50}$ | 0.2 and 1 mm |
| Sediment grain grading coef.   | $\sigma$ | 1.1       |
| Relative sediment density      | $s$    | 2.65       |
| Critical shield parameter      | $\theta_c$ | 0.05     |
Figure 2.16: Sketch showing the integration map.

Figure 2.17: Sketch showing the longshore sediment transport at \( L/4 \), \( L/2 \), \( 3L/4 \) and \( L \).
where $c$ is the migration speed and $a$ is the amplitude of the undulation.

### 2.3.7 Sinusoidal Shoreline Evolution Model

The idea of a shoreline evolution model is to model the further development of a small shoreline undulation in the time domain. This is done in order to study the evolution itself and to determine what the shape and dimensions of the fully developed undulation are. In the beginning of the development the sinusoidal undulation will grow as a sinusoidal shape because of the linearity of the system in the early stages. Therefore a natural extension of the stability analysis is to use the found growth rate (equation 2.16) to update the shoreline position while keeping the sinusoidal shape of the shoreline. This gives a simple evolution model which runs relatively fast and is very robust due to the fixed shape of the shoreline. Therefore a dynamic equilibrium can be reached with much less computational effort than when the shoreline is free to change its shape (see section 2.3.8). The drawback of the model is that the sinusoidal shape is not correct when the amplitude of the undulation becomes large, thus the model results can only be used as a first estimate of the equilibrium width.

Each morphological time step in the sinusoidal evolution model consists of the following steps:

- Compute sediment field using Mike21FM as described in section 2.3.6
- Integrate the sediment field to obtain longshore sediment transport as described in section 2.3.6
- Compute the change in the amplitude of the sinusoidal undulation using equation 2.19.
- Update the node positions by solving a Laplace equation as described in section 2.3.8
- Update the bathymetry using the chosen Dean profile.
- Start next time step

The amplitude of the sinusoidal shoreline is found as:

\[
\Delta a = \Delta t \cdot \sigma \tag{2.18}
\]
\[
a^{n+1} = a^n + \Delta a \tag{2.19}
\]

where $a$ is the amplitude of the shoreline undulation, $\Delta t$ is the time step size, $\sigma$ is the growth rate and the $n$ superscript denotes the $n$'th time step. The time step is constant in the model.
Figure 2.18: Sketch showing the parameters used to estimate the growth rate.

To compare the growth rate of the sine model with the growth rate found in the stability analysis, a robust estimate of the growth rate in the sine model should be found. This robust estimate of the mean growth rate of the undulations can be found by integrating the variation in the width of the undulation over time and approximating the area found by integration by the area of a trapezoid as shown in figure 2.18. The growth rate, $\sigma$, can then be estimated as:

$$
\sigma = \frac{w_{\text{max}}}{a-b} \quad (2.20)
$$

$$
b = \frac{2A}{w_{\text{max}}} - a \quad (2.21)
$$

$$
A = \int_{t=0}^{t=\text{end}} w(t) \, dt \quad (2.22)
$$

where $\sigma$ is the growth rate and the remaining parameters are shown in figure 2.18.

**Multiple Undulations**

Equation (2.19) can only be used for a single sinusoidal undulation, for multiple undulations (see figure 2.19) a slightly different approach is used, this is explained in the following.

The initial shoreline containing multiple undulations is described by:

$$
y = \sum_{i=1}^{i=N} a_i \sin \left( \frac{2\pi}{L} x \right) + b_i \cos \left( \frac{2\pi}{L} x \right) \quad (2.23)
$$
where \( y \) is the cross shore position of the shoreline, \( N \) is the number of wavelengths to be represented in the solution, \( a_i \) and \( b_i \) are constants, \( L \) is the length of the domain. Based on this shoreline, a mesh can be made as described in section 2.3.2.

Next the longshore sediment transport along the shoreline containing multiple undulations is found using the Mike21 modeling system. Then the longshore sediment transport can be approximated by a sum of sine functions with the same lengths as the ones used to describe the shoreline, thus:

\[
q_l = \sum_{i=1}^{N} \left[ A_i \sin\left( \frac{2\pi}{L} x \right) + B_i \cos\left( \frac{2\pi}{L} x \right) \right] + C \tag{2.24}
\]

where \( q_l \) is the approximate longshore sediment transport. \( A_i \), \( B_i \) and \( C \) are unknown constants, thus at least \( 2N + 1 \) equations are needed to determine these constants. These \( 2N + 1 \) equations are found by distributing \( 2N + 1 \) points along the shoreline where the longshore transport is known. This gives \( 2N + 1 \) linear equations for determg all \( A_i \) and \( B_i \) and the \( C \). It is also possible to use more than \( 2N + 1 \) points, thereby overdetermining the system, which means that the system is solved as an optimization problem instead of a regular system of equations. In the present work this latter approach has been used; all points along the shoreline are used when solving the optimization problem.

In this way an expression for \( q_l \) is determined. This expression can now be differentiated to give:

\[
\frac{dq_l}{dx} = \sum_{i=1}^{i=N} A_i \frac{2\pi}{L} \cos\left( \frac{i2\pi}{L} x \right) - B_i \frac{2\pi}{L} \sin\left( \frac{i2\pi}{L} x \right) \tag{2.25}
\]
Combining this with the one line model equation:
\[
\frac{\partial h}{\partial t} = \frac{1}{D_{cld}} \frac{\partial q_l}{\partial x}
\] (2.26)
we find:
\[
\frac{\partial h}{\partial t} = \frac{2\pi}{D_{ddL}} \sum_{i=1}^{i=N} i \left( A_i \cos \left( \frac{i2\pi}{L} x \right) - B_i \sin \left( \frac{i2\pi}{L} x \right) \right) \] (2.27)
An expression for the evolution of the amplitudes in equation 2.23 can then be found by comparing (2.27) and (2.23):
\[
a_{i}^{n+1} = a_{i}^{n} + \Delta t \frac{i2\pi}{D_{ddL}} (-B_{i}) \] (2.28)
\[
b_{i}^{n+1} = b_{i}^{n} + \Delta t \frac{i2\pi}{D_{ddL}} A_{i} \] (2.29)
where the \(n\) superscript defines the \(n\)'th time step.

### 2.3.8 Shoreline Evolution Model

The models described in the previous section can only be used to give a first estimate of the shape and dimensions of the shoreline during its growth because of the fixed shoreline shape. In the present section a model is described where the shape of the shoreline is able to move freely, this model is termed the shoreline evolution model.

Each morphological time step in the evolution model consists of the following steps:

- Compute sediment field using Mike21FM as described in section 2.3.6
- Integrate the sediment field to obtain longshore sediment transport as described in section 2.3.6
- Compute the displacement of the shoreline using a one-line model as described in section 2.3.8
- Check that the maximum distance between adjacent shoreline nodes is below the maximum allowed length and above the minimum allowed length. If not, insert or remove shoreline points.
- Update the node positions by solving a Laplace equation as described in section 2.3.8 if no shoreline points were added or removed. Otherwise make a new mesh based on the shoreline nodes as described in section 2.3.2.
- Update the bathymetry according to the chosen Dean profile.
- Correct the position of the nodes to improve sediment conservation as described in section 2.3.8
- Start next time step
The deformation of the undulation during its growth is obtained by solving a modified version of the classical oneline sediment conservation equation. This equation reads:

\[
\frac{\partial n}{\partial t} = \frac{1}{D_{cld}} \frac{\partial q}{\partial s} \tag{2.30}
\]

where \( n \) is the shore normal coordinate, \( t \) is time, \( s \) is the longshore coordinate and \( q \) is the volumetric longshore sediment transport flux including pore volume. In the classical version of the one-line equation the \( n \) and the \( s \) coordinates are fixed such that they represent the two horizontal directions. The problem with this approach is that such a model cannot model the evolution of a spit which we know is a possible shoreline shape from examples in nature and from Ashton and Murray (2006b). So in the present model the \( n \) and the \( s \) coordinates change direction along the shoreline as indicated in figure 2.20. This type of formulation was first proposed by LeBlond (1972) who points out that the formulation is only valid when the radius of curvature of the shoreline is large compared with the width of the surf zone.

Writing (2.30) on discrete form we find:

\[
\Delta n \Delta t = \frac{1}{D_{cld}} \Delta q \Delta s \tag{2.31}
\]

If we multiply \( \frac{\Delta q}{\Delta s} \) with \( \Delta s \) we get the volume of sediment deposited at the stretch of shoreline \( \Delta s \) per time step, \( \Delta t \).
For each element in the mesh the deposited volume per time step is found by taking the sum of the flux of sediment on each edge of the element:

\[ vol = \sum_{i=1}^{N} q_{fi} \]  

(2.32)

where \( q_{fi} \) is the flux across the \( i \)th edge of the element and \( N \) is the number of edges. The flux of sediment is known in the cell centers and must therefore be interpolated from the element centers to the element edges. This interpolation is known as reconstruction. The method used in Mike21 ST is also used here. For each edge the upwind and downwind element is identified. See figure 2.21 for definition.

\[ q_{e1} = q_1 x n_x + q_1 y n_y \]  

(2.33)

\[ q_{e2} = q_2 x n_x + q_2 y n_y \]  

(2.34)

\[ q_e = q_{e1} + q_{e2} \]  

(2.35)

If \( q_e \) is positive, element 1 is the upwind element. The flux is then determined as:

\[ q_f = ds (\alpha q_u + (1 - \alpha) q_d) \]  

(2.36)

where \( q_u \) is the upwind flux and \( q_d \) is the downwind flux and \( ds \) is the length of the edge. \( \alpha \) is determined by:

\[ \alpha = \tanh(0.5493 \max(q_1 q_2, q_2 q_1)) \]  

(2.37)

If there is a large difference between \( q_1 \) and \( q_2 \) the scheme is close to an upwind scheme.

Equation (2.31) can then be rewritten to:

\[ \frac{\Delta n}{\Delta t} = \frac{vol}{D_{cd} \Delta s} \]  

(2.38)

The term \( D_{cd} \Delta s \) can be seen as the area of the elements in front of the stretch of shoreline \( \Delta s \) projected onto the vertical axis. Thereby the move-
Chapter 2. Model Set-up and Stability Analysis

Figure 2.22: Sketch defining the interpolation vectors and distances

The movement of the stretch of shoreline can be written as:

\[
\frac{\Delta n}{\Delta t} = \frac{\text{vol}}{\sum_{i=1}^{N} (dA_{z})} \quad (2.39)
\]

\[
\frac{\Delta n}{\Delta t} = \frac{\sum_{i=1}^{N} \partial h \partial t i dA_{i}}{\sum_{i=1}^{N} (dA_{z})} \quad (2.40)
\]

where \( dA_{z} \) is the area of the \( i \)’th element projected onto the vertical plane parallel to \( \Delta s_{i} \), see figure 2.24. Equation (2.40) describes the movement of an edge of the shoreline, \( \Delta s \), however it is the movement of the shoreline nodes which is needed. The interpolation from the movement of the shoreline edge to the movement of the shoreline nodes is done by:

\[
\bar{dh} = \frac{\bar{dn}_{1} \Delta s_{1} + \bar{dn}_{2} \Delta s_{2}}{\Delta s_{1} + \Delta s_{2}} \quad (2.41)
\]

where the definition of \( \bar{dh}, \bar{dn}_{1}, \bar{dn}_{2}, \Delta s_{1} \) and \( \Delta s_{2} \) are shown in figure 2.22.

Mesh Movement

Once the displacement of the shoreline has been determined, the position of the nodes in the mesh must be updated. Because the displacement takes place in both horizontal directions this update is done using the same approach as in Jasak and Tuković (2006) where a Laplace equation is solved for the displacement of the nodes. The method is described below. The Laplace equation reads:

\[
\nabla \cdot (\gamma \nabla u) = 0 \quad (2.42)
\]

where \( \nabla \) is the gradient operator, i.e. \( \nabla = (\frac{\partial}{\partial x}, \frac{\partial}{\partial y}) \) in the present case where the mesh is to be moved in two dimensions \((x, y)\). \( \gamma \) is a diffusivity which can vary in the domain and \( u \) is the variable which is being solved for. In the present work \( \gamma = \frac{1}{\text{dist}} \) is used, where \( \text{dist} \) is the shortest distance to the shoreline. The movement of the individual nodes in the mesh is determined by solving (2.42) on the domain described by the mesh. The boundary conditions are periodic on the east and west boundaries, zero gradient on the north boundary and specified and equal to the shoreline displacement on
the southern boundary. This is done in two steps, one for the displacement in the \( x \)-direction and one for the displacement in the \( y \)-direction. Once the solution to (2.42) is obtained the new mesh positions are known, and the bathymetry of this new mesh can be set using (2.6).

The actual solution of the Laplace equation is done using the algorithm described in Alberty et al. (1998).

**Morphologic Time Step**

The morphological time step used in the model is dynamic. It is calculated based on the CLF criterion:

\[
\frac{c \Delta t}{\Delta s} \leq C
\]

where \( c \) is the characteristic velocity of the system and \( C \) is a constant usually taken to be smaller than 1. In the present system the characteristic velocity is found as \( c = \frac{\partial q}{\partial n} \), see Marieu et al. (2008). This velocity cannot be found analytically so it must be determined numerically. The obvious choice is:

\[
\frac{\partial q}{\partial n} = \frac{\Delta q}{\Delta n}
\]

\( \Delta q \) can be determined across the shoreline edge \( \Delta s \) making it the same \( \Delta q \) as in equation 2.31. \( \Delta n \) cannot be determined across \( \Delta s \) as it would always be zero due to the definition of \( s \) and \( n \), see figure 2.20. \( \Delta n \) becoming zero or small is a problem because this makes the characteristic velocity become very large resulting in small time steps. It has been found that the numerical scheme is stable when computing \( \Delta n \) as shown below, as long as the aspect ratio of the computational cells close to the shoreline is not larger than \( \approx 15 \).

The sketch showing the used quantities is seen in figure 2.23.

\[
\Delta n_1 = \Delta h_3 - \Delta h_2 \\
\Delta n_2 = \Delta h_3 - \Delta h_1 \\
\Delta n_3 = \Delta h_4 - \Delta h_2 \\
\Delta n_4 = \Delta h_4 - \Delta h_1 \\
\Delta n = \Delta s \cdot \max \left( \frac{\Delta n_1}{\Delta s_1}, \frac{\Delta n_2}{\Delta s_2}, \frac{\Delta n_3}{\Delta s_3}, \frac{\Delta n_4}{\Delta s_4} \right)
\]

**Volume Correction**

The shoreline is curved, therefore the movement of the shoreline using (2.40) induces a volume error. Furthermore the way the bathymetry is set after the movement of the nodes also induces a volume error because when the shoreline is moved, the path of the shortest distance to the shoreline may have changed for some location in the mesh. Thereby the depth at the location in question may change both due to the actual shoreline movement
Figure 2.23: Sketch showing quantities for computing $\Delta n$. The blue line is the shoreline and $\Delta s$ is the stretch of shoreline across which $\Delta n$ is being determined.

This change is accounted for, and due to the change in the path of the shortest distance (this change is not accounted for).

These volume errors result in a violation of the sediment conservation principle. This is very undesirable as the only thing we know for sure is that no sediment must be lost from the system. Therefore a correction procedure is applied to minimize the volume error. This procedure is an iteration, which consist of the following steps:

- Compute the volume between the original and the new mesh for each edge along the shoreline.
- Compute the volume error for each edge along the shoreline.
- Compute corrected positions of the nodes in new mesh.
- If the error is smaller than a tolerance or the maximum number of cycles has been reached, stop the iteration.

The volume between the original and new mesh for each edge along the shoreline is sketched in figure 2.24. The volume is determined numerically by constructing triangles around the volume using the existing nodes in the old and new mesh. Next the volume can be determined using the method described in Giaccari (2009).

The volume error is found by subtracting the found volume from the deposited volume:

$$er = V_{dep} - V$$

$$V_{dep} = |\bar{d}n|dA_z$$

(2.50)  
(2.51)
Figure 2.24: Sketch showing the volume between the original (full lines) and new mesh (dashed lines). The green areas count as positive and the red areas count as negative. On the bottom plot the projected area $dA_z$ is shown. It is the area of the cells belonging to the shoreline edge $\Delta s$, projected onto the plane which is parallel to $\Delta s$ and the vertical axis.
where $|\overrightarrow{dn}|$ is the length of $\overrightarrow{dn}$, found from 2.40. $dA$ is the area of the domain belonging to each stretch of shoreline e.g. the coherent colored areas in figure 2.16. $dA_z$ is the area of $dA$ projected onto the vertical axis ($z$) parallel to each stretch of shoreline, see figure 2.24.

Dividing the found error by $dA_z$ gives the correction for each edge along the shoreline:

$$d\overrightarrow{n}^* = \frac{er}{dA_z} \overrightarrow{n}$$

(2.52)

where $d\overrightarrow{n}^*$ is the correction which should be applied to each edge along the shoreline. $\overrightarrow{n}$ is the vector perpendicular to the shoreline edge pointing towards the interior of the domain, see figure 2.20. All the cells belonging to each shoreline edge are then given the found correction and the correction for all nodes can subsequently be found using interpolation. The interpolation scheme is described in Holmes and Connell (1989).

Creating a New Mesh

After a certain number of morphological time steps either: the mesh is so distorted that the volume correction cannot be performed without creating inverted or non-convex cells, or the distance between adjacent nodes on the shoreline is so large that a new node should be added to the shoreline, or the distance between adjacent nodes is so small that a shoreline node should be removed. In these cases a new mesh is created using the method described in section 2.3.2.
2.4 Stability Analysis

In this section the stability of a shoreline with an initial small perturbation is investigated. The aim of the stability analysis is to determine under which conditions the shoreline is unstable and to get an estimate of the width of an undulation in dynamic equilibrium. An undulation is in dynamic equilibrium when it migrates along the shoreline without changing its shape, this requires at the minimum that the growth rate, $\sigma$, of the undulation is zero.

The most unstable perturbation length is found by running the model for different perturbation lengths and determining which length gives the largest growth rate. First the dependence of the most unstable length on grid discretization is investigated. This is done in order justify the choice of discretization in the later sections and to get some understanding of what the choice of discretization means for the solution. Next the most unstable length is determined for different model parameters; these include variation with wave height, wave direction, wave directional spreading, Manning number, eddy viscosity and sediment size. Finally the amplitude of the undulation is increased to investigate if the growth rate becomes zero for a wide undulation.

2.4.1 Discretization Effect

There are two parameters governing the discretization, firstly the number of nodes on the shoreline, and secondly the constant $a$ in the advancing front, see section 2.3.2, which determines the number of points in the cross-shore direction.

First the longshore transport as function of cross shore discretization is found on a straight shoreline using the following parameters: $H_s = 1\, m$, $MWD = 55^\circ$, $d_{50} = 0.2\, mm$, $D_{cld} = 3H_s$ and $M = 45\, m^{1/3}/s$, the remaining parameters were set according to table 2.1. The total transport as function of $a$ (the cross-shore discretization in the mesh near the shoreline, see section 2.3.2) and the longshore discretization, $\Delta x$, is seen in figure 2.25. It is seen that the total longshore transport varies with the cross shore discretization for all $a > 0.5\, m$. Further it is noted that the longshore discretization also affects the total longshore transport; the finer the discretization, the larger the total longshore transport. Even though the effect is small it is important to keep in mind when creating the mesh for the evolution model.

Next the most unstable undulation length is found (using the stability analysis explained in section 2.3.6) as function of the cross shore discretization and the number of points on the shoreline. The following parameters are used: $H_s = 1\, m$, $MWD = 60^\circ$, $d_{50} = 0.2\, mm$, $D_{cld} = 4H_s$ and $M = 45\, m^{1/3}/s$, the remaining parameters are set according to table 2.1.
The variation of the most unstable length with the cross-shore discretization distance, \( a \), and the number of points along the shoreline, \( n \), is seen in figure 2.26. The most unstable length is seen to converge towards 4750 m as \( a \) is decreased and as \( n \) is increased for all cases except \( n = 10 \) where the most unstable length explodes when \( a \) is decreased; this could be because the aspect ratio of the computational cells becomes too large. It is noted that for \( n > 50 \) and \( a \leq 10 \text{ m} \) the error is 250 m, or 5%. Based on this relatively small error it is chosen to use \( a = 10 \text{ m} \) and \( n = 50 \) in the following.

### 2.4.2 Dependence on Wave Properties

The dependence of the growth rate on wave properties is considered. These properties include the mean wave direction, the directional spreading of the wave, the wave height and the wave steepness. Parameters that are not mentioned in the following are set according to the values from table 2.1.

First the directional spreading of the waves is varied (\( DSI = (5, 10, 20, 50 \) and 100)) together with the mean wave direction (\( MW D = 20^\circ \) to \( 90^\circ \) every \( 5^\circ \)) and the undulation length (\( L = 2000 \text{m} \) to \( 10000 \text{m} \) with intervals of 500m) for a constant wave height \( Hs = 1 \text{m} \). It is noted that increasing the directional spreading index, decreases the directional spreading of the waves.

Figure 2.27 shows a contour plot of the growth rate as function of \( MW D \) and \( L \) for \( DSI = 10 \). Note that the growth rate has been normalized with the maximum growth rate. It is seen that the minimum angle for an unstable perturbation is around \( 47^\circ \) for the longest perturbation (\( L = 10000 \text{m} \)) and
that the minimum length for an unstable perturbation is \( L = 2800 \text{m} \) for MWD = \( 75^\circ \). Further it is noted that the maximum growth rate is found for MWD = \( 75^\circ \) and \( L = 4500 \text{m} \).

Figure 2.28 shows the most unstable length as function of MWD for different directional spreading index, DSI. It is seen that the most unstable length increases for decreasing DSI and generally decreases for increasing MWD (although the picture is somewhat mixed for DSI > 20).

Next the wave height was varied (\( H_s = 1 \text{m}, 2 \text{m} \) and \( 3 \text{m} \)) together with the mean wave direction (MWD = \( 20^\circ \) to \( 90^\circ \) every \( 5^\circ \)) and the undulation length (\( L = 2000 \text{m} \) to \( 30000 \text{m} \) with varying intervals) while keeping the directional spreading constant (DSI = 10).

Figures 2.29 and 2.30 show contour plots of the growth rate normalized by the maximum growth rate for \( H_s = 2 \text{m} \) and \( H_s = 3 \text{m} \). These plots look similar to figure 2.27, only the location of the maximum growth rate is moved to longer lengths as the significant wave height is increased.

This behavior, where the most unstable undulation length increases for increasing wave height is also seen in figure 2.31. Here the most unstable undulation length is shown as function of MWD for \( H_s = 1, 2 \) and \( 3 \text{m} \). It is noted that the reason the lines for \( H_s = 2 \text{m} \) and \( H_s = 3 \text{m} \) stop at \( 55^\circ \) is that the most unstable length is longer than the longest undulation length (30000 m) which was used in the computation.

Finally the dependence of the wave steepness on the growth rate is studied by varying the peak wave period of the incoming waves and the undulation length keeping all other parameters constant and equal to the parameters shown in tables 2.1 and 2.2. The result is shown in figure 2.32; it shows
that the most unstable undulation length does not change much when the wave period is changed. However the growth rate is seen to increase for increasing wave periods. This is because the longshore sediment transport rate increases with increasing wave period, which in turn will increase the growth rate if the variation of the sediment transport rate along the shoreline is unchanged.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>value</th>
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<tbody>
<tr>
<td>Mean wave direction</td>
<td>MW(D)</td>
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<tr>
<td>Significant wave height</td>
<td>(H_s)</td>
</tr>
<tr>
<td>Peak wave period</td>
<td>(T_p)</td>
</tr>
<tr>
<td>Directional Spreading Index</td>
<td>(DSI)</td>
</tr>
<tr>
<td>Manning number</td>
<td>(M)</td>
</tr>
<tr>
<td>Eddy viscosity model</td>
<td>-</td>
</tr>
<tr>
<td>Smagorinsky constant</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 2.2: Wave and hydrodynamic parameters used unless stated otherwise.
Figure 2.28: Most unstable length as function of the mean wave direction, MWD for different wave directional spreading index, DSI.
Figure 2.29: Growth rate relative to maximum growth rate as function of the mean wave direction, $\text{MW D}$ and undulation length, $L$, for wave directional spreading index $\text{DSI} = 10$ and wave height $H_s = 2\text{m}$
Figure 2.30: Growth rate relative to maximum growth rate as function of the mean wave direction, MWD and undulation length, L, for wave directional spreading index DSI = 10 and wave height $H_s = 3$ m
Figure 2.31: The most unstable undulation length as function of the mean wave direction, $\text{MWD}$ for different significant wave heights, $H_s$, all for wave directional spreading index $DSI = 10$. 
2.4. STABILITY ANALYSIS

Figure 2.32: The growth rate as function of undulation length for different wave periods.
2.4.3 Dependence on Hydrodynamic Properties

The hydrodynamic properties are varied next: they include the bed roughness and the horizontal eddy viscosity. Both are specified in the Mike21FM HD model. The effect of changing these parameters is seen in figure 2.33 where the growth rate is shown for different Manning numbers and in figure 2.34 where the growth rate is shown for different turbulent eddy viscosity models. The different eddy viscosity models are: No eddy: The horizontal turbulent eddy viscosity is set to zero. Constant eddy: The horizontal turbulent eddy viscosity was constant equal to 0.002 m\(^2\)/s in the computational domain. Smagorinsky: The horizontal turbulent eddy viscosity model by Smagorinsky is used; see DHI (2009a) for further information. On figure 2.33 it is seen that the most unstable undulation length changes significantly when changing the bed roughness. The larger the roughness the shorter the most unstable length becomes. This agrees well with Ugocci et al. (2006) where it was found that including the inertia terms in the longshore current increased the length of the most unstable undulation. Increasing the Manning number means that the bed roughness length is reduced, thus increasing the effect of the inertia terms in the longshore current. This is seen by looking at the depth integrated momentum equation for the longshore current (in the horizontal dimension along the shoreline, \(x\)): 

\[
\frac{\partial(Du)}{\partial t} + \frac{\partial(Du^2)}{\partial x} = -gD \frac{\partial \eta}{\partial x} - \tau_b \frac{1}{\rho} \left( \frac{\partial S_{xx}}{\partial x} + \frac{\partial S_{xy}}{\partial y} \right) \tag{2.53}
\]

where \(x\) is the longshore coordinate, \(y\) is the cross-shore coordinate, \(t\) is the time, \(D\) is the water depth, \(u\) is the velocity of the longshore current, \(g\) is the acceleration due to gravity, \(\eta\) is the surface elevation, \(\tau_b\) is the bed shear stress, \(\rho\) is the density of water and \(S_{xy}\) and \(S_{xx}\) are the radiation stresses due to wave breaking. Assuming a stationary flow, the first term is zero, then we can rearrange to get:

\[
\frac{1}{\rho} \left( \frac{\partial S_{xx}}{\partial x} + \frac{\partial S_{xy}}{\partial y} \right) + gD \frac{\partial \eta}{\partial x} = \frac{\partial(Du^2)}{\partial x} + \frac{\tau_b}{\rho} \tag{2.54}
\]

From this equation it is clear that the pressure gradients and the radiation stress gradients driving the flow are balanced by the inertia term and the bed shear stress term, thus reducing the effect of the bed shear stress term will relatively increase the effect of the inertia term.

It is noted that the Manning numbers: 45, 32 and 20, are equivalent of a roughness height of: 3cm, 25cm and 4m. Thus the last value is only included to get a feel for what the length of the most unstable undulation is when the effect of the inertia terms is very small, since this value is unrealistic for the present system. Whether the actual roughness height should be 3cm or 25 cm is more debatable since the wave boundary layer will act as a roughness height for the longshore current, see Fredsoe and Deigaard (1992)
for estimates of the wave boundary layer thickness in combined waves and current.

Figure 2.34 shows that the actual choice for the horizontal turbulent eddy viscosity is of minor importance for the growth rate of the undulation.

2.4.4 Dependence on Sediment Properties

The sediment properties include both the median grain size, \(d_{50}\), the sediment size distribution represented by the variance of the grain size, \(\sigma\), and the density of the sediment. Only the effect of changing the median sediment diameter has been studied here because the variation in the other two quantities is usually minor. The median sediment diameter affects the model in two ways, directly through the specification of a different sediment size in the Mike21ST model, which in turn results in different transport rates for equal hydrodynamic forcing, and through the dependency of the steepness parameter \(A\) in the Dean profile, (equation 2.6) on the median sediment size. The growth rate as function of undulation length is seen for two sediment sizes in figure 2.35. The larger sediment size is seen to have the effect of shortening the length of the most unstable undulation from around 4500m to around 2000m. This is probably only due to the steepening of the beach.
Figure 2.34: The growth rate as function of undulation length for different turbulent eddy viscosity models.
profile due to the larger \( A \) in equation 2.6 because the sediment transport description is entirely local, i.e. convection of sediment is ignored. Therefore the longshore distribution of the sediment transport is expected to remain the same if only the sediment size changes which means the length of the most unstable undulation is also expected to remain the same. However the time scale of the morphologic development will decrease when reducing the sediment size due to the increase in the longshore sediment transport.

### 2.4.5 Dependence on Off-shore Phase Lag

In chapter 6 a phase lag between features on the off-shore contours and features on the shoreline is observed. The effect of this phase lag on the stability of the shoreline is studied in this section.

The phase lag is introduced by setting the water depth at a given offshore point equal to the water depth some distance downstream, see figure 2.36, where the \( \alpha_p \) is given by:

\[
\alpha_p = \frac{\text{phase}}{360} \cdot \frac{L}{y_{cld}}
\]

where \( L \) is the length of the undulation, \( \text{phase} \) is the phase in degrees between the shoreline and the contour at the closure depth, and \( y_{cld} \) is the
distance to the closure depth.

The growth rate as function of the mean wave direction, \( MW D \), and phase is shown in figure 2.37 for \( L = 5000 \) m. It is seen that introducing the phase lag does not change the undulation from being stable to being unstable for any wave direction. For \( MW D < 40^\circ \) a negative phase lag makes the undulation less stable, (the growth rate becomes less negative), whereas a positive growth rate makes the undulation more stable. For \( MW D > 55^\circ \) a phase \(< -40^\circ \) changes the undulation from unstable to stable, whereas a positive phase makes the undulation more unstable.

2.4.6 Expanding to Changing Wave Climates

The results from the previous section is for a constant forced wave climate, however these results can be expanded to varying wave climates by superposition of the growth rates from the model runs with the constant wave climates. This is justified in the linear regime.

The varying wave climates are reduced in this investigation to contain just 4 different directions as shown in figure 2.14, i.e:

\[-90^\circ < \Delta \alpha_1 < -45^\circ < \Delta \alpha_2 < 0^\circ < \Delta \alpha_3 < 45^\circ < \Delta \alpha_4 < 90^\circ \]  

(2.56)

see section 2.3.5.

As there are no differences between positive or negative angles for a sinusoidal undulation only the first two intervals are used in the following.
Figure 2.37: Growth rate relative to maximum growth rate as function of the mean wave direction and the phase between the undulation at the closure depth contour and the undulation at the shoreline.
Figure 2.38: The growth rate, $\sigma$ for mean wave direction of $30^\circ$ and $60^\circ$ together with the average growth rate for the first, $\sigma_u$, and second, $\sigma_s$, wave direction boxes.

The growth rate representative for the first box, $\sigma_u$, is found by averaging the growth rate over the mean wave direction, $MWD$, i.e.: 

$$\sigma_u = \frac{1}{45^\circ} \int_{MWD=-90^\circ}^{MWD=-45^\circ} \sigma d(MWD) \quad (2.57)$$

and likewise for the second box:

$$\sigma_s = \frac{1}{45^\circ} \int_{MWD=-45^\circ}^{MWD=-0^\circ} \sigma d(MWD) \quad (2.58)$$

On figure 2.38 $\sigma_u$ and $\sigma_s$ is shown together with $\sigma(MWD = 30^\circ)$ and $\sigma(MWD = 60^\circ)$ for $H_s = 1m$, $T_p = 5s$, $DSI = 10$ and the remaining parameters set according to table 2.1 and 2.2. It is seen that the average growth rate for the first box is well represented by the growth rate found using a constant $MWD = 60^\circ$ and the average growth rate for the second box is well represented by the growth rate found using a constant $MWD = 30^\circ$. This shows that the initial development of the shoreline can be described by a representative single wave direction in stead of the sum of many different wave directions.

The growth rate can now be found for different values of the parameter $U$ which represents the fraction of waves coming from the unstable wave
regime, as:

\[
\sigma(U) = U \cdot \sigma_u + (1 - U) \cdot \sigma_s
\]  

(2.59)

The result is shown in figure 2.39 and shows that for decreasing \( U \) the length of the most unstable undulation increases. This is in line with the results from the previous section, e.g. figure 2.27, since a decrease in \( U \) is equivalent of decreasing the mean wave direction from around 60° towards 45°.

### 2.4.7 Dependence on Undulation Width

The width of an unstable undulation is varied to see if the undulation becomes stable for a certain width. This can be used to give a first estimate of the width of an undulation in dynamic equilibrium with the wave forcing. The wave conditions are the ones specified in table 2.2 and the remaining parameters are taken from table 2.1. On figure 2.40 the growth rate is shown as function of the undulation length and amplitude; the most unstable undulation length as function of undulation amplitude is marked with the wide black line. It is seen that the length of the most unstable undulation grows as the amplitude of the undulation increases. Further it is noted that the line of zero growth rate is almost linear in the plot meaning that the aspect
Figure 2.40: The growth rate as function of undulation length and undulation amplitude. The wave forcing is seen in table 2.2.

ratio of the undulation giving zero growth rate is constant, i.e. aspect ratio = width/length. This aspect ratio is estimated to be between 0.35 and 0.4 and can be used to get a first estimate for the width of an undulation in dynamic equilibrium.

2.5 Sinusoidal Shoreline Evolution

In this section an estimate of the width of a shoreline undulation in dynamic equilibrium is obtained using the model described in section 2.3.7. An undulation in dynamic equilibrium is an undulation which migrates down the shoreline without changing its shape. For the sine model this happens when the longshore sediment transport at \( x = L/4 \) is equal to the longshore sediment transport at \( x = 3L/4 \), see figure 2.17 since this will make the growth rate zero according to equation 2.16.

2.5.1 Evolution of Single Undulation

Constant Wave Climate

First the sine model is run for an undulation with a length of 5000 m for
mean wave directions: $MWD = 55^\circ, 60^\circ$ and $65^\circ$, using a morphologic time step of 10 years. All other parameters are set according to tables 2.1 and 2.2. Keeping the length of the undulation constant for different mean wave directions is in contrast to the results from the stability analysis where it was found that the length of the most unstable undulation changes when changing the wave climate; however in this investigation the length is kept constant to isolate the effect of changing the wave climate from the effect of changing the undulation length.

The evolution of the width of the undulation for the three model runs is shown in figure 2.41. It is seen that the width is largest for the most oblique waves. The aspect ratio, of the undulations are 0.2, 0.3 and 0.4 for $MWD = 55^\circ, 60^\circ$ and $65^\circ$.

As noted above the length of the undulation should be changed when changing the mean wave direction. In the stability analysis (section 2.4.2) it was found that the length of the most unstable undulation varies with the mean wave direction such that for $MWD = 55^\circ$ the most unstable length is around 5500 m, for $MWD = 60^\circ$ the most unstable length is 4500 m and for $MWD = 65^\circ$ the most unstable length is 4000 m. Using these lengths the evolution of the undulation width is shown in figure 2.42. It is seen that in this case the widest undulation is for a mean wave direction of $55^\circ$ and the least wide undulation is for a mean wave direction of $65^\circ$. The aspect ratio is 0.3 for all three cases (0.29, 0.28 and 0.28 for $MWD = 55^\circ, 60^\circ$ and $65^\circ$).

The growth rate, $\sigma$, has been estimated using equation 2.22 for both the case with constant length and for the case where the most unstable length is used. The results are seen in table 2.3 together with the growth rates
found in the stability analysis. Firstly it is noted that the mean growth rates for the evolution is one order of magnitude larger than the growth rate found in the stability analysis. This is in line with what can be observed in figure 2.41 and 2.42 where it is seen that the undulations grow slowly in the beginning, fast in the middle and slow towards the end of the simulations. Secondly it is seen that the estimated growth rates are larger for the case with $L = 5000 \text{ m}$ than when using the most unstable length for $MWD = 60^\circ$ and $MWD = 65^\circ$. This is a little surprising; the explanation is found in the fact that for each mean wave direction, the largest mean growth rate is found for the longest undulation. If the growth of the undulation is divided into three parts: Part 1 is the slow growth in the beginning; Part 2 is the fast growth in the middle and Part 3 is the slow growth towards the end. An undulation growing to a large equilibrium width has a relatively larger Part 2 compared to an undulation growing to a small equilibrium width, this is evident when looking at figures 2.41 and 2.42. This has the effect of increasing the mean growth rate, and explains why the mean growth rate is largest for the longest undulation for each mean wave direction.

**Varying Wave Climate**

Next the sinusoidal evolution model is run for a range of varying wave climates, these are characterized by the $U$-parameter which represents the fraction of waves coming from the unstable wave regime, see section 2.3.5. The range of values used are: $U = 0.7 - 1.0$, while keeping the length of the
2.5. SINUSOIDAL SHORELINE EVOLUTION

Table 2.3: The mean growth rates for the different cases. The numbers in parenthesis are the growth rates found in the stability analysis.

<table>
<thead>
<tr>
<th>Mean wave direction</th>
<th>Most unstable length</th>
<th>Length = 5000 m</th>
</tr>
</thead>
<tbody>
<tr>
<td>55°</td>
<td>1.1 (0.18) m/year</td>
<td>0.8 (0.06) m/year</td>
</tr>
<tr>
<td>60°</td>
<td>2.0 (0.31) m/year</td>
<td>2.6 (0.09) m/year</td>
</tr>
<tr>
<td>65°</td>
<td>3.1 (0.42) m/year</td>
<td>4.1 (0.13) m/year</td>
</tr>
</tbody>
</table>

undulation in the model fixed at 5000 m; this is in contrast to the results from the stability analysis (section 2.4.6), where it was found that the length of the most unstable undulation increases for decreasing $U$. This choice has been made here to isolate the effect of the $U$-parameter from the dependence on the undulation length.

The evolution of the width of the undulation is shown in figure 2.43. For $U = 0.9$ and 1.0 the morphological time step is 10 years, and for the remaining runs the morphological time step is 1 year. This time step influences the variability of the width of the shoreline such that a larger time step results in a larger variability of the width. For $U = 1$ the width evolution is seen to agree with the result in figure 2.41. For $U = 0.9$ the variation in width is seen to be around 600 m, with a mean value for the width around 1100 m, the extreme variation in width is due to the large morphological time step of 10 years. For $U = 0.8$ the variation in width is less violent than for $U = 0.9$ due to a smaller morphological time step of 1 year. The width is seen to fluctuate around 700 m. The only results not violating the assumption of small undulation width is the simulation with $U \leq 0.75$. For $U = 0.75$ the width of the undulation is seen to vary between 20 m and 100 m. For $U = 0.7$ the undulations are slowly disappearing, i.e. the undulation is stable under the given wave climate. Thus it is seen that a narrow band of wave climates gives undulations in the linear regime when keeping the undulation length constant.

2.5.2 Evolution of Multiple Undulations

To see the effect of having multiple undulation in one domain, the extended sinusoidal model (see section 2.3.7) is run for the wave conditions defined by $U = 0.75$ and $A = 0.5$, thus the same conditions which gave a small amplitude equilibrium width in the case of a single sinusoidal undulation (section 2.5.1).

The initial shoreline for this model run is described by:

$$y = 5 \cos \left( \frac{2\pi}{5000} x \right) + 5 \cos \left( \frac{2\pi}{3000} x \right)$$

(2.60)

thus only two sinusoidal components are present in the initial shoreline, one with a length of 3000 m and one with a length of 5000 m.
Figure 2.43: The variation in undulation width as function of time using the sinusoidal evolution model for different values of $U$, i.e. the fraction of waves coming from directions larger than $45^\circ$ or smaller than $-45^\circ$.

The evolution of the amplitudes of the 10 longest wavelengths is shown in figure 2.44. The figure shows that the undulation with length 5000 m, does not grow very large, this is in agreement with the result from section 2.5.1. However it is also seen that the undulation with length 7000 m grows quite large, i.e. between 400 and 500 meters in amplitude. This behavior is expected from the stability analysis of the wave climates, i.e. see figure 2.39, where it is seen that for $U$ between 0.7 and 0.8 the most unstable length is between 6000 m and 8000 m.
2.5. SINUSOIDAL SHORELINE EVOLUTION

Figure 2.44: The evolution of the amplitude of the 10 longest wave lengths for the simulation case with $U = 0.75$ and $A = 0.5$. $U$ is the fraction of waves coming from directions larger than $45^\circ$ or smaller than $-45^\circ$. $A$ is the fraction of waves coming from the left.
2.6 Discussion

Previous work has shown the dependence of the most unstable undulation length on the off-shore extension of the shoreline undulation (Falqués and Calvete (2005)), on the width of the break zone (Falqués and Calvete (2005) and Uguccioni et al. (2006)) and on the inclusion of the inertia terms in the longshore current description (Uguccioni et al. (2006)). The results from these earlier works have been confirmed in the presented results.

For constant wave forcing it was found that: Firstly, the length of the most unstable undulation increases when the directional spreading of the incoming waves increases. Secondly, introducing a phase between the undulation on the shoreline and the undulation on the off-shore bathymetry was found to have a stabilizing effect for most wave directions. Thirdly it was seen that increasing the width of the undulation, increases the length of the most unstable undulation, and that the growth rate of the undulation can become zero or negative when the undulation width is increased. This indicates that the undulation width does have a maximum beyond which it will not grow.

The last results is confirmed in the results from the sinusoidal shoreline evolution model. Using this model it is predicted that the width of a shoreline undulation in dynamic equilibrium with the wave forcing depends mainly on the length of the undulation. For three different mean wave directions ($55^\circ$, $60^\circ$ and $65^\circ$) the ratio of the width and the length of an undulation in dynamic equilibrium is predicted to be 0.3.

For changing wave climates it is found that the length of the most unstable undulation increases for decreasing values of $U$ (fraction of waves from unstable directions). This means that, when the undulation is free to choose its preferred length it will choose a longer length when $U$ is decreased. This leads to a larger width, even though small equilibrium widths are found when keeping the length of the undulation constant and decreasing the $U$ parameter. Thus the presented sinusoidal model does not predict small undulation width for any wave climates when the undulation is free to choose its own length. This is in contrast to observations of naturally occurring shoreline undulations, where small undulation width are very common, see e.g. the data analysis in chapter 6, Stewart and Davidson-Arnott (1988) and Verhagen (1989).

This discrepancy between the observed undulation dimensions and the dimensions found using the sine model indicates that the physical process(es) responsible for limiting the undulation width is (are) not included in the model. Possible candidates are: Non-linear shape of the shoreline, phase lag between the offshore contours and shoreline undulations, steepening of the beach profile at crest and flattening of beach profile at trough of undulations, interaction between longshore bars and shoreline undulations and a sloping underlying geology such that the water depth beyond the depth of closure.
is not constant.
Chapter 3

Numerical Modeling of Shoreline Undulations: Model Results for Constant and Varying Wave Impact

3.1 Introduction

In the previous chapter a numerical method was presented to study longshore shoreline undulations. The stability of shorelines subject to oblique wave incidence was studied in order to determine under which conditions the shoreline is unstable and to determine the length of the initial undulations which grow in the linear regime for a range of different wave climates.

The present chapter uses the presented evolution model to study the the evolution of the shoreline undulation from the linear to the non-linear regime.

The evolution of shorelines subject to waves in the unstable regime has been studied before in Ashton and Murray (2006a). They used a CERC-type formulation to describe the longshore sediment transport and a one-line model for updating the shoreline position; the curvature of the shoreline was not included in the model. In their work four different shoreline shapes were found depending on the wave climate; these were termed cuspatate bumps, shoreline sand waves, flying spits and reconnecting spits, see figure 3.1.

The cuspatate bump shape and shoreline sand waves are shapes with no spits, the flying spits shape has spits growing towards the off-shore and the reconnecting spits shape have spits that grow towards the off-shore, but sometimes reconnect to the shoreline.

Thus it is seen that the model by Ashton and Murray (2006a) is successful in describing very large scale features, their development and the shoreline shapes resulting from a range of different wave climates. However
due to the assumptions in the model the initial development of shoreline undulations is not described correctly. As is shown in Uguccioni et al. (2006), in the type of model used by Ashton and Murray (2006a), the shortest possible undulation length will always be the one that grows the fastest. Only due to shadowing effects, where one undulation shadows the wave climate for the next undulation, and due to a time varying wave climate can meaningful shoreline shapes be obtained using this model.

As pointed out by Ashton and Murray (2006a) the resulting shoreline shape is always a spit for constant wave forcing in the unstable regime, when the curvature of the shoreline is ignored. This is easily realized: when an initial undulation grows, the difference in the shore-normal angle on the upstream and downstream side increases; this causes the growth rate of the undulation to increase further, which again will increase the difference between the shore-normal angles at the upstream and downstream sides, as shown on the two top sketches in figure 3.2. In this process the downstream point is more and more sheltered from the waves, thus reducing the longshore sediment transport further and further. So the sediment in-flux to the area between the upstream and downstream points keeps increasing, while the out-flux decreases. After a certain time, the angle between the shore-normal at the upstream point and the incoming waves, will be equal to the angle giving maximum longshore transport, $\alpha_c \approx 45^\circ$. At this point the downstream point will be completely shielded from the incoming waves; the
angle here is now above $90^\circ$ as sketched in figure 3.2, resulting in a very low longshore transport. The only solution for the morphology in this case is to create a spit on the downstream side of the undulation.

As shown by Petersen et al. (2008) the spit will be infinitely narrow in case the curvature of the shoreline is ignored; and as pointed out by Ashton and Murray (2006a), it will migrate in the direction giving maximum transport, i.e. $\alpha = \alpha_c$; this is also sketched in figure 3.2. Only by varying the wave climate is it possible to obtain spits with a finite width in the model by Ashton and Murray (2006a).

The mechanisms controlling the width of the spit are described in Petersen et al. (2008). A spit migrating with unchanging shape has accretion along the entire front of the spit, so the longshore sediment transport must be decreasing along the entire front of the spit as shown in figure 3.3. An interesting feature is that the longshore sediment transport is zero at the tip of the spit, in other words: the tip of the spit is located where the longshore sediment transport is zero. The waves cannot push the sediment farther than this point, because the front of the spit shadows the point from the incoming waves. Therefore changes which affect the distance along the front of the spit, which the incoming waves can reach, changes the width of the spit.

The evolution of shoreline undulations from small initial undulations to full non-linear shoreline shapes has also been modeled by Falqués et al. (2009), where the model described in Falqués et al. (2008) is used. The focus of the paper is the role of the cross-shore profile dynamics in relation to the development of shoreline undulations. A model results is presented, where a spit has developed at the downstream end of a shoreline undulation, the model breaks down at this point due to the development of a large hole in the embayment behind the spit, therefore a dynamic equilibrium was not obtained.

A detailed description of the development of shoreline undulations from an initial small undulation to a full non-linear shoreline shape in dynamic equilibrium has, to the authors knowledge, not previously been published. Therefore, a basic understanding of what governs the evolution of the shoreline undulations and the dimensions of the resulting shoreline shapes is still missing.

Scope of the present work

It is hypothesized that there are two main factors, which govern the evolution of the shoreline undulations. These are the angle between the shoreline and the incoming waves and the curvature of the shoreline. This is because, all other things being equal, the strength of the longshore current along a curving shoreline depends strongly on the angle between the incoming waves and the shoreline angle and on the curvature of the shoreline.
Figure 3.2: Sketch showing the development of the shoreline undulation if the curvature of the shoreline is ignored.
The scope of the present chapter is to confirm this hypothesis. The confirmation is sought by answering the following questions:

- How does the development of an initial small shoreline undulation into a fully non-linear shoreline shape occur?
- What is the equilibrium width of a shoreline undulation subject to a constant wave climate?
- Under what conditions does a spit form on the downstream end of the undulation?
- What are the shape and dimensions of the formed spit?

3.2 Numerical Methods

To study the questions posed above, the shoreline model described in chapter 2 is used to model the evolution of longshore shoreline undulations subject to: firstly constant wave climates and secondly to varying wave climates.

In order to quantify the answers to the questions posed, the following quantities regarding the morphology are defined (see figure 3.4):

- Width of the undulation, \( w_u \); this is the width of the undulation if the embayment behind a possible spit is filled up with sediment.
- Total width of the undulation, \( w_{u,tot} \); this is the width from the smallest \( y \)-value to the largest \( y \)-value.
CHAPTER 3. CONSTANT AND VARYING WAVE IMPACT

Figure 3.4: Sketch showing the parameters defined to quantify the evolution of the shoreline undulations.

- Width of the spit, \( w_s \).
- Total width of the spit, \( w_{s,tot} \).
- \( x \)-coordinate of the tip of the spit, \( x_{spit} \).
- \( y \)-coordinate of the tip of the spit, \( y_{spit} \).
- Phase between the location of the maximum in the longshore sediment transport and location of the crest of the shoreline undulation.
- Curvature of the shoreline at the crest of the undulation, \( \kappa_{crest} \).

The curvature of the shoreline at the crest is computed as:

\[
\kappa_{crest} \approx \frac{d^2y}{dx^2}
\]  

where \( x, y \) describes the position of the shoreline. Since the gradient of the shoreline is always small at the crest, this is a reasonable approximation.

3.3 Shoreline Evolution for Constant Wave Climate

3.3.1 Evolution of Shoreline Undulation

First the evolution of the shoreline subject to a constant wave climate is investigated. The used wave parameters are shown in table 3.1 together with the remaining parameters for the Mike21FM model. From figure 2.27 it is seen that an undulation with a length of 5000 m is unstable for the chosen wave climate, thus a single undulation with this length will grow in
3.3. EVOLUTION FOR CONSTANT WAVE CLIMATE

Table 3.1: Model parameters used when not stated otherwise.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wave parameters:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Significant Wave height</td>
<td>$H_s$</td>
<td>1 m</td>
<td></td>
</tr>
<tr>
<td>Mean Wave Direction</td>
<td>MW D</td>
<td>60 $^\circ$</td>
<td></td>
</tr>
<tr>
<td>Directional Spreading Index</td>
<td>$DSI$</td>
<td>10</td>
<td>-</td>
</tr>
<tr>
<td>Peak wave period</td>
<td>$T_p$</td>
<td>5 s</td>
<td></td>
</tr>
<tr>
<td>Breaking wave parameter</td>
<td>$\gamma$</td>
<td>0.8</td>
<td>-</td>
</tr>
<tr>
<td>Hydraulic parameters:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Manning number</td>
<td>M</td>
<td>20 32 45</td>
<td>$m^{1/3}/s$</td>
</tr>
<tr>
<td>Smagorinsky coef.</td>
<td>-</td>
<td>0.28</td>
<td>-</td>
</tr>
<tr>
<td>Sediment parameters:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sediment porosity</td>
<td>$p$</td>
<td>0.4</td>
<td>-</td>
</tr>
<tr>
<td>Sediment grain diameter</td>
<td>$d_{50}$</td>
<td>0.2 mm</td>
<td></td>
</tr>
<tr>
<td>Sediment grain grading coef.</td>
<td>$\sigma$</td>
<td>1.1</td>
<td>-</td>
</tr>
<tr>
<td>Relative sediment density</td>
<td>$s$</td>
<td>2.65</td>
<td>-</td>
</tr>
<tr>
<td>Critical shield parameter</td>
<td>$\theta_c$</td>
<td>0.05</td>
<td>-</td>
</tr>
</tbody>
</table>

The model and is therefore chosen. The amplitude of the initial undulation is 10 m.

The first result presented is a time stack of the evolution from the initial sinusoidal undulation during an asymmetric evolution to the point when a spit is formed on the downstream end. This result is depicted in figure 3.5. It is seen that the shoreline undulation keeps its sinusoidal shape up to around 100 years, where the asymmetric shape starts to develop; i.e. the slope of the undulation becomes mild on the upstream side and steep on the downstream side. This change is related to changes in the sediment transport on the downstream side due to the increase in the angle between the shoreline normal and the incoming waves, $\alpha$, this is investigated in detail later in the present section. It is observed that the undulation migrates 1.5 lengths to the left before the spit is formed.

Figure 3.6 shows the evolution of the undulation after the formation of a spit at the downstream end. First it is seen that the part of the shoreline, which is shadowed by the spit does not move at all after the spit is formed. In nature the bay which is formed behind the spit sometimes fill up by aeolian sediment transport, this mechanism is not included in the model; it can also fill up due to a variable wave climate as will be seen in section 3.4. Further it is noted that the tip of the spit is migrating with a more or less constant speed, however with fluctuations around this average speed. Next
Figure 3.5: The time stack of the evolution of an undulation subject to $H_s = 1\, \text{m}$, $T_p = 5\, \text{s}$, $MWD = 60^\circ$ and $DSI = 10$ from the initial small undulation to the formation of a spit. Note that 2 undulation are shown.
it is seen that the tip of the spit migrates a little towards the shoreline in the trough of the undulation before a new tip is formed. This new tip overtakes the old tip and in this process the periodic features on the shoreward part of the spit are formed. This process is related to the variation in the sediment transport around the tip of the spit as explained in the analysis of the sediment budget below.

Analysis of Sediment Budget During Evolution

The sediment budget along the undulation during its evolution can give valuable information regarding the behavior of the shape of the undulation. Therefore 4 points along the undulation are defined, at the crest, at the trough, and at the two zero crossing, see figure 3.7 top. After the development of the spit the points are defined on a truncated shoreline, where the truncated shoreline is the shoreline resulting from filling up the entire bay shoreward of the spit. An example of a truncated shoreline is also shown in figure 3.7 bottom.

The longshore sediment transport at the four points is shown as function of time in figure 3.8. The figure also shows the development in the width of the undulation for the whole shoreline, \( w_{u,\text{tot}} \), and the width of the undulation for the truncated shoreline, \( w_u \). Further the phase between the maximum in the longshore sediment transport and the maximum in the \( y \) coordinate of the shoreline is shown together with the curvature of shoreline at the crest of the undulation, \( \kappa_{\text{crest}} \). The shape of the shoreline at different times are shown on the bottom of the figure, with a dot showing the location in time of the shown shape.

As pointed out, the difference between the sediment transport at the upstream and downstream points determines if the undulation will grow or decay in the linear regime. A second requirement for growth is that the phase between the location of the maximum in the longshore sediment transport and the location of the crest of the undulation is positive as indicated in figure 3.4, i.e. the maximum in the longshore transport must be located upstream of the maximum in the \( y \)-coordinate of the shoreline. As seen in figure 3.4 a positive phase ensures that the gradient of the sediment transport is negative at the crest of the undulation. A negative gradient in the sediment transport means that more sediment is arriving at the crest than leaving the crest, thus sediment is being deposited at the crest thereby making the undulation grow.

When looking at the phase and the difference between the upstream and downstream longshore transport in figure 3.8 it is seen that the phase is positive, and there is a positive influx of the sediment to the top of the undulation throughout the simulation period. The positive phase can be explained from the \( q - \alpha \) diagram shown in figure 3.9 (left). When moving along a sinusoidal undulation the angle between the incoming waves and
Figure 3.6: The time stack of the evolution of an undulation subject to $H_s = 1\text{m}$, $T_p = 5\text{s}$, $MWD = 60^\circ$ and $DSI = 10$ after the formation of a spit at the downstream end. Note that 2 undulations are shown.
Figure 3.7: The location of the 4 points used in the analysis of the sediment budget along the undulation.
Figure 3.8: Top: The undulation width, \( w_u \), and total undulation width, \( w_{u,\text{tot}} \). Middle: The phase difference between the location of the maximum in the longshore sediment transport and the location of the crest of the undulation, and the curvature of the shoreline around the crest point, \( \kappa_{crest} \). Note that a smoothing filter has been applied to both in order to get smooth curves. Bottom: The longshore sediment transport at the four points and the change in shape of the shoreline.
Shoreline
Longshore sediment transport

Figure 3.9: Left: Sketch of longshore sediment transport, $q$ as function of angle between incoming waves and shoreline normal, $\alpha$. Right: Longshore transport along a sinusoidal shoreline undulation based only on the $q-\alpha$ diagram for both $\alpha > 45^\circ$ (full line) and $\alpha < 45^\circ$ (dotted line).

the normal to the shoreline, $\alpha$, changes due to the change in shoreline orientation. The angle is smallest at the upstream point and largest at the downstream point, thus according to the $q-\alpha$ diagram the longshore sediment transport should be largest at the upstream point and smallest at the downstream point when $\alpha \approx 45^\circ$. If the longshore transport was determined solely based on the $q-\alpha$ diagram the longshore transport along the sinusoidal shore is shown in figure 3.9 (right).

From figure 3.8 it is seen that the phase is large ($\approx 30^\circ$) in the beginning and it decreases as both the width of the undulation and the curvature of the shoreline at the crest of the undulation increases. The deviation from the ideal longshore sediment transport is mainly due to the effect of the curving bathymetry. From around 120 years and forward, the phase is fluctuating around $10^\circ$. These observations are in line with the behavior of the width of the undulation, which continues to grow. However, the width of the truncated shoreline does become almost constant after approximately 135 years, which shows that the undulation is actually not increasing its width anymore; the increase in width is an imprint of the spit growing in a slight offshore direction.

At the time when the phase starts fluctuating around $10^\circ$, the top and the upstream longshore sediment transport becomes almost constant. This indicates two things, firstly that the angle between the incoming waves and the angle of the shoreline in this area does not change and secondly that the curvature of the shoreline in this area does not change (which is also observed), since changes in these two parameters is what drives changes is the longshore sediment transport. Therefore the shape of the shoreline in the crest- and the upstream-area must be close to constant from this time on, i.e. if both the shoreline angle and the shoreline curvature are constant there is not room for many changes to the shoreline shape.

Slightly later in time (around 130 years) the sediment transport becomes
zero at the trough point. This is due to the formation of a spit which effectively shadows the bottom point from the waves. From this time on it is only the sediment transport at the downstream point, which fluctuates slightly. These fluctuation are linked to the formation of the quasi periodic features on the shore ward part of the spit. In figure 3.10 is shown a zoom of the lower figure in figure 3.8. On this figure it can be observed that the peaks in the fluctuating longshore sediment transport at the downstream point are related to periods, where the tip of the spit is growing towards the shoreline, and the troughs in the longshore sediment transport at the downstream point are related to periods, when a new spit is forming and overtaking the old tip. E.g. between 140 years and 145 years, a small trough in the longshore sediment transport is observed, and the development of a new spit tip is also seen; and other example is between years 154 and 159 where another small trough in the transport is observed and a new spit tip is also forming.

While the new spit is forming, the shoreline angle at the downstream point is close to being perpendicular to the original shoreline orientation, therefore the point is shadowed from the waves and does not receive much wave energy. Thus the longshore sediment transport is low. An example of this is seen in figure 3.10 between 150 years and 160 years. Once the new spit has formed, the angle decreases and thus the transport increases. The curvature of the shoreline around the downstream point supports the behavior, since the curvature is low before the spit has formed, and increases after it has formed. A large shoreline curvature focuses the waves, thereby increasing the longshore transport.

### 3.3.2 Wave Direction Effect

To see how different wave parameters affect the evolution of the shoreline undulation, models runs with mean wave directions $MW\theta = 52^\circ$, $MW\theta = 55^\circ$ and $MW\theta = 65^\circ$ have been done. In the following, the results from these runs are compared with the result from the previous section, where
3.3. EVOLUTION FOR CONSTANT WAVE CLIMATE

$MW D = 60^\circ$. For the case of $MW D = 52^\circ$, no spit forms at the downstream end, therefore no time stack is shown for this case. Because the evolution of the undulation until the formation of a spit is very similar in the three other cases, the time stacks depicted begin at this point. Figure 3.11 shows $MW D = 55^\circ$ and figure 3.12 shows $MW D = 65^\circ$.

It is noted that the evolution for $MW D = 65^\circ$ is very similar to the evolution for $MW D = 60^\circ$ (see figure 3.6). The evolution for $MW D = 55^\circ$ is different than the other two, because 16 years after the spit is formed, it reconnects to the shoreline in the trough of the undulation, hereafter a new spit is formed, which also reconnects to the shoreline.

The variation over time of the 7 quantities described in section 3.2 (width of the undulation, $w_u$; total width of the undulation, $w_u,tot$; width of the spit, $w_s$; total width of the spit, $w_s,tot$; $x$-coordinate of the tip of the spit, $x_{spit}$; $y$-coordinate of the tip of the spit, $y_{spit}$; phase between the location of the maximum in the longshore sediment transport and the crest of the undulation; curvature of the shoreline at the crest, $\kappa_{crest}$) are found for the four model runs ($MW D = (52^\circ, 55^\circ, 60^\circ$ and $65^\circ$)) when possible.

Figure 3.13 shows that the total width of the undulation only reaches a maximum in the simulation with $MW D = 52^\circ$ and $MW D = 55^\circ$. This
Figure 3.12: The time stack of the evolution of an undulation subject to $H_s = 1\text{m}$, $T_p = 5\text{s}$, $MWD = 65^\circ$ and $DSI = 10$ after the formation of a spit at the downstream end. Note that 2 undulations are shown.
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Figure 3.13: Width of the undulation, $w_u$, (dashed lines) and total width of the undulation, $w_{u,\text{tot}}$, (full lines) as function of time for different mean wave directions. The remaining properties were set according to table 3.1.

maximum is equal to the maximum of the undulation width. For the other two simulations the total width keeps growing as time goes, because the spit in these two cases grows in a slightly offshore direction. The total undulation width grows the fastest in the simulation with $MWD = 65^\circ$.

Figure 3.13 further shows that a maximum undulation width is almost reached for all four simulations: a small increase in $w_u$ at the end of the simulation is observed for $MWD = 60^\circ$ and $MWD = 65^\circ$. This width depends on the wave direction, for $MWD = 52^\circ$ the maximum width is around 510 m, for $MWD = 55^\circ$ it is around 1050 m, for $MWD = 60^\circ$ it is around 1180 m and for $MWD = 65^\circ$ it is around 1260 m. These widths correspond to aspect ratios of 0.10, 0.21, 0.24 and 0.25, where the aspect ratio is the ratio of the width of the undulation to the length of the undulation, $w_u/L$.

Figures 3.14 and 3.15 show that the width of the spit is highly variable in all cases, which can be related to the quasi periodic features on the shore ward side of the spit. Due to the large variability the mean values are not very accurate. The mean total width of the spits, $w_{s,\text{tot}}$, are 830 m, 840 m and 940 m for $MWD = 65^\circ$, $60^\circ$ and $55^\circ$. The mean width of the spits, $w_s$, are 530 m, 510 m and 730 m for for $MWD = 65^\circ$, $60^\circ$ and $55^\circ$. It is noted that the mean width of the spit for $MWD = 60^\circ$ is larger than for $MWD = 65^\circ$, this is attributed to the large variability of the width of the spit for $MWD = 65^\circ$.

The migration speed of the spit can be deduced from the change in the $x$-coordinate of the tip of the spit shown in figure 3.16. It is observed that
Figure 3.14: Width of the spit as function of time for different mean wave directions. The remaining properties were set according to table 3.1.

Figure 3.15: Total width of the spit as function of time for different mean wave directions. The remaining properties were set according to table 3.1.
Figure 3.16: Alongshore migration of the spit tip (full lines) and the undulation crest (dashed lines) for different mean wave directions. The remaining properties were set according to table 3.1.

The three curves are rather straight lines, and that the slope of three curves is very similar. From this it can be concluded that the migration speed of the spit does not depend on the incoming wave direction, and that the migration speed of the spit is rather constant during the migration. This is in agreement with what can be observed in figure 3.6.

The $y$-coordinate of the tip of the spit is shown in figure 3.17. It is observed that the location of the tip of the spit is very dynamic, this is due to the periodicity in the migration path of the spit tip as discussed in section 3.3.1; i.e. the tip of the spit migrates towards the trough of the undulation a certain distance until it is overtaken by a new spit tip which then grows towards the trough. Due to the relatively coarse discretization of the mesh around the tip of the spit, the instantaneous migration direction is not completely accurate in the present model. However the average migration direction is not controlled by the discretization around the spit of the tip, but by the shape of the longshore transport along the spit, which is sufficiently accurate in the present model as shown in section 3.3.5.

The angle between the original shoreline and the average migration direction of the spit can be found by looking at the $x$ and $y$ coordinates of the crest point. As seen in figure 3.16 and 3.18, the crest point of the undulation migrates alongshore together with the spit and can therefore be used to define the migration path of the spit. This has the advantage compared to using the coordinates of the tip of the spit, that the crest point fluctuates less than the tip of the spit and is therefore a more robust way of determining the migration direction.
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Figure 3.17: y-coordinate of the tip of the spit as function of time for different mean wave directions. The remaining properties were set according to table 3.1.

Figure 3.18: Example showing the migration path of the spit defines by the crest point.
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Figure 3.19: \((x, y)\) position of the crest point of the undulation for the different mean wave directions. The remaining properties were set according to table 3.1. Dashed lines are linear regression lines for each mean wave direction.

Figure 3.19 shows the \(x\) and \(y\) position of the crest point for the three mean wave directions which developed spits, together with the linear regression lines for each of the mean wave directions. From the gradient of the regression lines the angle between the original shoreline and the migration direction of the crest point can be found as \(\alpha_{crest} = \tan \left( \frac{dy}{dx} \right)\), where \(\frac{dy}{dx}\) is the gradient of the regression line. The angle between the original shoreline and the general migration direction of the crest, \(\alpha_{crest}\), has been found to be 1.1° for \(MWD = 55°\), 4.4° for \(MWD = 60°\) and 7.5° for \(MWD = 65°\). It is noted that the crest point for \(MWD = 55°\) is moving slightly away from the shoreline, this is surprising since the spit is continually reconnecting with the shoreline in the trough; the explanation is that the entire system is moving slightly away from the original shoreline location due to the formation of lakes when the spit reconnects.

The curvature of the shoreline around the crest point is shown in figure 3.20 for the four mean wave directions. It is seen that the curvature around the crest point fluctuates around a more or less constant value towards the end of each of the simulations. The absolute value of the curvature is largest for \(MWD = 65°\) and smallest for \(MWD = 52°\).

To compare the growth rate of the evolution model with the growth rate of the sinusoidal evolution model and the growth rate found in the stability analysis, the robust estimate of the mean growth rate described in section 2.3.7 (equation (2.22)) is used. It is noted that the width, \(w\), in equation (2.22) is taken to be \(w_u\) and not the total width \(w_{u,tot}\). This is because the total width does not reach a constant value in all cases due to the off-shore...
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Figure 3.20: The curvature of the shoreline around the crest point of the undulation, for the different mean wave directions. The remaining properties were set according to table 3.1.

Table 3.2: The mean growth rates for the different mean wave directions for the stability analysis, the sinusoidal evolution model and the evolution model.

<table>
<thead>
<tr>
<th>$MWD$</th>
<th>Stability Analysis</th>
<th>Sinusoidal model</th>
<th>Evolution Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>55°</td>
<td>0.18 m/year</td>
<td>1.1 m/year</td>
<td>3.2 m/year</td>
</tr>
<tr>
<td>60°</td>
<td>0.31 m/year</td>
<td>2.0 m/year</td>
<td>7.1 m/year</td>
</tr>
<tr>
<td>65°</td>
<td>0.42 m/year</td>
<td>3.1 m/year</td>
<td>9.9 m/year</td>
</tr>
</tbody>
</table>

The result is shown in table 3.2 together with the growth rates for the sine mode and the stability analysis, note that $MW D = 52^\circ$ is not shown as it was not included in the other models. It is noted that both the sine model and the evolution model gives mean growth rates that are an order of magnitude larger than the growth rates found in the stability analysis. This is expected since the growth rate is much smaller in the beginning of the evolution than later on, as seen in figure 3.13. Further it is noted that the evolution model gives mean growth rates that are around three times larger than the growth rates from the sine model.

3.3.3 Wave Spreading Effects

Next the dependence on the spreading of the incoming waves is studied by running the model for directional spreading indices $DSI = 25$ and $DSI = 40$ for $MW D = 60^\circ$. It is noted that $DSI = 10$ is equal to a directional
3.3. EVOLUTION FOR CONSTANT WAVE CLIMATE

standard deviation of 17.2°, $DSI = 25$ is equal to a directional standard deviation of 11.2° and $DSI = 40$ is equal to a directional standard deviation of 8.9°. The remaining parameters are set according to table 3.1. As in the previous section, these runs are compared with the model run from section 3.3.1. The time stacks of the shoreline evolution for the two cases are shown in appendix B, since they resemble the time stacks shown in the previous sections.

The width of the undulation and the total width of the undulation is seen in figure 3.21 for three model runs. The width is seen to increase with decreasing wave spreading (for increasing $DSI$). This is because an increase in the wave spreading at $MW D = 60°$ will increase the amount of waves coming from directions smaller than the critical angle for shoreline instability, thus decreasing the final width of the undulation. The same effect explains why the undulation grows the fastest for the smallest wave spreading.

The width of the spit is shown in figure 3.22, where it is observed that the width of the spit decreases for decreasing wave spreading. For a given spit shape a smaller wave spreading results in the waves reaching a shorter distance along the front of the spit as shown in figure 3.23. As explained in section 3.1 this results in a decrease in the width of the spit because the waves cannot push the sediment as far along the front of the spit.

The alongshore migration of the tip of the spit is shown in figure 3.24. It is observed that the migration speed of the spit does depend on the wave spreading. Further it is noted that the average alongshore migration speed
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Figure 3.22: Width of the spit, $w_s$, as function of time, for different directional spreading indices $DSI$. The remaining parameters were set according to table 3.1.

Figure 3.23: Sketch showing how a wave climate with a smaller directional spreading, $\sigma_2$, is shadowed more by the spit than a wave climate with a larger directional spreading, $\sigma_1$, and therefore cannot generate a longshore sediment transport along the entire spit, resulting in a spit with a smaller width.
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Figure 3.24: Alongshore migration of the spit (full lines) and the crest of the undulation (dashed lines) for different directional spreading indices $DSI$. The remaining parameters were set according to table 3.1.

of the spit and of the undulation crest is the same.

Next the average migration direction of the spit is determined based on the migration direction of the crest point shown in figure 3.25. The average migration direction is found to be $4.4^\circ$ for $DSI = 10$, $7.6^\circ$ for $DSI = 25$ and $8.0^\circ$ for $DSI = 40$.

The curvature of the shoreline around the crest point is shown in figure 3.26 for the different directional spreading indices. The absolute value of the curvature is largest for $DSI = 40^\circ$ and smallest for $DSI = 10^\circ$.

3.3.4 Undulation Length Effect

In the previous sections the undulation length is kept constant at 5000 m. This is in order to be able to isolate the effect of the mean wave direction and of the directional wave spreading. However this approach suffers from the fact that if e.g. the mean wave direction is changed, the length of the most unstable undulation also changes, as is seen in section 2.3.6. This can have an important impact on the shape of the resulting shoreline.

In the present section the mean wave direction is varied as in section 3.3.2, but the length of the undulation is chosen based on the most unstable length found in the stability analysis in section 2.3.6. In table 3.3 the most unstable undulation length corresponding to three different mean wave directions are shown for the default model parameters from table 3.1. To save computational time the amplitude of the initial shoreline undulation was 200 m in all three cases.

The time stacks of the shoreline evolution for the three cases are shown
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Figure 3.25: \((x, y)\) position of the crest point of the undulation for the different direction spreading indices, \(DSI\). The remaining properties were set according to table 3.1. Dashed lines are linear regression lines.

Table 3.3: The mean wave directions and corresponding most unstable undulation length for the default model parameters shown in table 3.1.

<table>
<thead>
<tr>
<th>Mean Wave Direction</th>
<th>Most Unstable Length, [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>55°</td>
<td>5500</td>
</tr>
<tr>
<td>60°</td>
<td>4500</td>
</tr>
<tr>
<td>65°</td>
<td>4000</td>
</tr>
</tbody>
</table>
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The width of the undulation, \( w_u \) (dashed lines), and total width of the undulation, \( w_{u,tot} \) (full lines) as function of time for the three cases, where mean wave direction is varied, and the undulation length is the most unstable length for each mean wave direction. The remaining parameters were set according to table 3.1.

in appendix B, since they resemble the time stacks shown in the previous sections.

The width of the undulation, \( w_u \), is shown for the three cases in figure 3.27, where it can be observed that the largest width is found for a mean wave direction \( MWD = 55^\circ \) and the smallest width is found for \( MWD = 65^\circ \). This is in contrast to the findings in section 3.3.2 where the opposite is seen, i.e. the smallest width is found for \( MWD = 55^\circ \) and the largest width is found for \( MWD = 65^\circ \). This difference arises because the length of the undulation is not kept constant in the present section. Furthermore, it is observed that in all three cases the gradient of the total width of the undulation is constant towards the end of the simulation period. This is due to the spit growing in a slight off-shore direction in all three cases. The observed widths correspond to aspect ratios of 0.23 for the cases with \( MWD = 65^\circ \) and \( MWD = 60^\circ \). For the case with \( MWD = 55^\circ \) the aspect ratio is 0.22.

The total width of the spit, \( w_{s,tot} \), is shown for the three cases in figure 3.28 together with the width of the spit, \( w_s \). The total width of the spit is seen to be largest for mean wave direction \( MWD = 55^\circ \) and smallest for \( MWD = 65^\circ \). This is in line with the results in section 3.3.2. For the width of the spit, \( w_s \), the picture is the same; i.e. largest width is for \( MWD = 55^\circ \) and smallest width is for \( MWD = 65^\circ \) in line with the results in section 3.3.2.

The alongshore migration of the spit tip is shown in figure 3.29. It is
observed that the spit tip migrates fastest for a mean wave direction of $MWD = 65^\circ$ and slowest for $MWD = 55^\circ$. This is in contrast with the results from section 3.3.2 where the alongshore migration speed was the same for all three mean wave directions.

The position of the crest of the undulation is shown in figure 3.30 together with linear regression lines for each case. The regression lines have only used data points after the creation of a spit. The gradients of the regression lines have been used to determine the migration direction of the crest point quantified by $\alpha_{crest}$; the angle between the original shoreline and the general migration direction of the crest. $\alpha_{crest}$ has been found to be $1.9^\circ$ for $MWD = 55^\circ$, $3.2^\circ$ for $MWD = 60^\circ$, and $3.9^\circ$ for $MWD = 65^\circ$.

The curvature of the shoreline around the crest point is shown in figure 3.26 for the three cases. It is seen that the curvature around the crest point fluctuates around a more or less constant value towards the end of each of the simulations. The absolute value of the curvature is largest for $MWD = 65^\circ$ and smallest for $MWD = 55^\circ$.

One additional model run was performed for $MWD = 50^\circ$ and $L = 7000$ m. For this run the cross-shore discretization had to be reduced in the middle of the simulation to ensure stability, therefore the results from this run are not included in the present results, but the time stacks can be seen in appendix B.
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Figure 3.29: Alongshore migration of the spit, for the three cases, where the mean wave direction is varied, and the undulation length is the most unstable length for each mean wave direction. The remaining parameters were set according to table 3.1.

Figure 3.30: $(x,y)$ position of the crest point of the undulation, for the three cases, where the mean wave direction is varied, and the undulation length is the most unstable length for each mean wave direction. The remaining properties were set according to table 3.1. Dashed lines are linear regression lines.
3.3.5 Grid Resolution Effects

The results presented in this section have all been obtained on a mesh with 35 points on the shoreline and $a = 10$ m in the grid generation, see section 2.3.2. This is by no means a good resolution of the shoreline and the problem is clearly larger when a spit is formed, since the spit is a smaller features that is being described with the same resolution as describes the shoreline. It was tried to vary the resolution in the model so the points on the shoreline were closer on the spit, but this was never successful. Figure 3.32 shows the reason, why this was never successful; from the figure it is clear that a varying longshore grid resolution results in a varying longshore sediment transport, which will result in a morphologic change, which can then feed back to the hydrodynamics and so on.

The rather rough resolution is chosen out of necessity; the computational cost for increasing the number of points on the shoreline is extreme in the model. This is because the finer resolution not only affects the number of computational cells, and therefore increases the cost of computing a single time step, but also reduces the morphological time step, thereby increasing the number of time steps before a dynamic equilibrium is obtained. This means one has to accept a given resolution in order to obtain any results at all from the evolution model, which also means that the uncertainty of the model results can be quite severe.

To get an estimate of the impact of the resolution in the model the run
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Figure 3.32: The longshore sediment transport as function of longshore coordinate for a computation where the longshore grid resolution varies along the shoreline as shown, i.e. $\Delta x$ is the distance between the shoreline points.
with $MWD = 65^\circ$ (see section 3.3.2) is repeated once using $n = 20$ and $a = 15$ m in the grid generation process and once using $n = 40$ and $a = 10$.

Figure 3.33 shows the width and total width of the undulation for all three runs. Some differences are observed: The undulation in the $N = 20$ model grows slower, but to a larger width than the undulation in the $N = 35$ and $N = 40$ runs. Furthermore, the total width of the undulation is increasing faster in the $N = 20$ run compared with the $N = 35$ and $N = 40$ runs.

The two results from the finer models are quite similar both regarding the width of the undulation and regarding the angle between the original shoreline and the migration direction of the spit. However, the difference in resolution between them is also small.

### 3.4 Shoreline Evolution for Changing Wave Climate

#### 3.4.1 Implementation of Changing Wave Climate

The effect of changing wave climates is studied using the concept of the $U$ and $A$ parameters defined by Ashton and Murray (2006a). $U$ is the fraction
of waves coming the unstable wave regime meaning directions larger than \(45^\circ\) or smaller than \(-45^\circ\), i.e. from direction intervals \(\Delta \alpha_1\) and \(\Delta \alpha_4\) in figure 3.34. \(A\) is the fraction of waves coming from the left, meaning the fraction of waves from directions smaller than \(0^\circ\), i.e. from direction intervals \(\Delta \alpha_1\) and \(\Delta \alpha_2\) in figure 3.34.

Every time the mean wave direction is changed in the simulation, a random number is generated, if the number is larger than \(U\) then a direction from the stable wave regime is chosen, if the number is smaller than \(U\) a wave direction from the unstable wave regime is chosen. A second random number is generated to choose which side the wave should come from. If the number is smaller than \(A\), the wave is chosen to come from the left, meaning that \(MWD < 0^\circ\), if the number is larger than \(A\) the wave is chosen to come from the right, meaning that \(MWD > 0^\circ\).

In the Mike21FM model, changing the mean wave direction between two morphological time steps means that the longshore current must be accelerated or decelerated, which takes time. Therefore the model runs a lot faster when the mean wave direction is not changed very often (or not at all as in the previous section). This means that an approach where the mean wave direction is changed at every new morphological time step is very time consuming in the present model.

Therefore a faster approach is used in the following. Instead of picking a random wave direction in each interval, a representative wave direction for each interval is chosen. This choice is made based on the growth rates shown in figure 2.38. This figure shows that the average growth rate for direction interval \(\Delta \alpha_4\) is well represented by the growth rate for \(MWD = 60^\circ\), and that the average growth rate the direction interval \(\Delta \alpha_3\) is well represented by the growth rate for \(MWD = 30^\circ\). This indicates that in the linear regime there will be very little difference between using a random wave direction in the interval \(45^\circ - 90^\circ\) at every new morphological time step, and using \(60^\circ\) for all the directions that fall in this interval. In the same way, using \(MWD = 30^\circ\) instead of a random mean wave direction between \(0^\circ\) and \(45^\circ\) should give almost the same result in the linear regime. It is well known from i.e. Hanson (1989) that this is not the case in the non-linear regime, but it is a necessary simplification due to the long simulation times of the present model. Lastly, a time step, \(\Delta T_{MWD}\), is introduced; it determines...
Table 3.4: Overview of the main simulations made with changing wave climates. "NS" means that no spit is formed during the computation. "S" means that a spit is formed during the computation and - means that this combination of \( U \) and \( A \) was run not.

<table>
<thead>
<tr>
<th>( U )</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>NS</td>
<td>NS</td>
<td>NS</td>
</tr>
<tr>
<td>0.9</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>NS</td>
<td>S</td>
<td>S</td>
</tr>
<tr>
<td>1.0</td>
<td>NS</td>
<td>NS</td>
<td>NS</td>
<td>S</td>
<td>S</td>
<td>S</td>
</tr>
</tbody>
</table>

how often a new value for the mean wave direction is computed.

### 3.4.2 Evolution of Shoreline Undulations

All wave parameters and model parameters are kept constant except the wave direction, the other model parameters are set according to table 3.1. Model simulations are made for the combinations of \( U \) and \( A \) shown in table 3.4 all for \( \Delta T_{MWD} = 100 \) days. Some of them are also run for \( \Delta T_{MWD} = 10 \) days. The results are presented in the following.

The evolution of the width and the total width of the shoreline undulation for the simulations from table 3.4 is shown in figure 3.35. The figure shows that the width depends greatly on the \( U \); the width decreases for decreasing \( U \), whereas the width does not depend on the \( A \) parameter. However for \( U = 1 \) the difference between the curves for the different values of \( A \) is the smallest, and for \( U = 0.8 \) this difference is largest. This behavior can be explained by the fact that for \( U = 0.8 \) the wave climate is changing more often than for \( U = 1.0 \). For \( U = 1.0 \) it is noted that the simulation with \( A = 0.5 \) gives the largest width around 1500 m which gives an aspect ratio of 0.3 (the length is 5000 m) which is slightly lower than the value found using the stability diagram in figure 2.40 where a value between 0.35 and 0.4 was found, but equal to the aspect ratio found using the sinusoidal evolution model in section 2.5.1.

For \( U = 0.8 \) an equilibrium has not yet been reached in any of the simulations. From result of the simulation with \( U = 0.8 \) and \( A = 1 \) it seems like it may never happen; due to the large value of \( \Delta T_{MWD} = 100 \) days, the stochastic behavior of the mean wave direction causes the width of the undulation to fluctuate around a certain width with a large amplitude of the fluctuation. Adding the wave directions in the \( U = 0.8 \) and \( A = 1.0 \) run gives \( MWDA = 0.8 \cdot 60^\circ + 0.2 \cdot 30^\circ = 54^\circ \), thus the simulation can be compared with the simulation from section 3.3.2 with \( MWD = 55^\circ \). It is then seen that there is a large difference in the evolution, where the undulation in the simulation with the constant wave climate grows much faster and to a greater width than the simulation with the varying wave
climate, even if their average wave directions are within one or two degrees of each other. This illustrates how important the varying wave climate is for the time scale and the final width; the time scale is too fast and the width too large in the simulation with the constant wave direction.

The effect of changing the $\Delta T_{MW D}$ on the evolution of the shoreline shape is seen in figure 3.36. It is observed that a spit is formed in both simulations; the first spit, which forms in both simulations, reconnects to the shoreline and therefore disappears again. The development of the spit is much faster in the simulation with the large value of $\Delta T_{MW D}$.

The effect of changing $\Delta T_{MW D}$ on the width of the undulation is seen in figure 3.37, where four cases are shown for $A = 0.8$ and 0.9 and $\Delta T_{MW D} = 10$ and 100 days. $U = 1$ for in all cases. Not surprising a larger variability is seen on the width of the undulation for the larger $\Delta T_{MW D}$.

3.4.3 Dependence on Grid Resolution

To evaluate the dependency of the results on the grid resolution the simulation with $U = 1$, $A = 0.5$ and $N = 35$ is repeated for two coarser grids with $N = 10$ and 20 points, where $N$ is the number of points along the shoreline. All other parameters are kept constant in the simulations. Figure 3.38 shows the evolution of the width of the shoreline undulation during the three simulations. It is seen that the predicted maximum width becomes lower as the number of shoreline points is reduced. The difference in maximum width between 10 and 35 points is 300 m equal to around 20% of the undulation width with 35 shoreline points. Further it is noted that the better the resolution the faster the growth rate of the undulation, i.e. the width increases faster, when using 35 shoreline points; this is due to the better resolution of the longshore gradients in the longshore current, which results in a larger growth rate of the undulation.
Figure 3.35: The evolution of undulation width for simulations from table 3.4. Dashed lines are total width, \( w_{u,tot} \) and full lines are width \( w_u \).
Figure 3.36: Top: Time stack of the shoreline evolution for the simulation with $U = 1$, $A = 0.8$ and $\Delta T_{MW D} = 10$ days. Bottom: Time stack of the shoreline evolution for the simulation with $U = 1$, $A = 0.8$ and $\Delta T_{MW D} = 100$ days.
Figure 3.37: The evolution of undulation width for simulations with $U = 1$ and different $A$ and $\Delta T_{MWD}$ (in days). Dashed lines are total width, $w_{u,tot}$ and full lines are undulation widths, $w_u$. 
3.4. EVOLUTION FOR CHANGING WAVE CLIMATES

Figure 3.38: The evolution of undulation width for $U = 1$ and $A = 0.5$ for different number of shoreline points, $N$. 

The graph shows the evolution of undulation width over time for different numbers of shoreline points, $N$. The x-axis represents time in years, while the y-axis represents width in meters. The graph includes lines for $N = 10$, $N = 20$, and $N = 35$, each colored differently to distinguish between the number of shoreline points.
3.5 Physical Mechanism Governing Dimensions of Shoreline Shapes

3.5.1 Types of Shoreline Shapes

The shape and dimensions of the longshore shoreline undulations depend heavily on the incoming wave conditions. Basically three types of shoreline shapes emerge in the present results: undulation with no spit, undulation with flying spit and undulation with reconnecting spit. The flying spit is a spit, which stays a spit after its formation, i.e. it does not reconnect with the shoreline, whereas the reconnecting spit is a spit, which reconnects with the shoreline, hereafter a new spit is created. The last two terms were used before by Ashton and Murray (2006a) as described in section 3.1.

Which type of undulation that develops depends on the direction of the incoming waves and on the variation in the direction of the incoming waves. The goal of the present section is to determine the physical mechanisms which control the type and dimensions of the shoreline undulations which develop.

As shown in section 3.1, when the curvature of the shoreline is ignored, a spit will always develop in the case of constant wave forcing, but is this also true when including the curvature? The effect of the curvature of the shoreline is to focus the waves in areas with large convex curvature, thus increasing the longshore sediment transport in these areas. With this effect it could be possible for the morphology to increase the curvature of the shoreline on the downstream section and thereby increase the longshore sediment transport in the area, thus avoiding the formation of a spit. This scenario is most likely to happen, when the mean wave direction is close to the critical angle, i.e. $\approx 45^\circ$.

The results presented in the present section show that a shoreline shape without a spit is possible for a constant wave forcing, when the mean wave angle is reduced to $52^\circ$ and the length of the undulation is kept constant at $L = 5000$ m. However, keeping the length of the undulation constant, when reducing the mean wave direction, is in violation with the results from the stability analysis in section 2.3.6; these show that the most unstable undulation length increases, when the mean wave direction is decreased. For the case where the undulation length is set according to the results from the stability analysis for each value of the mean wave direction, a spit develops for all mean wave directions tested ($50^\circ, 55^\circ, 60^\circ$ and $65^\circ$). This indicates that when the shoreline is subject to waves coming from only one direction, the resulting shoreline shape will be a spit no matter what the mean wave angle is, as long as the mean wave angle is in the unstable regime.

For the case of varying wave climate all three possible shapes are formed: When all waves are coming from the unstable wave regime, i.e. for $U = 1$, a spit is formed for $A = 0.8, 0.9$ and $1.0$, whereas no spit is formed for
A = 0.5, 0.6 and 0.7. So it is seen that with 30% or more of the wave energy coming from the opposite direction, the formation of a spit is prevented in the model, see table 3.4.

When the fraction of waves coming from the unstable wave regime is reduced to $U = 0.9$, the formation of a spit is prevented already for $A = 0.8$, and for $U = 0.8$ the spit formation is prevented even for $A = 1$. Waves from the stable regime are better at inhibiting the development of spits, than waves from the opposite direction. It is noted that the length of the undulation was kept constant at $L = 5000$ m, when the $U$ parameter was changed. This could potentially explain part of the suppression of the spit development; as seen in section 3.3.2, if the undulation length is much shorter than the most unstable undulation length, no spits forms even in the case of constant wave forcing. However from the stability analysis, figure 2.39, it is found that for $U = 0.8$, the most unstable length is $L = 5500$ m, thus within 10% of the $5000$ m which was used. For $U = 0.9$ the most unstable length is exactly the one which was used, $L = 5000$ m.

Since the formation of a spit changes the morphology and hydrodynamics rather drastically, it is natural to divide the evolution of a shoreline undulation into four parts: before the formation of a spit, after the formation of a spit, the prevention of spit development and destruction of the spit.

### 3.5.2 Before formation of spit

Before the formation of a spit, the shoreline undulation grows to a certain width in the linear regime; in this regime the sinusoidal shape is not changed. During this period the curvature at the crest of the shoreline undulation increases steadily, and non-linear effects become more and more important for the evolution of the undulation. After a certain growth, an asymmetric shape starts to develop, where the slope is milder on the upstream side of the undulation and steeper on the downstream side of the undulation. At this point the phase between the maximum in the longshore sediment transport and the crest of the undulation is reduced, thereby showing the effect of the increased curvature at the crest point. The steeper slope on the downstream side of the undulation causes the trough of the undulation to receive less wave energy due to the shadowing effect. Therefore the sediment transport is greatly reduced in the trough and the formation of a spit begins. From this point on, the shape of the shoreline on the upstream side of the undulation does not change. The width of the undulation is therefore established, this width increases for increasing length of the undulation and increases for increasing mean wave direction. The increase in the undulation width for increasing mean wave direction happens because the shoreline undulation moves further and further into the unstable wave regime when the mean wave direction increases, thus a larger curvature at the crest of the undulation is needed in order to stabilize the undulation growth. This larger
curvature can only be obtained by increasing the width of the undulation. The same mechanism is responsible for the increase in undulation width for increasing undulation length: when the undulation length is increased, a larger width is needed to have the same curvature. The two effects are observed in figure 3.39. The reason the width decreases for increasing mean wave direction for \( L = \text{MUL} \) is that the most unstable undulation length decreases for increasing mean wave direction. Thus it is seen that when the undulation is free to choose its length, the effect of the length on the width of the undulation is strongest such that the undulation width decreases for increasing mean wave direction.

3.5.3 After formation of spit

After the formation of a spit, the tip of the spit starts migrating towards the trough of the shoreline undulation. This migration is so fast that the rest of the spit cannot keep up, therefore a new spit tip emerges and overtakes the old spit tip. This new spit tip then starts migrating on the same path as the old spit tip; towards the trough of the undulation. From this periodic behavior the average alongshore migration of the spit is found to be the same as the average migration of the crest of the undulation, thus the spit is migrating with the same speed as the crest of the undulation.

The migration direction of the spit is controlled by the mean wave direc-
3.5. GOVERNING MECHANISMS

The deviation of the migration direction from the theoretical migration direction when the curvature of the shoreline is zero, $\beta$, as function of the curvature of the shoreline at the crest of the undulation, $\kappa_{crest}$. See caption in figure 3.39 for details on the different data points.

The width of the spit depends mainly on the angle between the mean wave direction and the migration direction of the spit as shown in figure 3.42. As this angle is increased, the waves reach a shorter distance along the front of the spit because of the shadowing effect as sketched in figure 3.43. The curvature of the shoreline along the spit increases when the mean...
Figure 3.42: The width of the spit, $w_s$, as function of the mean wave direction minus the migration direction of the spit. See caption in figure 3.39 for details on the different data points.

wave direction is increased, this also reduces the distance along the spit a wave with a given angle can reach, this is also sketched in figure 3.43. As explained in section 3.1 when the waves reach a shorter distance along the front of the spit, the width of the spit reduces.

The two mechanisms also explain why increasing the directional spreading index narrows the spit; when the directional spreading index is increased the directional spreading is decreased, thereby the wave climate contains less waves from small directions and thus the mean wave reaches a shorter distance along the spit. At the same time the curvature of the shoreline along the spit increases for increasing directional spreading index thereby further reducing the distance along the front of the spit the waves reach.

3.5.4 Prevention of Spit Development

As was seen in section 3.4, even relatively small amounts of wave energy ($\approx 20 - 30\%$) from directions other than the main wave direction can prevent the development of a spit on the shoreline undulation.

In the most general way this behavior can be understood in terms of the morphodynamic equilibrium concept. When the hydrodynamics change on a given morphology, the morphology does not react instantly to these changes; moving sediment around takes time. In the beginning the changes are large because the morphology is far from being in equilibrium with the new hydrodynamics, but slowly the morphology changes towards the new equilibrium, and the morphodynamic changes become smaller. This concept is equivalent to how Wright and Short (1984) explain the change from one
3.5. GOVERNING MECHANISMS

Figure 3.43: Sketch showing that a wave with a smaller angle, $\alpha_2$, reaches farther along the spit than a wave with a larger angle, $\alpha_1$, due to the shadowing effect and that on a spit with a larger shoreline curvature, $\kappa_1$, both waves reach a shorter distance along the spit than on a spit with a smaller shoreline curvature, $\kappa_2$. Note that for simplicity, refraction of the waves has been ignored.

beach state to another on their figure 12, when the size of the incoming waves change.

Looking at the behavior on a more detailed level, a change in the mean wave direction will cause a change in the longshore sediment transport, which in turn will change the erosion and deposition pattern along the shoreline. As will be seen in the following, these changes can all be explained by the two mechanisms so well known by now: firstly, the angle between the shoreline and the mean wave direction, $\alpha$, and secondly, the curvature of the shoreline, $\kappa$.

In figure 3.44 the longshore sediment transport, and the resulting change in shoreline position along an asymmetric shoreline, is shown for four different mean wave directions, $MWD = -60^\circ$, $MWD = -30^\circ$, $MWD = 30^\circ$, and $MWD = 60^\circ$. The four directions represent each of the four different direction intervals shown in figure 3.34. The asymmetric shoreline shape has developed freely in a model with a mean wave direction of $MWD = -60^\circ$.

When the mean wave direction is unchanged, i.e. $MWD = -60^\circ$, the longshore sediment transport is in quasi-equilibrium with the shoreline shape; the shape of the longshore sediment transport resemble the shape of the shoreline undulation. The maximum in the longshore sediment transport is located slightly upstream of the crest of the undulation, and slightly downstream of the location where the angle between the mean wave direction and the shoreline angle, $\alpha$, is $45^\circ$. This deviates from the $q-\alpha$-diagram, figure 3.9; it is due to the curvature of the shoreline as discussed in the previous section. The erosion/deposition pattern is relatively small and enhance the shape of the shoreline further; the largest deposition is on the steepest part of the downstream side of the undulation and the erosion is happening on the gentle upstream side of the undulation. Thus the undulation is migrating alongshore without changing its shape dramatically, but still promoting the increase in the asymmetric shape, which eventually will lead
to the growth of a spit on the downstream side.

When the mean wave direction is changed to $-30^\circ$, it is found that the longshore transport is not in quasi-equilibrium with the shoreline shape. The combined effect of an angle between the mean wave direction and the shoreline angle of $\alpha \approx 45^\circ$ and a large curvature results in a substantial maximum in the longshore transport on the steep downstream side of the undulation. Again the maximum in the longshore transport is slightly downstream of the location of $\alpha = 45^\circ$ and again the deviation is due to the curvature effect. The minimum in the longshore sediment transport is located on the gentle upstream slope, where $\alpha$ is smallest. The largest deposition is happening on the gentle part of the downstream side of the undulation, close to the trough, and the erosion is taking place on and around the crest of the undulation. Thereby the trough is filling up, the crest is being eroded and the asymmetry is being reduced; the undulation and its features are being smoothed out.

When the mean wave direction is changed to $30^\circ$, the peak in the the longshore sediment transport, is in fact a minimum because most of the longshore transport is going in the negative direction. A change in the direction of the transport is observed on the steep downstream slope of the undulation, because the angle between the shoreline and the mean wave direction changes sign at this location. The maximum in the longshore sediment transport is located on the gentle downstream slope very close to where $\alpha = 45^\circ$, due to the small curvature of the gentle slope the deviation from the $q-\alpha$ diagram is very small. It is noted that $\alpha = 45^\circ$ also in the trough of the undulation, but here the longshore transport is smaller due to the concave curvature of the shoreline in the trough. The minimum of the longshore sediment transport is located on the steep upstream slope, where $\alpha$ is smallest. From the erosion/deposition pattern it is observed that the steepest part of the upstream side of the undulation experiences erosion, while there is large deposition in the trough and erosion on the crest of the undulation. The asymmetric shoreline shape is therefore disappearing very fast, and the undulation itself is being smoothed out.

When the mean wave direction is changed to $60^\circ$, the longshore sediment transport is now negative on the entire shoreline. The maximum in the longshore sediment transport is located slightly upstream of the undulation crest, and slightly downstream of the location of $\alpha = 45^\circ$. In a second location close to the trough of the undulation $\alpha = 45^\circ$ also, and here the longshore sediment transport is very small due to the large concave curvature of the shoreline here. The minimum of the longshore transport is located on the steep upstream slope, right where $\alpha$ is smallest. It is observed that the steepest part of the downstream side of the undulation experiences erosion, and some deposition happens in the trough, while the crest and the downstream side of the undulation experiences deposition. The asymmetric shoreline shape is therefore being smoothed out, but the undulation itself is
Figure 3.44: The shape of the longshore sediment transport, the erosion/deposition pattern and the angle between the shoreline and the mean wave direction, $\alpha$, along a shoreline with an asymmetric shape, previously subject to waves with a mean waves direction of $MWD = -60^\circ$, now subject to waves $MWD = -60^\circ$, $MWD = -30^\circ$, $MWD = 30^\circ$ and $MWD = 60^\circ$. The dotted lines show the location of zero longshore sediment transport.


3.5.5 Destruction of Spit

In the previous section, the prevention of the development of a spit was studied, in the present section the case is different in the way that the spit has already developed when the change in wave direction occurs. This changes things because a spit cannot be smoothed out in the same way as an asymmetric undulation.

On figure 3.45 the longshore sediment transport, and the resulting change in shoreline position along a shoreline undulation with a spit is shown for four different mean wave directions: $MW D = -60^\circ$, $MW D = -30^\circ$, $MW D = 30^\circ$ and $MW D = 60^\circ$. The four directions represent each of the four different direction intervals shown in figure 3.34. The shoreline has evolved freely to the shown shape in a model forced by a mean wave direction of $MW D = -60^\circ$.

When the mean wave direction is unchanged in the model, i.e. $MW D = -60^\circ$, it is seen that the maximum in the longshore sediment transport is located very close to the crest of the undulation, and the minimum in the longshore sediment transport is located in the embayment behind the spit. Further it is observed that the shape of the longshore sediment transport resembles the shape of the shoreline, if the embayment behind the spit is collapsed onto the shoreline. This indicates that the morphology is not far from being in equilibrium with the longshore sediment transport, and therefore with the hydrodynamic forcing. This is expected since the morphology has had a long time to react to the wave forcing. The erosion/deposition pattern is such that the shape of the shoreline is promoted, i.e. the spit grows and the gentle upstream slope of the undulation is slowly eroded.

When the mean wave direction is changed to $MW D = -30^\circ$, it is observed that the longshore sediment transport has a pronounced maximum, located downstream of the undulation crest, on the middle part of the spit where $\alpha \approx 45^\circ$. This has the result that the entire undulation is undergoing erosion except for the tip of the spit where a lot of deposition is happening. In fact more sediment is being deposited around the tip of the spit for $MW D = -30^\circ$ than for $MW D = -60^\circ$. The direction in which the spit grows is changed towards the trough of the undulation for $MW D = -30^\circ$. So the effect on the spit of changing the mean wave direction to $MW D = -30^\circ$, is to increase the growth rate of the spit and change its migration direction.

When the mean wave direction is changed to $MW D = 30^\circ$, it is seen that the direction of the longshore sediment transport changes direction on the spit, where $\alpha$ changes sign. The direction of the longshore sediment transport is therefore negative on the entire undulation, except around the tip of the spit. The consequence of this transport pattern is that the spit
Figure 3.45: The shape of the longshore sediment transport, the erosion/deposition pattern and the angle between the truncated shoreline and the mean wave direction, $\alpha$, along a shoreline with a spit, previously subject to waves with a mean waves direction of $MWD = -60^\circ$, now subject to waves with $MWD = -60^\circ$, $MWD = -30^\circ$, $MWD = 30^\circ$ and $MWD = 60^\circ$. The dotted lines show the location of zero longshore sediment transport.
is now subject to erosion, whereas there is deposition behind the tip of the spit, meaning that the spit now grows directly towards the shoreline on the shoreward side of the embayment. On the gentle slope of the undulation, the longshore sediment transport is in the negative direction, with a maximum very close to where $\alpha \approx 45^\circ$. Near the trough of the undulation $\alpha$ is reduced and the curvature of the shoreline is concave, which means that the longshore sediment transport is reduced, leading to deposition in the trough. As the spit starts to shield the waves in the embayment behind the spit, the transport goes to zero and large deposition happens meaning that the shoreline here is moving towards the tip of the spit, thereby helping to close the mouth of the embayment.

When the mean wave direction is $MW D = 60^\circ$, the longshore sediment transport is negative along the entire undulation and spit, except along the seaward part of the shoreline in the embayment, where $\alpha$ changes sign. The longshore sediment transport has its maximum just upstream of the crest of the undulation. In this case there is erosion on the entire spit including the tip of the spit, and deposition on the shoreline in the embayment both on the seaward and landward side, thus the mouth of the embayment is also closing in this case, however slightly slower than for $MW D = -30^\circ$. As for the case with no spit, there is deposition on the crest of the undulation and on the gentle downstream side.

To sum up the above analysis, a change in the mean wave direction promotes the growth of the shoreline features associated with the new mean wave direction. In the case where a spit is present this means that the spit changes its migration direction. When the new mean wave direction is larger than $\approx -45^\circ$, the new migration direction is towards the shoreline on the landward side of the embayment. This eventually causes the spit to reconnect with the shoreline thereby ending its days as a spit. Whether this will happen with a varying wave climate depends on when the mean wave direction changes back to the original direction. When this happens the development of a new spit tip may be promoted before the old spit tip merges with the shoreline. Therefore the time between the changes in the wave direction, $\Delta T_{MW D}$, influences the resulting shoreline shape which is seen in figure 3.37.

### 3.6 Discussion

The evolution of longshore shoreline undulations has been studied using a numerical model describing the longshore sediment transport along arbitrarily shaped shorelines, thereby permitting the shoreline to evolve freely in the model.

It is found that for constant wave forcing in the unstable wave regime, a spit will always develop on the downstream end of the undulation, when
the undulation is able to choose its length freely. The average migration
direction of the spit is found to depend on the direction of the incoming
waves and on the curvature of the shoreline at the crest of the undulation:
For a larger angle of the incoming waves the spit migrates farther away
from the shoreline; for a larger curvature at the undulation crest the spit
migrates less away from the shoreline. It is possible for the spit to migrate
towards the shoreline resulting in it reconnecting with the shoreline in the
trough of the undulation after a certain time. The width of the resulting
undulation and the width of the spit are both found to depend on the angle
of the incoming waves and on the length of the undulation. When allowed to
choose its length freely, the width of the undulation increases for decreasing
angle of the incoming waves, because the chosen length increases.

For varying wave climates three types of shoreline shapes develop in the
model, depending on the varying wave climate: undulation with no spit,
undulation with flying spit and undulation reconnecting spit. It is found
that relatively small amounts of wave energy from directions other than the
main wave direction (20-30%), prevents the formation of a spit.

The three types of shoreline shapes can be related to the shoreline shapes
found by Ashton and Murray (2006a) shown in figure 3.1. The terms flying
spits and reconnecting spits have been taken from their work even though
the shapes are not completely identical. The difference is that the curvature
of the shoreline was ignored in their model, meaning that the migration
direction of their spits is directed farther towards the off-shore compared to
the results in the present work. Ashton and Murray (2006a)’s figure 9 shows,
which types of shoreline shapes are obtained for different wave climates.
Their highest value of the fraction of waves from the unstable regime is
$U = 0.75$, whereas in the present work the lowest value tested is $U = 0.8$,
the best values to compare are therefore these two. In their model, spits
are found for values of the $A$ down to $A \approx 0.57$ for $U = 0.75$; $A$ being the
fraction of waves from one side. In the present work $U = 0.8$ gives no spits
for any value of $A$, whereas for $U = 0.9$, $A = 0.8$ gives no spits, but $A = 0.9$
gives reconnecting spits. The reason for this large discrepancy between the
two models is probably due to the way spits grow in the two models. As
shown by Petersen et al. (2008), in a shoreline model, where the curvature of
the shoreline is ignored, the fastest growing spit is the infinitesimally narrow
spit, which grows infinitely fast (the finite width of the spits in Ashton and
Murray (2006a) is due to the changing wave climate). This is not the case
in a model, which takes the curvature of the shoreline into account, here the
spit growing in the model is finite and likewise is the growth rate. Therefore
the growth rate of the spits in the model by Ashton and Murray (2006a)
is much larger than in the present model. Due to the higher growth rate,
spits can form under wave climates that are more adverse to the formation
of spits in their model compared to the present model.

The results from the model are compared to field observations in a de-
etailed fashion in the next chapter. However one example is given here in figure 3.46. The figure shows the island of Baia dos Tigres on the west coast of Namibia together with a shoreline shape that was evolved in the present model using a mean wave direction of $65^\circ$ and the default model parameters from table 3.1. The crossing of the shoreline in the model result is possible due to the location of the boundary in the periodic domain, which is located on the gentle upstream slope close to the crest of the undulation; thereby the model does not experience a crossing shoreline.

It is noted that the wave climate, the beach profile and the sediment are not the same in the model and at Baia dos Tigres, so a direct comparison should not be made. With this in mind it is observed that the shape of the shoreline from the model does resemble the observed shoreline. Some differences are also apparent, namely that the curvature around the crest of the undulation is larger at Baia dos Tigres, than it is in the model, a possible explanation for this is differences in the steepness of the beach profile.

To sum up, the hypothesis stated in the introduction, that the shapes and dimensions of shoreline undulations subject to waves from the unstable regime is controlled by two parameters, namely the angle between the shoreline and the incoming waves, and the curvature of the shoreline, has been confirmed by the analysis of the results in the present chapter.
Figure 3.46: Top: Island of Baia dos Tigres. Bottom: Shoreline from evolution model using a mean wave direction of $MW\ D = 65^\circ$ and the default model parameters from table 3.1.
Chapter 4

Numerical Modeling of Shoreline Undulations: Model Results for Multiple Undulations

4.1 Introduction

In the previous chapter the numerical model developed in chapter 2 was used to simulate the evolution of shoreline undulations subject to constant and varying wave climates. Thereby the different types of shoreline shapes - and the dimensions of the shoreline shapes - which evolve in the model, were determined for a range of different wave climates. However, for all simulations the domain in the model is a period domain containing just one shoreline undulation, thus assuming alongshore periodicity of both the shape and the size of the shoreline undulations. For naturally occurring shoreline undulations these are more stochastic. In this chapter the model domain is therefore expanded to include more than one shoreline undulation, thereby allowing some alongshore variability in the size and shape of the shoreline undulation.

Two cases are studied in the present chapter, firstly the case of random sinusoidal undulations in a periodic domain, and secondly the case of a single beach bump in a periodic domain which is much longer than the beach bump it-self.
4.2 Numerical Methods

4.2.1 Model Domain

Ideally the investigations were carried out on a domain some hundreds of kilometers in length in order to accommodate as large a range of undulation wave-lengths as possible. A shoreline in a periodic domain of finite length \( L \) described by \( n \) points on the shoreline can contain undulations with lengths: \( L, L/2, L/3, \ldots, L/(0.5 \cdot n) \). Most of these undulations are not of interest because they are much shorter than the shortest length growing due to the shoreline instability. This means that in order to describe two undulations with lengths that are similar, the domain has to be quite long, i.e. in order to describe both an undulation with \( L = 4000 \text{ m} \) and one with \( L = 5000 \text{ m} \) in one domain, the domain needs to be 20000 \text{ m} long. Such a domain is able to describe undulations with lengths 20000, 10000, 6667, 5000, 4000, 3333 \text{ m} and so on. In the following the evolution of undulations in such a domain is considered.

4.2.2 Wave Length Analysis

A shoreline described by \( n \) points in the \((x, y)\) domain, can be decomposed into \( N = (n - 1)/2 \) sinusoidal components such that:

\[
y = \sum_{i=1}^{i=N} \left[ a_i \sin \left( \frac{2\pi}{L} x \right) + b_i \cos \left( \frac{2\pi}{L} x \right) \right] + c \quad (4.1)
\]

This gives a total of \( n \) unknowns, i.e. \((n - 1)/2 \) \( a_i \)’s, \((n - 1)/2 \) \( b_i \)’s and one \( c \). \( y \) is known at \( n \) locations giving a total of \( n \) linear equations that can be solved.

From figure 4.1 it can be realized that the amplitude of each component can be found as:

\[
amp_i = \sqrt{a_i^2 + b_i^2} \quad (4.2)
\]

and the phase of each component is:

\[
\theta_i = \tan^{-1} \left( \frac{a_i}{b_i} \right) \quad (4.3)
\]

This can be used to analyze which undulation lengths are growing and which ones are decaying in the domain, which can otherwise be very difficult due to the interaction of undulations with different lengths.

4.3 Evolution of Random Sinusoidal Undulations

For the first investigation the initial shoreline is described by random sinusoidal undulations:
4.3. EVOLUTION OF RANDOM SINUSOIDAL UNDULATIONS

Figure 4.1: The amplitude and phase of a sinusoidal component given by: \( y = a_i \sin(\theta) + b_i \cos(\theta) \).

\[
y = \sum_{i=1}^{i=n} h a_i \sin \left( \frac{2\pi}{L} x \right) \pm h b_i \sin \left( \frac{2\pi}{L} x \right)
\]

where \( h = 10 \) m is the initial amplitude of all components, \( a_i \) is a random number between 0 and 1, \( b_i = \sqrt{1 - a_i^2} \), \( L = 20000 \) m and \( \pm \) is randomly plus or minus. This gives an initial shoreline with sediment distributed evenly on the \( n \) longest lengths which can be described in the domain. Here \( n = 10 \) is used, so the lengths are between 2000 m and 20000 m.

The shoreline evolution model has been run for mean wave directions \( MW D = 50^\circ \) and \( MW D = 60^\circ \) for the case of constant wave climate, and for \( A = 0.5 \) and \( U = 1.0 \) for the case of varying wave climate. \( A \) is the fraction of waves coming from either side and \( U \) is the fraction of waves coming from the unstable wave regime, see section 2.3.5. The remaining model parameters are set accordingly to table 3.1. The length of the most unstable undulation is known from the stability analysis to be around 4500 m for a mean wave direction of \( MW D = 60^\circ \) and increasing up to 7000 m for \( MW D = 50^\circ \) (see figure 2.28).

The evolution of the 10 longest sinusoidal components for the simulation with \( MW D = 50^\circ \) is shown in figure 4.2. It is seen that the amplitude of the components with lengths shorter than 5000 m decreases rapidly right in the beginning of the simulation, and that the fastest growing component are 6666 m and 10000 m long. This corresponds well with the result from the stability analysis: undulations with lengths shorter than 5000 m are stable, and the most unstable undulation was around 7500 m long, i.e. between the two fastest growing sinusoidal components in the present case.

For the case of \( MW D = 60^\circ \) the evolution of the 10 longest sinusoidal components is shown in figure 4.3. It is seen that the component growing fastest is 5000 m long in agreement with the stability analysis. Further it
Figure 4.2: Evolution of the amplitude of the 10 longest waves lengths present in the simulation with $L = 20000$ and $MW D = 50^\circ$

is seen that the amplitude of the component with a length of 5000 m has stabilized at the end of the simulation, while the component with a length of 6666 m is still growing a little. The time stack of the simulation is shown in figure 4.4. Four undulations are observed on the shoreline towards the end of the simulation, in agreement with figure 4.3. Further it is noted that spits have developed at the downstream end of the two shortest undulations (3600 m and 4100 m long), whereas spits have not yet developed on the two longer undulations (6100 m and 6200 m long).

For the case of $U = 1.0$ and $A = 0.5$ the evolution of the amplitude of the 10 longest components is shown in figure 4.5 and the time stack of the evolution is depicted in figure 4.7. Figure 4.6 is a zoom of figure 4.5 at the start of the evolution. It is noted that in the beginning of the evolution, the fastest growing components are $L = 4000$ m and $L = 5000$ m. $L = 3333$ m is the shortest component which grows in the simulation. As the amplitudes increase, the amplitudes of the undulations with lengths $L = 4000$ and $L = 3333$ m start decreasing, and the $L = 6667$ m grows as fast as the $L = 5000$ m towards the end of the simulation.

On the time stack in figure 4.7 it is seen how the shorter undulation merge with longer undulations, thereby making the longer undulation even longer. Due to the relatively high resolution in this model, it has not been possible to get further in the shoreline evolution than the results indicate due to computation time.

Therefore a model with a much coarser resolution is run to investigate if
4.3. EVOLUTION OF RANDOM SINUSOIDAL UNDULATIONS

Figure 4.3: Evolution of the amplitude of the 10 longest wave lengths present in the simulation with $L = 20000$ and MW $D = 60^\circ$.

Figure 4.4: Time stack of the evolution of the shoreline for the simulation with a domain length of $L = 20000$ and constant mean wave direction MW $D = 60^\circ$. The remaining parameters were set according to table 3.1.
Figure 4.5: Evolution of the amplitude of the 10 longest waves lengths present in the simulation with a total domain length of $L = 20000$ and wave climate parameters $A = 0.5$ and $U = 1.0$.

Figure 4.6: Zoom of figure 4.5.
Figure 4.7: Time stack of the evolution of the shoreline for the simulation with a domain length of $L = 20000$ and wave climate parameters $A = 0.5$ and $U = 1.0$. 

4.3. EVOLUTION OF RANDOM SINUSOIDAL UNDULATIONS
undulations continue to merge and the dominating undulation length therefore continues to increase as time goes. The model is the one described in section 2, with the change that the sediment volume correction step is omitted. That will have some slight effect on the shape of the shoreline, but should not effect the mechanism presently studied.

The length of the domain has been chosen to be 100 km, the longshore discretization is 1000 m and the cross-shore discretization is 20 m inside the surf zone and 100 m outside the surf zone. The beach profile is a Dean beach profile with a steepness parameter of 0.08. The wave conditions are characterized by \( U = 0.7 \) and \( A = 0.5 \), using a significant wave height of 1 m, a peak wave period of 5 s and a directional spreading index of 20. The initial shoreline shape is:

\[
y = \sum_{i=25}^{i=40} \sin \left( \frac{i \cdot 2\pi L}{x} \right)
\]

(4.5)

The time stack of the shoreline evolution is shown in figure 4.8. It is noted that the initial shoreline is barely visible on the time stack and that the length of the dominant undulation continues to grow as time goes, i.e. undulations keep merging and thereby become longer and longer. This is also observed in figure 4.9, where the evolution of the 14 longest wave lengths are shown. It is observed that before year 1750, the undulations with wave lengths 7,692 m and 7,143 m dominate. These lengths agree with the result from the stability analysis for varying wave climates, namely figure 2.39; even though this figure is for a directional spreading index of \( DSI = 20 \) the change in length from \( DSI = 20 \) to \( DSI = 10 \) is relatively small as seen in figure 2.28. After year 1750, the dominating lengths are 14,286 m and 16,667 m and the amplitudes of the previously dominating lengths decrease. After year 6000, the undulations with lengths 20,000 m and 25,000 m begin to take over.

### 4.4 Evolution of a Sand Engine for Beach Nourishment

In this section the case of a single beach bump on an otherwise straight shoreline subject to different wave climates is studied. This problem is inspired by a new way of making beach nourishments currently being investigated in the Netherlands and in Denmark called the sand engine. The idea is that instead of placing sand along the entire stretch of shoreline you want to nourish, the sand is placed at the upstream end of the shoreline; the waves and currents will then redistribute the sand along the downstream shoreline in a natural fashion. The shoreline evolution model is used to model the shoreline evolution for this case for waves in both the stable and the unstabilized conditions.
Figure 4.8: Time stack of the shoreline evolution for a 100 km long shoreline subject to waves specified by $U = 0.7$ and $A = 0.5$. 
Figure 4.9: Evolution of the amplitudes of the 14 longest wave lengths for the shoreline evolution for a 100 km long shoreline subject to waves specified by $U = 0.7$ and $A = 0.5$. 
4.4. EVOLUTION OF A SAND ENGINE

The time stack of the shoreline evolution with a beach bump with length $L = 5000 \text{ m}$ and width of $100 \text{ m}$. The mean wave direction is $MW D = 30^\circ$.

Figure 4.10: The time stack of the shoreline evolution with a beach bump with length $L = 5000 \text{ m}$ and width of $100 \text{ m}$. The mean wave direction is $MW D = 30^\circ$.

This is done by considering the evolution of a shoreline with an initial beach bump, representing a beach nourishment. The initial beach bump is taken to be $5000 \text{ m}$ long and shaped like a cosine function:

$$y = a \cos \left( \frac{2\pi}{L_n} \right) + a \quad (4.6)$$

where $2a$ is the total width of the beach nourishment and $L_n$ is the length of the beach nourishment.

Two cases are considered: $MW D = 30^\circ$ and $MW D = 60^\circ$, all other parameters are set according to table 3.1. The length of the domain is set to $20000 \text{ m}$.

The time stacks for the evolution of the shoreline for the two cases is shown in figures 4.10 and 4.12. Figure 4.10 shows that the width of the beach bump decreases while the length increases as time goes; no migration of the beach bump is observed. This is in agreement with the analytical solutions by Dean (2002), where the beach bump only migrates along the shoreline if the sediment used for the nourishment is of a different size than the sediment on the natural beach.

Next the length of the beach bump is reduced to investigate if an increase in the curvature of the shoreline at the crest of the undulation results in an alongshore migration of the undulation. As seen in figure 4.11 this is not the case.

Figure 4.12 shows that, for the case with $MW D = 60^\circ$, new undulations grow on the shoreline at the downstream end, and both the original and
CHAPTER 4. MULTIPLE UNDULATIONS

Figure 4.11: The time stack of the shoreline evolution with a beach bump with length $L = 500$ m and width of 100 m. The mean wave direction is $MWD = 30^\circ$. The cyan dots show the center of gravity of the undulation at each time step.

the new undulations migrate in the down drift direction. It is noted that the undulation downstream of the original beach bump is the one with the largest amplitude at the last shown time step. This is because its length has been chosen freely and the length is therefore closer to the most unstable length found in the stability analysis (around 4500 m) than the length of the original undulation.

The evolution of the amplitudes for the 10 longest lengths for the case with $MWD = 60^\circ$ is shown on figure 4.13, 4.14 and 4.14 for different lengths and widths of the the beach nourishment. It is observed that for all cases the component with a length of 5000 m is the fastest growing. However the second fastest growing component varies depending on the length of the original undulation, for an original length of 5000 m it is 6667 m, whereas for an original length of 2500 m it is 4000 m.

Still, the results show that it is the wave forcing and not the configuration of the beach bump which determines the length of the undulations which evolve downstream from the beach bump.
Figure 4.12: The time stack of the shoreline evolution with a beach bump with length $L = 5000$ m and width of 100 m. The mean wave direction is $MW D = 60^\circ$.

Figure 4.13: The evolution of the amplitude of the 10 longest wavelengths for the case of a beach bump with a width of 100 m, undulation length of $L = 5000$ m. The mean wave direction of $MW D = 60^\circ$. 
Figure 4.14: The evolution of the amplitude of the 10 longest wavelengths for the case of a beach bump with a width of 200 m and a length of $L = 5000$ m. The mean wave direction is $MWD = 60^\circ$.

Figure 4.15: The evolution of the amplitude of the 10 longest wavelengths for the case of a beach bump with a width of 100 m and a length of $L = 2500$ m. The mean wave direction is $MWD = 60^\circ$. 
4.5 Discussion of the calculated results

For a constant wave climate the initial shoreline with random sinusoidal shoreline undulations evolves into 4 undulations in the domain which is 20000 m long. The undulations which can be observed on the shoreline at the end of the simulation are two longer ones (6100 m and 6200 m) and two shorter ones (3600 m and 4100 m), even though the most unstable undulation length is 4500 m for the given wave climate and beach profile, and the length of the undulation with the largest amplitude in the component analysis is also 5000 m. Whether this discrepancy is only due to the limited time of the simulation, and the undulations will all become 5000 m long given enough time, or if the length of the undulations has been locked due to the formation of spits at the downstream end is still an open question because the amplitudes of some of the lengths in the component analysis are still growing at the end of the simulation. However due to the long time it takes before the shoreline attains a possible equilibrium, shorelines in nature are expected to be found in the state described above. Thus when looking at naturally occurring shoreline undulations, the length of the observed undulations is not expected to be exactly the length of the most unstable undulation due to the interaction of multiple undulations.

For a variable wave climate with equal amounts of wave energy coming from both sides, in the unstable wave regime, i.e. for \( U = 1.0 \) and \( A = 0.5 \), the length of the undulation with the largest amplitude keeps increasing as time goes, in agreement with Ashton and Murray (2006a). The reason for this behavior is by Ashton and Murray (2006a) attributed to the shadow effect, i.e. the mechanism where larger undulations shield smaller undulations from the incoming waves and thereby limit their growth. It turns out that this is only part of the explanation as explained in the following:

In the beginning of the evolution, undulations grow according to the growth rates found in the linear stability analysis, such that the one which grows fastest in the evolution model is the one which has a length equal to the most unstable length found by the stability analysis. After a certain time, shorter undulations stop growing. This stop can only be attributed to the shadow effect, because the amplitude at which the stop in growth happens, is much smaller than the amplitude at which the growth stops when an equivalent undulation is alone in a periodic domain. When the amplitudes of the undulations increase, the length of the fastest growing undulation increases according to the stability analysis (see figure 2.39). This means that undulations which are slightly longer now grow slightly faster. After a certain time these slightly longer undulations have grown so much larger than the slightly shorter undulations, that the shadow effect can come into play and the slightly shorter undulations therefore start decreasing in amplitude.

From this it is seen that the shift in the length of the dominating un-
dulation to longer and longer lengths is due to the combined effect of the shift to longer lengths of the most unstable undulation when the amplitude is increased and the shadow effect.

Comparing the shape of the undulations in the present work with the shape of the undulations in Ashton and Murray (2006a) for wave climates with $A = 0.5$, it is seen that the undulations in their work are more flat in the trough than the undulation in the present work. This discrepancy is perhaps due to difference in the sea bed beyond the depth of closure, which is flat in the present model and sloping in their model.

For the case of a single beach bump on a straight and uniform shoreline, an incoming wave direction in the unstable regime triggers the formation of undulations on the downstream shoreline; both the original undulation and the triggered undulation migrate along the shoreline while they grow in amplitude. The length of these downstream undulations is found not to depend on the dimensions of the initial undulation, but only on the incoming wave climate; this length is in agreement with the most unstable length from the stability analysis. For the case of incoming waves in the stable regime, it is found that the initial undulation is smoothed out by the waves. No alongshore migration of the undulation is observed; in agreement with analytical solutions to the same problem, see Dean (2002).

It is noted that the presented results are for a uniform shoreline with unlimited amounts of sand. In reality, beach nourishments are usually performed at locations where there is not enough sand available in the coastal profile, otherwise why perform a beach nourishment? This means that the actual longshore sediment transport upstream of the initial undulation may be very small because there is no sand available in the coastal profile. In this case alongshore migration of the initial undulation is expected for waves from both the stable and unstable wave regime.

Further it is noted that the coastal profile is constant in the present model, assuming that cross-shore processes act faster than the longshore processes. Kristensen et al. (2010) modeled the evolution of a near shore beach nourishment using a model similar to the present shoreline evolution model. The difference between the two models is that in their model the coastal profile is steeper in front of the beach nourishment than along the natural beach. In that case a longshore migration of the beach nourishment was observed.
Chapter 5

Comparisons with Observations

5.1 Introduction

In this chapter the model presented in chapter 2 is used to predict the size and shape of shoreline undulations on two naturally occurring shorelines based on the wave climate, the sediment size and the coastal profile at each location. The first shoreline is on the west coast of Namibia and the second shoreline is on the west coast of Denmark.

5.1.1 West Coast of Namibia

The undulations on the west coast of Namibia are shown in figure 5.1. As briefly described in chapter 1 these undulations are around 60 km long and between 10 and 12 km wide. Down drift spits are clearly observed on the two northerly undulations; on the most southerly there are indications of a spit existing at an earlier stage, i.e. the very light colored areas look like they have been covered with water in a historic embayment. The embayments between the spits and the shoreline on the northerly undulations are seen to fill up with sediment, probably mostly from aeolian sediment transport, which just in-land on the dune field can be so large that the town at Walvis Bay needs protection from the drifting sand, Leroux (1974).

The literature discussing these undulations is limited, however the spit on the most northerly undulation (near Walvis Bay) has been studied earlier by Elfrink et al. (2003) and by Schoonees et al. (1999). Elfrink et al. (2003) focused on reasons for erosion on the spit. The harbor inside Walvis bay is the most important harbor in Namibia and it is protected from the ocean waves by the spit, thus erosion of the spit is a serious problem. The study speculates that the erosion of the spit is due to a change in the wind direction and therefore wave direction to a more southern direction. This speculated
Figure 5.1: Close-ups of the three longshore undulations on the west coast of Namibia.
change in wave direction promoted the growth of new spits further up drift, thus starving the original spit of sediment and thereby leading to erosion on the old spit. The spit is estimated to grow approximately 15 m/year and the yearly longshore sediment transport is estimated to be 1M m$^3$/year.

Schoonees et al. (1999) focused on the probability of breaching of the spit; the study did not find this plausible.

Hughes et al. (1992) studied the vulnerability of Walvis bay to rising sea levels. The migration rate of the spit was estimated to be 17 m/year from old photos and sea maps and a CSIR (Council for Scientific and Industrial Research, South Africa) report is cited, where the longshore sediment transport rate was estimated to be 2M m$^3$/year on ocean side shoreline of the spit protecting Walvis Bay.

5.1.2 West Coast of Denmark

The shoreline undulations at Srd. Holmslands Tange on the West Coast of Denmark are described in detail in chapter 6, where the undulations on Srd. Holmslands Tange were found to be around 5 km and the width around 100 m.

Other shoreline undulations on the Danish West coast are seen east of Hirtshals, an image from the home page of the Danish coastal authorities is shown in figure 5.2 where the undulations are visible. Unfortunately no detailed bathymetric surveys are available for this area, but the westerly undulation is estimated to be around 4 km long and 350 m wide from the image.
5.1.3 Scope of Chapter

The scope of the present chapter is to use the numerical model to predict the size and shape of shoreline undulations on two natural shorelines. A stability analysis is used to determine the most unstable length of the shorelines taking into account the local wave climate, bathymetry and shoreline orientation; thereby enabling a comparison between the model predictions and field observations. Further the evolution model is used to predict the width and shape of the shoreline undulations on the shoreline west of Namibia, leading a comparison between the model predictions and field observations. Further the evolution model is used to predict the width and shape of the shoreline undulations on the shoreline west of Namibia.

5.2 Numerical Methods

A description of the stability analysis is found in section 2.3.6 and a description of the evolution model is found in section 2.3.8. For both the stability analysis and the evolution model, the main model inputs are: the wave climate, the beach profile and the sediment size.

The model needs averaged wave conditions. Because the waves are used to drive the longshore sediment transport and this longshore sediment transport historically is taken to depend on the wave height to some power around 3 (see e.g. Komar and Inman (1970)), the average wave conditions are found by weighting the wave conditions with the wave height cubed.

The average mean wave direction is found as:

\[
W_x = H^p_s \cdot \cos(90^\circ - MWD) \\
W_y = H^p_s \cdot \sin(90^\circ - MWD) \\
MWD_m = 90^\circ - \tan^{-1}\left(\frac{Wy}{Wx}\right)
\]

where \(H_s\) is the significant wave height, \(p\) is 3 and \(MWD\) is the mean wave direction and \(MWD_m\) is the average mean wave direction, both are measured from true north.

The average significant wave height is found as:

\[
H_{sm} = \left(\frac{H_s}{f}\right)^{1/p}
\]

where \(H_{sm}\) is the mean significant wave height, \(H_s\) is the significant wave height and \(f\) is the average of \(f\).

Finally the average peak wave period is found as:

\[
T_{pm} = \frac{\sum (H^p_s \cdot T_p)}{\sum H^p_s}
\]

where \(T_{pm}\) is the mean peak wave period and \(T_p\) is the peak wave period.
The model also needs an average beach profile, i.e. a single beach profile is used along the entire shoreline in the model. When no measurements of the beach profile exist, it is chosen to use a Dean type beach profiles, such that the water depth out to the closure depth is given by:

$$z = A \cdot y^m$$

(5.6)

where $z$ is the vertical coordinate of the bathymetry and $y$ is the cross-shore coordinate of the bathymetry. $A$ is the steepness of the profile and $m = 2/3$. The steepness of the profile must be determined from either sea map data or bathymetric surveys for each location. Beyond the closure depth the water depth is constant, this is the case both when using a measured profile and when using a Dean profile.

### 5.3 West Coast of Namibia

#### 5.3.1 Available Data

The main model inputs are the wave climate, the beach profile and the sediment size. These data are available from several sources. The off-shore wave climate is available both in Elfrink et al. (2003) and in Bosman and Joubert (2008). In the present work the off-shore wave climate is taken from Bosman and Joubert (2008), it is shown in figure 5.3 at a water depth of 136 m. It agrees well with the off-shore wave climate in Elfrink et al. (2003). For the off shore wave climate the averaged wave properties are found to be: Average mean wave direction, $MW\, D_m = 203^\circ$, average significant wave height, $H_{sm} = 2.4$ m and average peak wave period, $T_{pm} = 12.7$ s.

In connection with the work presented in Elfrink et al. (2003) an ADCP was deployed closer to shoreline at the Pelican Point in 30 m water depth, the ADCP was only in the water from 27-10-2001 to 13-11-2001. The measured wave climate is shown in figure 5.4. For the measured wave climate the averaged wave properties are found to be: $\alpha = 229^\circ$, $H_{sm} = 1.0$ m and $T_{pm} = 10.9$ s. It is noted that the difference in the average mean wave direction is approximately $26^\circ$ from the off-shore to the near shore, and that the wave period and wave height are also quite different.

Using linear wave theory and Snell’s law for wave refraction the change in wave direction is found to be $24^\circ$, (using $T = 12.7$ s for the wave period); close to the $26^\circ$ found above. The discrepancies are probably due to the rather short deployment time for the ADCP. It is chosen to use the off-shore wave climate for the present study.

The sediment size is constant in the domain and is taken from Schoonees et al. (1999) who found a median sediment size of 0.35 mm close to Pelican point which is the tip of the spit at Walvis Bay. The beach profile is assumed to be a Dean profile; the steepness of the Dean profile is found on the map.
Figure 5.3: The offshore wave climate west of Namibia, from Bosman and Joubert (2008).

Figure 5.4: The wave climate measured at 30 m water depth, off the Pelican Point.
shown in figure 5.5, it is observed that in the trough of the undulation the 30 m depth contour is $\approx 4.5$ km off-shore, whereas at the crest of the undulation the 30 m contour is $\approx 2$ km off-shore. On figure 5.6 two Dean profiles are drawn, one with a steepness of 0.2 and one with a steepness of 0.12. It is seen that these intersect the 30 m depth contour at around 2 km and 4 km corresponding to the values found on the sea map; thus these two steepnesses are investigated together with the average of 0.16. It is noted that on the spit near Pelican Point the beach profile is steeper and it is clear that the beach profile is by no means constant along the entire coastline. This is in contrast to the model which uses a constant beach profile along the entire shoreline.

The closure depth, or the depth to which the undulations affect the bathymetry is seen from figure 5.5 to be between 30 m and 50 m, as the 30 m contour is clearly undulating with the shoreline, whereas the 50 m contour only weakly undulates with the shoreline. This suggest the closure depth to be around 40 m.

Using Snell’s law and linear wave theory the mean direction of off-shore wave direction can be transferred to smaller water depths. The shoreline normal direction is taken to be $273^\circ$, thus the angle between the off-shore mean waves direction and the shoreline angle is $70^\circ$. Due to wave refraction the angle is reduced to $57^\circ$ at a depth of 50 m; at 40 m it is $52^\circ$ and at 30 m it is $45^\circ$.

### 5.3.2 Stability Analysis

The stability analysis described in section 2.3.6 is performed in order to determine the length of the most unstable undulation length on this stretch of shoreline on the west coast of Namibia.

For the stability analysis the parameters shown in table 5.1 are kept constant, while varying the mean wave direction ($MW D = 50^\circ, 55^\circ$ and $60^\circ$), the steepness of the beach profile ($A = 0.12, 16$ and 0.2) and the closure depth ($D_{dcl} = 30, 40$ and 50 m).

The mesh is constructed as described in section 2.3.2. The alongshore distance between shoreline points is 750 m meters, and the cross-shore discretization distance in the surf zone (out to a water depth of $4 \cdot H_s$) is 50 m for $A = 0.12$, 38 m for $A = 0.16$ and 25 m for $A = 0.2$; the cross-shore discretization distance outside the surf zone is 250 m for $A = 0.12$, 188 m for $A = 0.16$ and 125 m for $A = 0.2$.

The result of the stability analysis is shown on figures 5.7 to 5.9, where the most unstable undulation length is shown for the three beach profile steepness’. It is noted that the most unstable undulation length is almost linearly dependent on the closure depth for all profile steepness’ and mean wave directions. The most unstable undulation length is around the observed value of 60 km for a number of combinations of closure depth, profile
Figure 5.5: Map showing a few off-shore contours, from sea maps
5.3. WEST COAST OF NAMIBIA

Figure 5.6: Dean profile \((z = Ay^m)\) for steepness parameters: \(A = 0.2\), \(A = 0.16\) and \(A = 0.12\), all for \(m = 2/3\).

Table 5.1: Parameters used for modelling the shoreline on the west coast of Namibia.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Wave parameters:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Significant Wave height</td>
<td>(H_s)</td>
<td>2.4 m</td>
</tr>
<tr>
<td>Directional Spreading Index</td>
<td>(DSI)</td>
<td>10</td>
</tr>
<tr>
<td>Peak wave period</td>
<td>(T_p)</td>
<td>12.7 s</td>
</tr>
<tr>
<td>Breaking wave parameter</td>
<td>(\gamma)</td>
<td>0.8</td>
</tr>
<tr>
<td><strong>Hydraulic parameters:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Manning number</td>
<td>(M)</td>
<td>32 (m^{1/3}/s)</td>
</tr>
<tr>
<td>Smagorinsky coeff.</td>
<td>-</td>
<td>0.28</td>
</tr>
<tr>
<td><strong>Sediment parameters:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sediment porosity</td>
<td>(p)</td>
<td>0.4</td>
</tr>
<tr>
<td>Sediment grain diameter</td>
<td>(d_{50})</td>
<td>0.35 mm</td>
</tr>
<tr>
<td>Sediment grain grading coef.</td>
<td>(\sigma)</td>
<td>1.1</td>
</tr>
<tr>
<td>Relative sediment density</td>
<td>(s)</td>
<td>2.65</td>
</tr>
<tr>
<td>Critical shield parameter</td>
<td>(\theta_c)</td>
<td>0.05</td>
</tr>
</tbody>
</table>
steepness and mean wave direction, these are seen in table 5.2. For the steepest beach profile, \( A = 0.2 \), all lengths are shorter than the observed; further it is noted that for this profile steepness the most unstable length does not increase when the mean wave direction decreases for \( D_{cld} = 40 \) m and 50 m, however this discrepancy is within the resolution of the most unstable length, which is 5000 m.

Comparing the combinations in table 5.2 with the mean wave direction at the different water depths (which varies due to depth refraction) it is seen that only the combinations with a closure depth of 40 m and 50 m are relevant. This is because for \( D_{cld} = 30 \) m, the mean wave direction becomes 45\( ^\circ \), which is right on the limit between the stable and the unstable wave regime. Thus we are left with the combinations 2 and 3. Both of these combinations fit well with the averaged mean wave direction at their respective depths. Note how the waves refract \( \approx 5^\circ \) between 40 and 50 m water depth, and there is 5\( ^\circ \) between the mean wave direction for the two combinations. So, when the beach profile steepness is \( A = 0.16 \) and when depth induced wave refraction is taken into account, the same most unstable undulation length comes out of the stability analysis for both \( D_{cld} = 40 \) m and \( D_{cld} = 50 \) m.

### 5.3.3 Shoreline Evolution

The evolution model described in section 2.3.8 is run for the three combinations of beach profile steepness, closure depth and mean wave direction shown in table 5.2. All other parameters are set according to table 5.1.

The mesh for each simulation is made according to section 2.3.2, the
Figure 5.8: Most unstable undulation length, \( L \) as function of closure depth and mean wave direction for the case with a beach profile steepness parameter \( A = 0.16 \).

Figure 5.9: Most unstable undulation length, \( L \) as function of closure depth and mean wave direction for the case with a beach profile steepness parameter \( A = 0.2 \).

Table 5.2: The combinations of beach profile steepness, closure depth and mean wave direction which gives a fastest growing undulation length in the stability analysis, which resembles the observed length of 60 km.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Combi. 1</th>
<th>Combi. 2</th>
<th>Combi. 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beach profile steepness, ( A )</td>
<td>0.12</td>
<td>0.16</td>
<td>0.16</td>
</tr>
<tr>
<td>Closure depth, ( D_{cld} )</td>
<td>30</td>
<td>40</td>
<td>50</td>
</tr>
<tr>
<td>Mean wave direction, ( MW D )</td>
<td>50°</td>
<td>50°</td>
<td>55°</td>
</tr>
<tr>
<td>Most unstable length, ( L )</td>
<td>65 km</td>
<td>60 km</td>
<td>60 km</td>
</tr>
</tbody>
</table>
longshore discretization is 1500 m, for all three cases, and the cross-shore discretization inside the surf zone (to a depth of $4 \cdot H_s$) is 100 m for the case with $A = 0.12$ and 80 m for the two cases with $A = 0.16$. Outside the surf zone the cross-shore discretization is 500 m for the case with $A = 0.12$ and 400 m for the two cases with $A = 0.16$.

The amplitude of the initial undulation is 2400 m for Combination 2 and 2600 m for Combination 1 and 3, this gives aspect ratios close to 0.04 for all undulations.

**Time Evolution of the Shoreline Undulation**

The time stack of the shoreline evolution for Combination 1 is shown in figure 5.10 before the development of a spit, and in figure 5.11 after the development of a spit. It is noted that it takes around 13,000 years before the formation of a spit. At this point the undulation has migrated one length in the alongshore direction. This is less that the undulations migrated before the spit formation in section 3.3.1, this discrepancy is due to the larger initial aspect ratio of the undulations in the present section, which is used in order to save computational time. When the initial width is relatively larger, the initial shape of the undulation is closer to forming a spit, thereby the time it takes before a spit is created is reduced and thus the alongshore length the undulation migrates before the a spit is created is also reduced.

Likewise the time stack of the shoreline evolution for Combination 2 is shown in figure 5.12 before the development of a spit, and in figure 5.13...
Figure 5.11: Time stack of the shoreline evolution after the development of a spit on the downstream end of the undulation for the Combination 1 case in table 5.2.
Table 5.3: Undulation width, $w_u$, Spit width, $w_s$, migration speed, $c$, and longshore sediment transport, $q$, at the end of the shoreline evolution simulations for the three combinations from table 5.2.

<table>
<thead>
<tr>
<th>Combi.</th>
<th>$w_u$ (m)</th>
<th>$w_s$ (m)</th>
<th>$c$ (m/year)</th>
<th>$q$ (M m$^3$/year)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Combi. 1</td>
<td>15,000</td>
<td>6000</td>
<td>6.2</td>
<td>2.1</td>
</tr>
<tr>
<td>Combi. 2</td>
<td>13,500</td>
<td>4500</td>
<td>4.3</td>
<td>1.7</td>
</tr>
<tr>
<td>Combi. 3</td>
<td>15,000</td>
<td>2400</td>
<td>3.9</td>
<td>1.6</td>
</tr>
</tbody>
</table>

After the development of a spit. For this combination the spit formation happens between year 10,000 and 11,000 and after migrating only 2/3 of the undulation length.

Finally, the time stack of the shoreline evolution for Combination 3 is shown in figure 5.14 before the development of a spit, and in figure 5.15 after the development of a spit. For combination 3 the spit development also happens between year 10,000 and year 11,000 and after migrating around 2/3 of an undulation length.

**Morphologic Parameters**

Table 5.3 shows the undulation width, $w_u$, the spit width, $w_s$, the migration speed of the spit, $c$, and the longshore sediment transport rate, $q$, for the three combinations from the table 5.2. Plots of the evolution of these parameters are shown in appendix C. Note that Combination 1 is mainly included to see the sensitivity of the model to various parameters, since it was found that the angle between the averaged mean wave direction and the shoreline orientation was not large enough for the waves to be in the unstable regime for this combination, see section 5.3.2.

It is noted that all three combinations have undulation widths between 13.5 and 15 km. The observations show undulation widths in the range 8-12 km, thus the model over predicts the undulation width by minimum 10-25%.

Further it is noted that the width of the spit is almost twice as large for Combination 2 compared with Combination 3. The difference between the two is the closure depth and the mean wave direction. The observed spits have widths ranging from 1 to 2.2 km. So clearly combination 3 is closest to the observations, and is very close to being within the range of the observations, whereas the spit for Combination 2 is minimum a factor 2 too wide.

The migration rates of the spits are seen to be around 4 m/year for both combinations 2 and 3. As mentioned in section 5.1.1, the migration speed of the spit at Walvis Bay was found to be around 15 m/year in Elfrink et al. (2003). The alongshore migration speed of the southern most undulation can be estimated using the position the wreck of "Eduard Bohlen", a ship...
Figure 5.12: Time stack of the shoreline evolution until the development of a spit on the downstream end of the undulation for the Combination 2 case in table 5.2.

Figure 5.13: Time stack of the shoreline evolution after the development of a spit on the downstream end of the undulation for the Combination 2 case in table 5.2.
which sunk in 1909. Figure 5.16 shows the position of the ship relative to the shoreline today. From the figure, the alongshore migration rate is estimated to be 13 m/year by assuming that the undulation migrates with unchanging form; if the width of the undulation is growing the actual migration speed will be smaller. So it is found that the migration speeds in the model are between 20 and 33% of the observed migration speeds.

The longshore sediment transport rates from the model are similar for Combination 2 and 3. This is because even though they have different mean wave directions the difference in closure depth means that the waves refract more for Combination 3 than for Combination 2, leading to similar longshore sediment transport rates. Comparing the rates from the model with the rates found in Elfrink et al. (2003) and in Hughes et al. (1992) which was 1M and 2M m$^3$/year, we find that the transport rates in the model model are right between these two observations.

The largest discrepancies between the model results and the observations are the migration rates. These discrepancies can be explained partly by the fact that the model over predicts the undulation width and the width of the spit, and partly by the fact that the model assumes a constant beach profile along an undulation, whereas the observed undulations clearly have steeper profiles at the crests and shallower profiles at the troughs of the undulations. If the undulation width is over predicted by 50% in the model, the migration rate will be underpredicted by 50% if the longshore transport is correct and the beach profiles are also correct. This can be seen from equation 2.17. The effect of the steepness of the profile is more difficult to estimate; by assuming
Figure 5.15: Time stack of the shoreline evolution after the development of a spit on the downstream end of the undulation for the Combination 3 case in table 5.2.
that the profile becomes infinitely steep at the crest, a beach profile which is
4 km wide (distance from shoreline to closure depth) on an undulation which
is 10 km wide, the reduction in the sediment volume needed to migrate the
undulation is 20 %, see figure 5.17 \((\frac{0.5w^p D_{chl}}{wD_{chl}})\). Thereby the migration speed
will also be underestimated by 20 %. The real effect is much smaller because
the profile at the crest is not infinitely steep.
Figure 5.17: Sketch showing the effect of the profile steepness on the migration speed.
5.4 West Coast of Denmark

5.4.1 Available Data

For the Danish West Coast the wave climate, sediment size and beach profiles have been measured by the Danish Coastal Authorities (in Danish Kystdirektoratet).

The wave climate is measured at a number of places, for the present study the wave climate measured at station 2041 shown on 6.3 is used for the Srd. Holmslands Tange, and the wave climate measured at the station west of Hirtshals shown in figure 5.2 is used for the shoreline east of Hirtshals.

For Srd. Holmslands Tange the yearly average wave conditions are found as described in section 5.2: Average mean wave direction, $\alpha = 305^\circ$, average significant wave height, $H_{\text{sm}} = 1.8 \text{ m}$ and average peak wave period, $T_{\text{pm}} = 6.1 \text{ s}$. The shoreline orientation is $263^\circ$, so the angle between the direction of the shoreline and the direction of the yearly mean wave direction is $42^\circ$. This is just the critical angle giving maximum longshore transport, which means that if the waves refract before reaching the undulations they will be in the stable regime. The wave gauge which measured the data is located 14 km off-shore, which means that other processes than depth induced wave refraction can affect the waves on their way to the shoreline. These processes include wind forcing and current induced wave refraction. This means that there is some doubt regarding the direction of the waves when they arrive at the shoreline; a parameter study is therefore made in the stability analysis to observe the effect of different wave directions.

For the shoreline east of Hirtshals, the yearly average wave conditions are found as described in section 5.2: Average mean wave direction, $\alpha = 297^\circ$, average significant wave height, $H_{\text{sm}} = 1.5 \text{ m}$ and average peak wave period, $T_{\text{pm}} = 6.9 \text{ s}$. The shoreline orientation is $358^\circ$, so the angle between the direction of the shoreline and the direction of the yearly mean wave direction is $61^\circ$.

The sediment size is known from Kystdirektoratet (1999), and the median grain size is $0.2 \text{ mm}$ at Srd. Holmslands Tange and $0.18 \text{ mm}$ east of Hirtshals.

The coastal profiles have been measured every 1-5 years every one kilometer along the Danish West coast since the 1950’s. In figure 5.18 the colored areas shown the areas that have been measured together with the profile line numbers at the boundaries of the shoreline sections studied.

Figure 5.19 shows the mean coastal profile based on coastal profiles measured by KDI every 1-5 years since the 1950’s, only profiles from year 1970 to year 2000 and profile lines from 5700 to 5810 were used to make the mean; figure 5.18 shows the location of these profile lines. Three Dean profiles have also been draw in figure 5.19 with different steepness parameters.

Figure 5.20 shows the mean coastal profile based on beach profiles mea-
Figure 5.18: The location of the profile lines used to make the mean profiles at Srd, Holmslands Tange (5700-5810) and Hirtshals (1300-1500). The coastal area covered by all the profile lines measured by Kystdirektoratet every 1-5 years is seen as the colored area.
Chapter 5. Comparisons with Observations

Figure 5.19: The mean beach profile at Srd. Holmslands Tange

The mean coastal profiles from the two locations both have a distinct shift in the steepness of the profile, at Srd. Holmslands Tange the shift happens around 4.5 meters water depth, and at east of Hirtshals the shift happens around 3.5 meters. According to Inman et al. (1993) this is the natural shape of beach profiles due to the change in the wave conditions inside and outside the surf zone. The reason the change in profile steepness is not included in the original formulations by Bruun (1954a) and Dean (1991), is postulated by Inman et al. (1993) to be because the coastal profiles they examined did not reach far enough beyond the surf zone.

5.4.2 Stability analysis

Srd. Holmslands Tange

The stability analysis is again performed as described in section 2.3.6 for Srd. Holmslands Tange first. Table 5.4 shows the parameters used for the stability analysis. Two types of beach profiles are used, a Dean profile...
with the steepness’ from the table, and the average measured profile shown in figure 5.19. For all case the selected beach profile is only used for water depths smaller than the depth of closure, beyond this depth, the water depth is constant and equal to the depth of closure.

Figure 5.21 shows the most unstable undulation length for different values of the mean wave direction and beach profiles. It is noted that the resolution of the most unstable undulation length is 1000 m. Further it is noted that for a mean wave direction of $MW D = 50^\circ$, using the real beach profile gives a most unstable undulation length, which is between the most unstable undulation length of the steep and the shallow Dean profile. For the smallest closure depth, $D_{cl} = 5$ m, the most unstable undulation length for the measured profile is closest to the most unstable undulation length of the steep Dean profile, whereas for the larger closure depth, it is closest to the most unstable undulation length of the shallow Dean profile. The difference is probably due to the fact that the shift in the measured profile from the steep profile to the shallow profile happens around 4.5 m, thus when the closure depth is 5 m, only the steep part of the measured profile is actually used when setting the bathymetry. When the closure depth is increased to 7 m, more of the shallow part of the measured profile is used when setting the bathymetry, thereby the results should resemble the results obtained on
Table 5.4: Model parameters used for the stability analysis for Srd. Holmsland Tange.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Wave parameters:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Significant Wave height</td>
<td>$H_s$</td>
<td>1.8 m</td>
</tr>
<tr>
<td>Mean Wave Direction</td>
<td>$MWD$</td>
<td>50, 55 &amp; 60 $^\circ$</td>
</tr>
<tr>
<td>Directional Spreading Index</td>
<td>$DSI$</td>
<td>10</td>
</tr>
<tr>
<td>Peak wave period</td>
<td>$T_p$</td>
<td>6.1 s</td>
</tr>
<tr>
<td>Breaking wave parameter</td>
<td>$\gamma$</td>
<td>0.8</td>
</tr>
<tr>
<td><strong>Hydraulic parameters:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Manning number</td>
<td>$M$</td>
<td>32 $m^{1/3}/s$</td>
</tr>
<tr>
<td>Smagorinsky coef.</td>
<td>-</td>
<td>0.28</td>
</tr>
<tr>
<td><strong>Sediment parameters:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sediment porosity</td>
<td>$p$</td>
<td>0.4</td>
</tr>
<tr>
<td>Sediment grain diameter</td>
<td>$d_{50}$</td>
<td>0.2 mm</td>
</tr>
<tr>
<td>Sediment grain grading coef.</td>
<td>$\sigma$</td>
<td>1.1</td>
</tr>
<tr>
<td>Relative sediment density</td>
<td>$s$</td>
<td>2.65</td>
</tr>
<tr>
<td>Critical shield parameter</td>
<td>$\theta_c$</td>
<td>0.05</td>
</tr>
<tr>
<td><strong>Mesh:</strong></td>
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<td></td>
</tr>
<tr>
<td>Dean profile steepness</td>
<td>$A$</td>
<td>0.1 &amp; 0.08</td>
</tr>
<tr>
<td>Closure depth</td>
<td>$D_{cbd}$</td>
<td>5 &amp; 7 m</td>
</tr>
<tr>
<td>Cross-shore discretization</td>
<td>$a$</td>
<td>10 m</td>
</tr>
<tr>
<td>Number of shoreline points</td>
<td>$n$</td>
<td>51 -</td>
</tr>
</tbody>
</table>
the shallow bathymetry more than before.

When comparing the most unstable lengths presented in figure 5.21 with the observed lengths on Srd. Holmslands Tange (≈ 5-6 km), it is observed that only when the closure depth is 5 m can lengths of ≈ 5 km be obtained in the model. Further it is required that the mean wave angle at this depth forms an angle with the shoreline of 55° or more. It can be argued that the first requirement is satisfied at Srd. Holmslands Tange, in section 6 it is found that the observed undulations are observed on the beach face at the 5 m depth contour, however the data which are analyzed do not reach farther off-shore, so it is not known if this depth is in fact be 7 meters or larger. Note that in the present work the closure depth is the depth to which the shoreline undulations can be felt on the bathymetry, this depth does not have to be the same depth as the original definition by Hallermeier (1981).

It is harder to argue that the second requirement is satisfied on Srd. Holmslands Tange. The measured wave climate shows a weighted yearly mean angle to the shoreline of 42° on 25 meters water depth, even with the help of wind forcing and current induced refraction it is hard to make this angle increase to 55° when moving the waves from 25 m to 5 m.
Table 5.5: Model parameters used for the stability analysis for Hirtshals.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wave parameters:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Significant Wave height</td>
<td>$H_s$</td>
<td>1.5 m</td>
</tr>
<tr>
<td>Mean Wave Direction</td>
<td>$MWD$</td>
<td>50 &amp; 60°</td>
</tr>
<tr>
<td>Directional Spreading Index</td>
<td>$DSI$</td>
<td>10</td>
</tr>
<tr>
<td>Peak wave period</td>
<td>$T_p$</td>
<td>6.9 s</td>
</tr>
<tr>
<td>Breaking wave parameter</td>
<td>$\gamma$</td>
<td>0.8</td>
</tr>
<tr>
<td>Hydraulic parameters:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Manning number</td>
<td>$M$</td>
<td>32 $m^{1/3}/s$</td>
</tr>
<tr>
<td>Smagorinsky coef.</td>
<td>-</td>
<td>0.28</td>
</tr>
<tr>
<td>Sediment parameters:</td>
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<td></td>
</tr>
<tr>
<td>Sediment porosity</td>
<td>$p$</td>
<td>0.4</td>
</tr>
<tr>
<td>Sediment grain diameter</td>
<td>$d_{50}$</td>
<td>0.2 mm</td>
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<td>Sediment grain grading coef.</td>
<td>$\sigma$</td>
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<tr>
<td>Relative sediment density</td>
<td>$s$</td>
<td>2.65</td>
</tr>
<tr>
<td>Critical shield parameter</td>
<td>$\theta_c$</td>
<td>0.05</td>
</tr>
<tr>
<td>Mesh parameters:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Closure depth</td>
<td>$D_{cd}$</td>
<td>5 &amp; 7 m</td>
</tr>
<tr>
<td>Cross-shore discretization</td>
<td>$a$</td>
<td>10 m</td>
</tr>
<tr>
<td>Number of shoreline points</td>
<td>$n$</td>
<td>31 -</td>
</tr>
</tbody>
</table>

Increasing the closure depth reduces the problem of satisfying the second requirement because of the large effect of the depth induced wave refraction on the angle of waves. However, increasing the closure depth results in an increase in the length of the most unstable undulation to lengths much longer than the observed 5 km; with a closure depth of 7 m the smallest length of the most unstable undulation is 9 km.

**Hirtshals**

Next the stability analysis is performed for the shoreline east of Hirtshals. Table 5.5 shows the parameters used for this stability analysis. The average of the measured profiles shown in figure 5.20 is used, i.e. no Dean profile is used.

Figure 5.22 shows the most unstable undulation length as function of closure depth and mean wave direction for the model parameters from table 5.5. It is seen that the shortest most unstable undulation length is 6000 m, for a mean wave direction of $65^\circ$ and a closure depth of 5 m.
Figure 5.22: The most unstable undulation length as function of closure depth, for different mean wave directions, MWD, using the mean measured beach profile and the weighted yearly average wave conditions for Hirtshals, see table 5.5.
The most unstable undulation length as function of closure depth, for different mean wave directions, MWD, using the mean measured beach profile at Hirtshals and a significant wave height of 1 m, a peak wave period of 5 s and a directional spreading index of 10.

The size of the waves is now reduced to investigate if this result in a most unstable undulation length of $\approx 4$ km, the reduced waves have a significant wave height of 1 m and a peak wave period of 5 s. Using these wave conditions, the most unstable undulation length is shown in figure 5.23 for different mean wave directions and for closure depths of 3 m and 4 m. It is seen that an undulation length of 4 km is obtained in the model for closure depths between 3 and 4 meters and mean wave directions larger than $55^\circ$.

5.4.3 Analysis

The results from the stability analysis of the shoreline at Srd. Holmslands Tange and east of Hirtshals show that the observed lengths of the shoreline undulations on these two shorelines are only predicted by the stability analysis when the closure depth is around 5 m at Srd. Holmsland and between 3 and 4 meter at Hirtshals; i.e. just beyond the location where the average beach profile changes its slope in each location. The angle of the incoming wave at this depth is usually quite small due to depth induced wave refraction; meaning that the waves are not in the unstable wave regime at this
5.4. WEST COAST OF DENMARK

Figure 5.24: The change in angle between incoming waves and shoreline, \( \alpha \) due to depth induced wave refraction for different wave periods.

depth.

Figure 5.24 shows the angle between the shoreline and the incoming waves as function of water depth for different wave periods. All waves start with an angle to the shoreline of \( 89^\circ \) in 30 meters water depth, which is a reasonable average depth for the Danish part of the North Sea. At 5 m water depth it is seen that only waves with period of 5 s and smaller have angles larger than \( 50^\circ \). Thus only short waves can reach a depth of around 5 m with a large enough angle to be in the unstable wave regime.

At Srd. Holmslands Tange the wave climate has been analyzed again, this time taking only waves with periods less than 5 s and 4 s. The average mean wave direction for waves with periods smaller than 4 s was \( \approx 58^\circ \), and average mean wave direction for waves with periods smaller than 5 s was \( \approx 52^\circ \). For both cases only waves in the direction interval \(-100^\circ < \alpha < 100^\circ\) (\( \alpha \) is the angle between the shoreline and the incoming waves), were used and no weighting of the wave directions was applied. These smaller waves (wave periods smaller than 4 s or 5 s) will not be able to move the sediment off-shore to a large depth, which agrees well with the small closure depth needed in the model in order predict the observed undulation length at Srd. Holmslands Tange.

In chapter 6 the average wave climate, i.e. without any weighing with
the significant wave height, is used to compare Srd. Holmslands Tange with
the stability analysis by Falqués and Calvete (2005), and it is concluded that
we are right on the limit between a stable and an unstable shoreline.

In the present chapter, the longshore undulations at Srd. Holmslands
Tange can be explained by the shoreline instability, if the waves responsible
for the undulations are the small “everyday” waves, and not the larger storm
waves; the latter will refract so much that they are in the stable wave regime
before reaching the depth where shoreline undulations are imprinted on the
bathymetry, and if the undulation should reach farther off-shore, the most
unstable lengths become much longer than the observed undulations.

The major problem with the above hypothesis is: Why do the large storm
waves not smooth out the shoreline undulations? One possibility is that the
outer longshore breaker bar dissipate most of the energy of the large storm
waves before it reaches the shoreline, whereas the small waves responsible
for the shoreline undulations pass the outer breaker bar unaffected.

For the case of the shoreline east of Hirtshals the situation is complicated
by the location of the shoreline stretch, which is right next to Hirtshals
harbor; located right at the 90° change in shoreline orientation. Therefore
the assumption of an infinitely long shoreline, which is implied in the periodic
boundary condition used in the model, is not valid. Further more, due to
the 90° corner, the wave climate cannot be assumed to be uniform in the
alongshore direction which is done in the model.

Based on the above, the observed shoreline undulations at Hirtshals
could be the results of some processes other than the shoreline instability
which forms the basis for the stability analysis. Possible local processes are:
Downstream erosion next to Hirtshals harbor and non-uniform alongshore
off-shore wave conditions due to the 90° change in shoreline orientation right
at Hirtshals harbor.

5.5 Discussion

The results in the present chapter point in opposite directions: For the
shoreline on the west coast of Namibia, running the stability analysis and
the evolution model with the weighted average wave climate and mean beach
profile gives undulation length, widths and shapes which predict the ob-
served shoreline undulations remarkably well. For the shoreline on the west
cost of Denmark running the stability analysis with the weighted average
wave climate and mean beach profile gives undulation lengths which are
more than twice as large as the observed undulation lengths.

For the shoreline at Namibia, the observed undulation length is around
60 km, which was perfectly predicted in the stability analysis by two realistic
combinations of wave conditions, beach profiles and closure depths. Running
the shoreline evolution model with these two combinations gave undulation
widths of 13.5 and 15 km, somewhat larger than the observed values of 10-12 km. The predicted width of the spit for the two combinations was only slightly too large for one combination and over predicted by a factor 2 for the other combination. The migration speed of the spit and of the undulations was under predicted in the model, probably due to the over prediction of both the undulation width and the width of the spit, but perhaps also because the model assumes a constant beach profile along the entire undulation, whereas the observations show that the beach profile is steep at the undulation crest and shallow at the undulation trough.

For the shoreline on the west coast of Denmark the stability analysis was only able to predict undulation lengths in the correct range for the small "every day" waves and not for the larger storm waves.
CHAPTER 5. COMPARISONS WITH OBSERVATIONS
Chapter 6

Coastline undulations on the West Coast of Denmark: Off-shore extent, relation to breaker bars and transported sediment volume.

The present chapter is an article which has been accepted in Coastal Engineering. There are two co-authors, namely Prof. J. Fredsoe and Civil Engineer S.B. Knudsen.

6.1 Introduction

Longshore shoreline undulations are observed on many shorelines around the world. They are periodic features on the shoreline with lengths ranging from hundreds of meters to many kilometers, widths ranging from tens of meters to hundreds of meters or kilometers and time scales of migration of years to decades. Examples in the literature include: the Danish North Sea coast (Bruun, 1954a), the Holderness coast of East England (Pringle, 1985), the Dutch coast (Verhagen, 1989), Long Point in Lake Erie, Canada (Stewart and Davidson-Arnott, 1988), Southampton Beach, New York (Thevenot and Kraus, 1995), the El Puntal spit in Spain (Medellin et al., 2008) and the Northern shoreline of Port Stephens estuary, Australia (Vila-Concejo et al., 2009).

The undulations have implications for the coast: The beach narrows in the troughs of the undulations so the dunes become more vulnerable to erosion, see Ruessink and Jeuken (2002). Coastal structures like groins may be undermined from behind when the trough passes, since the shoreline
undulations may pass a groin field unaffected as described by Verhagen (1989).

A possible explanation for the existence of the longshore undulations is an instability mechanism of an otherwise straight and uniform coastline exposed to very oblique wave incidence. This instability is related to the fact, that a maximum in the longshore sediment transport exists for a critical wave angle around 45°. This is the case for both the energetic approach like the CERC formulation (Komar and Inman, 1970) and a deterministic approach like that by Deigaard et al. (1986), as shown by Ashton and Murray (2006a) who found the critical angle to be 42°. If the angle of the incoming waves is larger than the critical angle, a reduction in the wave angle will lead to an increase in the longshore sediment transport. Therefore any small undulation will have a larger longshore sediment transport on the upstream side \( q_u \) than on the downstream side \( q_d \) sketched in figure 6.1. This means that any small undulation will grow in size, making the shoreline unstable under oblique wave incidence. From figure 6.1 it is further seen that if the incoming wave direction is below the critical angle, any undulations will disperse since the longshore sediment transport on the down drift side \( q_d \) will then be larger than the longshore sediment transport on the up drift side \( q_u \).

Some controversy exist regarding where \( \alpha \) should be measured. Original formulations of the CERC model for longshore sediment transport were formulated in terms of the wave angle at breaking. This lead to rejection that the instability could exist at all because most waves will have refracted so much when they reach their break-point that the angle \( \alpha \) will always be below the critical angle of \( \approx 45° \). However Ashton and Murray (2006a) used formulations of the CERC model in terms of the deep water wave angle to show that due to wave height variation along a shoreline undulation, the instability can exist even though the angle at breaking is below the critical angle. Based on the above it can be realized, that a fundamental requirement for the instability to exist is that the angle between the incoming waves and the shoreline is larger than the critical angle at the water depth, where the waves start to feel the shoreline undulation.

The instability was first outlined by Grijm (1960). He used an approximate mathematical analysis to show that if the incoming waves arrive at the shoreline with an angle equal to the one giving maximum longshore transport the resulting shoreline shape is either straight or what he terms cusps. His assumptions were somewhat crude, among others the sea bed was assumed to be flat, so wave refraction was not accounted for. LeMéhauté and Soldate (1978) determined under which conditions a straight shoreline is unstable taking refraction into account using Snell’s law.

Ashton et al. (2001) did a more complete investigation of the instability. In this work the longshore littoral drift was determined using the CERC formulation approach for the description of the longshore transport. Waves
were transformed from off-shore to breaking using linear theory for refraction and shoaling and assuming parallel depth contours (again Snell’s law). An implication of the use of Snell’s law for the wave refraction is that the shoreline undulations are assumed to extend all the way out to the depth of closure used in the model.

The effect of non-parallel depth contours on wave refraction in conjunction with the instability was first included in an analysis by Falqués and Calvete (2005), who looked at the diffusivity and instability of sandy coastlines. Their work is a linear stability analysis of a uniform shoreline subject to very oblique wave incidence described using a one-line model approach. The main new contribution was the determination of the most unstable perturbation length which, among others parameters, scales with the break zone width with a scaling factor of 50-100. Further it was found that in order for the shoreline to be unstable, the shoreline undulations must extend further off-shore than a certain minimum distance, otherwise the calculations predict the shoreline to be stable. This is easily realized by the following considerations: if the undulations are only felt a short distance away from the shoreline, the incoming waves will be turned to smaller angles by refraction, and assuming the incoming waves to be longshore uniformly distributed, they may already have been turned to smaller angles, i.e. to the stable regime - before they can feel the longshore non-uniformity due to the undulations. In this case an instability analysis will predict the coast to be stable. This problem is discussed by Falqués et al. (2011).

Uguccioni et al. (2006) included inertia terms in the longshore current in their linear stability analysis of the most unstable perturbation length. They found the most unstable length to increase when including the inertia terms and confirmed that the length of the most unstable perturbation scales with break zone width.

As described above, the shoreline only becomes unstable in the modeling
if the shoreline undulations extend some distance offshore. To the authors
knowledge, no field data concerning the offshore extent of the shoreline un-
dulations exist. This is because a dataset covering a large area and time
span is needed to be able to identify and follow shoreline undulations in
time and space, further the data set needs to have a relatively dense spatial
and temporal resolution. In Denmark, the Danish Coastal Authority (KDI)
usually surveys the coastal profile along lines perpendicular to the shoreline,
with a mutual distance of 1 km every 1-5 years. This resolution is too coarse
to identify and follow the shoreline undulations with a reasonable accuracy.
However 2 detailed surveys at the Danish West Coast are available, where
the survey lines are only 100 m and 200 m apart, and the survey lines have
been surveyed twice or more per year for 9 and 4 years. These two data
sets can be used to provide insight into the offshore extent of the shoreline
undulations.

The shoreline undulations can be viewed as a pile of sand migrating
down the shoreline as shown in figure 6.2. The migration of the pile of
sand contribute to the longshore sediment transport with the product of
the volume per alongshore unit length times the migration velocity. Due to
the missing knowledge on the offshore extent of the shoreline undulation, it
has so far not been possible to estimate how much sediment is transported
alongshore in this fashion.

The described modeling of the shoreline undulations all ignores the ex-
istence of the longshore breaker bars. This contradicts some of the fields
studies where the breaker bars are seen to have at least some role in the

Figure 6.2: Definition of the local sediment volume which is seen as the gray area
on the right figure. The closure depth is computed using the relationship given in
formation and evolution of the shoreline undulations, e.g. see Davidson-Arnott and Heyningen (2003). Further studies of the relation between the longshore breaker bars and the shoreline undulations are therefore needed. The detailed data sets can be used to give further insights in this area.

Scope of present work

The aim of the present investigation is to present an analysis of bathymetric data from the West Coast of Denmark. The net longshore sediment transport along this part of coastline is between 1.5 and 2.2 million m$^3$/year (Kystdirektoratet, 2001). On this shoreline, shoreline undulations are observed on an otherwise straight and uniform coast. The undulations have a spatial scale of kilometers and the time it takes one undulation to migrate its own wavelength is years or decades. The undulations on one of the stretches of shoreline are believed to be caused by the instability due to oblique incident waves as explained above.

The main goals of the analysis are:

- How far off-shore can you feel the shoreline undulations?
- What is the implication of breaker bars on the undulations?
- How much will the migrating undulations contribute to the total net longshore sediment transport?

6.2 Data and Environmental Setting

6.2.1 Bathymetric Data

Three data sets of bathymetric surveys have been used in the present analysis. Each of the bathymetric surveys consists of a number of survey lines perpendicular to the coast, along which the bathymetry is measured. This is repeated at given time intervals in order to observe changes in the shoreline and in the shore face. Each subset of data measured at the same time is referred to as a time frame in the following, note that this might not be obtained on exactly the same date, but will usually be within one or two days. The first dataset consists of survey lines along the entire west coast of Denmark. These lines are located every 1 km and have been surveyed every 1-5 years since 1957. This dataset has only been used to generate a mean shoreline position since both the temporal and spatial resolution is too crude to identify the longshore shoreline undulations looked for in the present case. The other two datasets are from more detailed surveys conducted by KDI on two different sections of the west coast of Denmark, see figure 6.3. The northern located section is Husby and this section is 3km long while the southern section is Srd. Holmslands Tange which is 11 km long. The data set at Srd. Holmslands Tange is limited to depths between
CHAPTER 6. FIELD DATA ANALYSIS

Figure 6.3: The location of the used data. Wave data was available at Station 2041 (located 14 km offshore at 25 m water depth), and at station 2031 (located 4 km offshore at 17.5 m water depth).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Srd. Holmslands Tange</th>
<th>Husby</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of survey lines</td>
<td>56</td>
<td>31</td>
</tr>
<tr>
<td>Distance between lines</td>
<td>200 m</td>
<td>100 m</td>
</tr>
<tr>
<td>Number of surveys</td>
<td>12</td>
<td>26</td>
</tr>
<tr>
<td>Start date</td>
<td>26-1-2005</td>
<td>21-4-1999</td>
</tr>
<tr>
<td>End Data</td>
<td>27-1-2009</td>
<td>7-5-2008</td>
</tr>
<tr>
<td>Northern most line (UTM North)</td>
<td>6236730</td>
<td>6201330</td>
</tr>
<tr>
<td>Southern most line (UTM North)</td>
<td>6233730</td>
<td>6190390</td>
</tr>
<tr>
<td>Length of shoreline section</td>
<td>11km</td>
<td>3km</td>
</tr>
<tr>
<td>Length of survey lines</td>
<td>$\approx 700$m</td>
<td>$\approx 1000$m</td>
</tr>
</tbody>
</table>

Table 6.1: Summary of the two data sets

approximately $+4$ and $-5$ m, corresponding to 700 m offshore whereas the data set at Husby is limited to depths between approximately $+4$ and $-9$ m, corresponding to 1000 m offshore. A summary of the two data sets is seen in table 6.1.

6.2.2 Hydrodynamic Data

Measurements of the wave conditions are available at two locations shown in figure 6.3 as station 2031, located 4 km offshore and station 2041 located 14 km offshore. The two stretches of coastline in the detailed areas have very similar hydrodynamic conditions, the yearly mean significant wave height at the two locations are both 1.30 m, and the yearly mean peak wave period is 4.7 s at point 2031 and 4.5 s at point 2041. The main difference in the
6.2. DATA AND ENVIRONMENTAL SETTING

Figure 6.4: Left: Wave rose for the station closest to Husby, i.e. station 2031 at 17.5 m water depth. Right: Wave rose for the station closest to Srd. Holmslands Tange, i.e. station 2041 at 25 m water depth. The shoreline orientations are shown with the hatched lines.

conditions is the incoming wave direction which is more northerly at point 2041 than at point 2031. In the following the wave climate measured at point 2031 is taken to represent the wave climate at Husby, and the wave climate measured at point 2041 is taken to represent the wave climate at Srd. Holmslands Tange. The normal to the shoreline (the shore-normal) is turned towards the southwest at Srd Holmslands Tange compared to Husby. This means that the waves approach the shoreline at Srd. Holmslands Tange significantly more oblique than at Husby. This is seen in figure 6.4 where wave roses are drawn for both locations and the coastline orientation is also depicted. It is seen that significantly more wave energy approach the shoreline with an oblique angle at Srd-Holmslands Tange than at Husby.

Wave roses for the 6 months period preceding each of the time frames at Srd. Holmslands Tange are seen in figure 6.5. For time frames 0-4, 6-7 and 11, Srd. Holmslands Tange receives a substantial amount of wave energy from oblique directions, i.e. with a large angle to the shoreline. For time frames 5 and 8-10 less waves arrive with an oblique angle.

6.2.3 Beach Classification

The classification of beaches is commonly done by the surf-scaling parameter, see e.g. Wright and Short (1984). The surf scaling parameter is defined as:

\[ \epsilon = a_b \frac{\omega^2}{g \tan^2(\beta)} \] (6.1)

Where \( a_b \) is the breaker amplitude, \( \omega \) is the wave frequency, \( g \) is the gravity and \( \beta \) is the beach slope. Depending on the value of \( \epsilon \) the beach is
Figure 6.5: Detailed wave roses. For each time frame a wave rose is shown for the preceding 6 months period. The numbers refer to the time frame.
characterized as either: reflective ($\epsilon < 2.5$), intermediate ($2.5 < \epsilon < 20$) or dissipative ($\epsilon > 20$). This has implications for the type of longshore bars and shoreline features one expects to find on the beach, see Aagaard and Masselink (1999). For the Srd. Holmslands Tange area the beach slope is around 0.0125 and at Husby the beach slope is around 0.020. Using the yearly mean significant wave height and yearly mean peak wave period the surf scaling parameter is found to be 3000 at Srd. Holmslands Tange and 1200 at Husby. Both of these indicate a dissipative beach state. Using the longest waves measured, $T_p = 14s$ with the mean wave height the values reduce to 340 and 130, still in the dissipative beach regime. The median sediment diameter, $d_{50}$, at Srd. Holmslands Tange is around 0.2mm and at Husby around 0.35mm (Kystdirektoratet, 1999). The Dean parameter, $\Omega$ is also used to characterize the beach state (Wright and Short (1984)), it is defined as:

$$\Omega = \frac{H_b}{w_s T}$$ (6.2)

Where $H_b$ is the breaking wave height, $w_s$ is the fall velocity of the sediment and $T$ is the wave period. Using the yearly mean wave conditions and the median sediment diameter stated above, the Dean parameter is found to be 23 at Srd. Holmslands Tange and 9 at Husby, implying dissipative beach states with longshore uniform breaker bars.

### 6.2.4 Tide

Both beaches are characterized as micro tidal with tidal ranges below 1m (around 0.8 m at Srd. Holmslands Tange and 0.7 m at Husby (Farvandsvæsset, 2011).

### 6.3 Offshore Extent of Shoreline Undulations

To determine the offshore extent of the shoreline undulation, the undulations must first be identified. Next the undulations on the shoreface must be identified, whereafter the correlation between the undulations on the shoreface and on the shoreline can be found. Based on these correlations, the off-shore extent of the shoreline undulations can be estimated.

#### 6.3.1 Identification of Shoreline Undulations

In order to identify the shoreline undulation, the position of the shoreline must first be identified. Therefore a procedure for the identification of depth contours on the shoreface must be found. A coastal profile does not monotonically decrease in the offshore direction, among other things because of the presence of longshore bars. This complicates matters when trying to
identify the location of depth contours on the shoreface. The procedure applied in this study is a robust one as compared to the commonly used zero crossing technique. The calculation is performed using a volume approach: Each contour is found by integrating the volume of sediment between two limiting depths, the upper limit is 0.5 m larger than the contour in question and the lower limit is 0.5 m smaller. The calculated volume between the upper and lower limit is converted into a single coherent layer with the same thickness as the interval and extending offshore from a fixed location on the beach. An illustration of the procedure is sketched in figure 6.6. A result of the procedure is that the width of possible longshore bars is added to the position of the depth contour placing the contour further offshore than it really is, as seen on the figure. The advantage of the procedure is that the depth contour does not suddenly jump from inside the longshore bar to outside the longshore bar if the trough becomes filled with sediment. The absence of these jumps eases the interpretation of the results.

The shoreline together with the depth contours at +4 m to -5 m with an interval of 1 m is extracted from all three data sets. The calculation is done using an in-house software called KIMENU/Kystlinier available at KDI. Next the undulations of the shoreline were identified. These are defined as the deviation from the mean shoreline such that:
6.3. OFFSHORE EXTENT OF SHORELINE UNDULATIONS

Figure 6.7: Definition of the undulation width.

\[ w' = w - \bar{w} \]  \hspace{1cm} (6.3)

where \( w \) is the shoreline position, \( w' \) is the shoreline undulation, and \( \bar{w} \) is the mean shoreline position, see figure 6.7. The undulations, \( w' \) are all based on the long term mean - the "West Coast data" - since the mean cannot be obtained accurately enough from the detailed surveys due to the short duration of these (9 years at Husby and 4 years at Srd. Holmslands Tange).

Time stacks of the shoreline undulations have been created for both areas under investigation. The time stacks show the shoreline undulations at different time steps on the vertical axis, so the shoreline features moving alongshore can be identified. Conversely if a feature does not move alongshore, it can be rejected as a permanent feature not contributing to the mean shoreline, because the mean shoreline is based on the "West Coast data" with a lower spatial resolution than the detailed datasets. Figure 6.8 shows the time stack for Husby. It can be observed that no features are present at all time steps, i.e. most features are seen only on two or three consecutive shorelines before disappearing again. A trough can be seen at \( t=0 \) at a longshore coordinate equal to 2250 m, and this trough travels south, (to the left on the figure), and can be followed until it disappears around \( t = 500 \) days at longshore coordinate equal 500m, thus traveling around 1200 meters per year. The length of the feature can hardly be identified. Figure 6.9 shows the time stacks for the Srd. Holmslands Tange area. In this area the shoreline features are clearly visible and can be followed in time. For instance the crest seen at \( t=0 \) longshore coordinate = 2500 m can be followed all the way to \( t = 1400 \) where it is located at longshore coordinate = 1000 m, thus moving at a rate of around 370 m/year. There are 2 features present at all times indicating a feature length of 5-6 km.
Figure 6.8: Time stack of the shoreline evolution at Husby. North is to the right on the figure.
6.3. OFFSHORE EXTENT OF SHORELINE UNDULATIONS

Figure 6.9: Time stack of the shoreline evolution at Srd. Holmslands Tange. North is to the right in the figure.
6.3.2 Imprint of Shoreline Undulations on Shoreface

To study the relation between the shoreline undulations and features on the shoreface, possible features on the shoreface must first be identified. This is done in the same manner as for the shoreline undulations, namely the position of each depth contour is found relative to the mean shoreline position such that:

\[ y' = y - \bar{w} \]  \hspace{1cm} (6.4)

where \( y \) is the position of the depth contour, \( y' \) is the position of the depth contour relative to the mean shoreline and \( \bar{w} \) is the mean shoreline position, see figure 6.10.

Figures 6.11 and 6.12 show examples of the computed depth contours together with the shoreline for Husby and Srd. Holmslands Tange. These figures clearly demonstrate the existence of coherent shoreline features. For Husby the computed depth contours along the shoreline are seen in figure 6.11. It is noted that the shoreline feature around 800m can be seen on the time stack at longshore coordinate = 800m, time = 300 days in figure 6.8. In figure 6.11 it is seen that the feature does not reach further out in the profile than to a depth between 2 and 3 meter, due to the fact that the -3 meter contour continues to move away from the shoreline when moving from right to left after the shoreline has begun to recede at 800m. An equivalent figure for the Srd. Holmslands Tange area is depicted in figure 6.12, on this figure the feature observed on the shoreline contour at longshore distance 2000 m is the feature which can be identified on all time frames in figure 6.9. On the present figure it is seen all the way out to the -5 meter contour; however there seems to be a phase shift between the shoreline undulation and the undulation at the -5-m depth contour.

To quantify the coherence, scatter plots of the position of the depth contours, \( y' \) as function of the shoreline position, \( w' \) were made. Next the
6.3. OFFSHORE EXTENT OF SHORELINE UNDULATIONS

Figure 6.11: Undulating depth contours along the mean shoreline at Husby.

A linear relationship between the two parameters was found for each time step using a least squares method. This implies finding the best linear relationship between $y'$ and $w'$ shown in figure 6.10, i.e. $S$ and $y_0$ are optimized in the linear relationship

$$\hat{y}' = S \cdot w' + y_0$$

(6.5)

Where $w'$ is the shoreline position and $\hat{y}'$ is the predicted position of the depth contour, $S$ is the slope of the linear relationship and $y_0$ is a constant in the linear relationship. If the amplitudes of the undulations on $y'$ and $w'$ are equal, $S$ will be one. $y_0$ is a measure of the average distance between the shoreline and the depth contour, this value has not been used in the present work. From the least squares fit a $R^2$-value can be computed to represent the goodness of the fit. This value is computed by:

$$R^2 = 1 - \frac{S_{err}}{S_{tot}}$$

(6.6)

$$S_{tot} = \sum (y'_i - \bar{y})^2$$

(6.7)

$$S_{err} = \sum (y'_i - \hat{y}_i)^2$$

(6.8)

(6.9)

Where $\bar{y}$ is the average of $y'$. For a perfect fit $R^2 = 1$ (in this case the error
Figure 6.12: Undulating depth contours along the mean shoreline at Srd Holmslands Tange.
An example of a scatter plot of the position of a depth contour as function of the shoreline position is shown in figure 6.13, where the position of the -4-m depth contour as function of shoreline position at Srd. Holmslands Tange is depicted for all time frames. The scatter is large, but the overall trend is also visible. The time average and time minimum of the $S$ and $R^2$ over the measuring periods (4 years at Srd. Holmslands Tange and 9 years at Husby) were calculated for each depth contours. The result is shown for Srd. Holmslands Tange in figure 6.14 and 6.15, and for Husby in figure 6.16 and 6.17.

It is seen that for Husby the minimum $S$ is positive for depths 1m, 0m and -1m, whereas at Srd. Holmslands Tange the minimum $S$ is positive for depths: 2m to -4m (+3m is also positive, but very close to 0). This indicates that at Husby, the shoreline features are confined to an area close to the shoreline, whereas at Srd. Holmslands Tange the features on the shoreline are imprints of the beach profile moving on- and off-shore between +2 m and -4 m.
Figure 6.14: Time-minimum S and $R^2$ for each depth contour for Srd. Holmslands Tange.
6.3. OFFSHORE EXTENT OF SHORELINE UNDULATIONS

Figure 6.15: Time-average $S$ and $R^2$ for each depth contour for Srd. Holmslands Tange.
CHAPTER 6. FIELD DATA ANALYSIS

Figure 6.16: *Time-minimum* $S$ and $R^2$ for each depth contour for Husby.

Figure 6.17: *Time-average* $S$ and $R^2$ for each depth contour for Husby.
6.3. OFFSHORE EXTENT OF SHORELINE UNDULATIONS

6.3.3 Phase Shift

At Srd. Holmslands Tange is can be observed from figure 6.12, that the depth contours seems to shift their position in the down-drift direction along the coast when moving offshore. The calculated values of the correlation shown in figure 6.14 to 6.17 have neglected this phase shift.

In the following an attempt is made to include this effect: The phase shift is quantified by shifting each depth contour back and forth alongshore and then choosing that phase shift, which gives the largest $R^2$ value for the linear regression between the shoreline position and the shifted position of the depth contour. This analysis was performed for both Srd. Holmslands Tange and Husby; however the results for Husby did not show any trends and are therefore not presented here.

For Srd. Holmslands Tange both the original position and the shifted position of the depth contours for time frame 2 are shown in figure 6.18. By defining the phase shift shown in figure 6.10 as positive, it is noted that the phase shifts seen in figure 6.18 is positive offshore, and negative on-shore, indicating that the features on the shoreface form an angle to the shoreline different than 90 degrees.

An example of a scatter plot of the phase shifted position of a depth contour as function of the shoreline position is shown in figure 6.19, where

Figure 6.18: Example showing the depth contours (dotted lines) and the shifted depth contours (full lines). This is for time frame 2.
the phase shifted position of the -4-m depth contour as function of shoreline position at Srd. Holmslands Tange for all time frames is plotted. Comparing this figure to figure 6.13, it is seen that the points are concentrated in a narrower band for the shifted contour positions than for the original positions, thus indicating a better correlation.

The phase shift as function of the depth contour is depicted for all time frames in figure 6.20. It can be observed that the phase shift is primarily positive for the off-shore contours and negative for the onshore contours. For the offshore contours a maximum in the phase shift is seen at the -3-m depth contour.

During the computation of the phase shift, the $S$ and $R^2$ for the phase-shifted contours are computed for all time frames and all depth contours and plotted in figure 6.21, which demonstrate that the time-average $S$ increases due to the phase-shifting for all depth contours. Further it is seen that both $S$ and $R^2$ have maximum values at the 0-m depth contour and generally decrease away from this depth for both positive and negative depths; the exception being at -4m where a local maximum in both $S$ and $R^2$ can be observed.

To represent the time variation of the $S$ and the $R^2$ values, the average over the depth contours is computed for both the $S$ and the $R^2$ values for each time frame. This contour-averaging is made for both the original and
phase-shifted depth-contour positions. The result is presented in figure 6.22. The phase shifted depth contour positions gives the largest $S$ for all time frames except the last one. Further it is seen that for the original contour positions, the $R^2$ (and to some extent also the $S$) is larger for the last 5 time frames compared with the first 6 time frames. This difference is almost removed by applying the phase shifting.

6.4 Shoreline Features and Longshore Bars

In figure 6.23-6.25 the measured bathymetry is shown for all time frames for the Srd. Holmslands Tange area. The longshore bars are clearly visible on the figures, and furthermore it can be observed that the bar system is moving towards the left (or south) in the figures, which is also the direction of the net sediment transport.

To quantify the relationship between the longshore bars and the shoreline undulations, the longshore bars must in one way or another be quantified. Two obvious quantities defining a longshore bar are its height and its offshore location. The location is chosen as the distance from the mean shoreline position to the highest point on the longshore bar and the height of the bar is chosen as the vertical distance from the trough to the highest
Figure 6.21: The time averaged $S$ and $R^2$ values for each depth contour for both the original and phase shifted contour positions.
6.4. SHORELINE FEATURES AND LONGSHORE BARS

Figure 6.22: The mean of the $S$ and the $R^2$ values for the correlation between the position of the depths contours and the shoreline position at Srd. Holmslands Tange area for all time frames.
Figure 6.23: Bathymetry of upper shoreface for time frame 0-3 at Srd. Holmslands Tange. Distances and depths are in meter.
Figure 6.24: Bathymetry of upper shoreface for time frame 4-7 at Srd, Holmslands Tange. Distances and depths are in meter. The colormap is the same as in figure 22a.
Figure 6.25: Bathymetry of upper shoreface for time frame 8-11 at Srd. Holmslands Tange. Distances and depths are in meter. The colormap is the same as in figure 22a.
An example showing three different longshore bars is shown in figure 6.26. All bars found in the analysis of the profile data are shown with black crosses. Longshore bars which are believed to belong to any of the three main bars are given the symbol of the bar, i.e. see the legend in the figure. As can be seen it is not always evident which of the bar positions actually belong to a continuous longshore bar and which do not.

The position of the bars as function of the longshore coordinate is depicted in figure 6.27 and 6.28. A tendency for the bar system to move to the south (left on the figures), is observed when following the crest of the middle bar at 7000m from the top plot down in figure 6.27, indicated with an arrow on the figure. The same direction of movement is seen when following the trough of the inner bar located around 2000m on the top plot of figure 6.27, also indicated with an arrow, it has moved to around 800m on the bottom plot of the same figure.
Figure 6.27: Position of longshore bars for time frames 0-5. The legend is the same as in figure 6.26.
Figure 6.28: Position of longshore bars for time frames 6-11. The legend is the same as in figure 6.26.
From figure 6.28 it can be observed that the connected longshore bars have a tendency to be located further offshore in the southern part of the measured domain than in the northern part, therefore the first analysis to be performed is to find the linear relationship between the longshore coordinate and the bar position for each time frame; these linear relations are shown as the straight lines in figure 6.27 and 6.28. The $S$ and the $R^2$ values of the linear fits is computed for all time frames. The $R^2$ value is between 0.55 and 0.88 for the inner bar, between 0.77 and 0.94 for the middle bar and between 0.12 and 0.87 for the outer bar. Furthermore $S$ is negative for the inner and middle bar whereas it is positive at three time frames for the outer bar, this can also be observed in figures 6.27 and 6.28.

Next the linear relation between the shoreline position and the bar position both relative to the mean shoreline is found. In order to correct the bar position for its mean dependency on longshore coordinate the linear relation is subtracted from the bar position giving a corrected bar position. Next the linear relation is determined between the corrected bar position and the shoreline position, both relative to the mean shoreline.

The same procedure regarding $S$ and $R^2$ is applied for the correlation between the shoreline position and both the corrected and uncorrected bar positions. In Table 6.2 is shown the mean and standard deviation of the computed $S$ and $R^2$ values.

<table>
<thead>
<tr>
<th></th>
<th>Mean $S$</th>
<th>Std $S$</th>
<th>Mean $R^2$</th>
<th>Std $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inner bar corrected</td>
<td>0.63</td>
<td>0.30</td>
<td>0.40</td>
<td>0.20</td>
</tr>
<tr>
<td>Inner bar</td>
<td>1.79</td>
<td>0.37</td>
<td>0.72</td>
<td>0.10</td>
</tr>
<tr>
<td>Middle bar corrected</td>
<td>0.67</td>
<td>0.42</td>
<td>0.30</td>
<td>0.20</td>
</tr>
<tr>
<td>Middle bar</td>
<td>0.06</td>
<td>1.08</td>
<td>0.06</td>
<td>0.22</td>
</tr>
<tr>
<td>Outer bar corrected</td>
<td>0.80</td>
<td>1.52</td>
<td>0.18</td>
<td>0.38</td>
</tr>
<tr>
<td>Outer bar</td>
<td>1.66</td>
<td>1.63</td>
<td>0.38</td>
<td>0.37</td>
</tr>
</tbody>
</table>

Table 6.2: The mean and standard deviations (std) of the correlations between the shoreline position and the bar position.
6.5 SEDIMENT FLUX DUE TO UNDULATIONS

Figure 6.29: The slope and $R^2$-value of the correlation between the corrected bar position and the shoreline position for the middle bar for all time frames.

Regarding any possible correlation between the bar height and the shoreline position no trends are observed in the data and those results are therefore not presented here.

6.5 Amount of Transported Sediment in Shoreline Features

It is well known that rhythmic bed forms create a non-uniform variation in the sediment transport, see e.g. Fredsoe (1982) regarding sand waves. In the case of sand waves the sediment transport is usually proportional to the local height of the bed-form. In the case of rhythmic coastline undulations an equivalent result can be obtained using the volume of sediment in an alongshore feature. The sediment conservation equation reads:

$$\frac{\partial q}{\partial x} = -\frac{\partial X}{\partial t}$$  \hspace{1cm} (6.10)

where $q$ is the volumetric longshore sediment transport including pore volume, $X$ is the volume of sediment per alongshore unit in the local coastal profile, $x$ is the longshore coordinate and $t$ is the time, see figure 6.2.

For an alongshore feature migrating with unchanging shape, the volume
X can be written as:

\[ X = X(x - at) \quad (6.11) \]

where \( a \) is the migration speed of the feature. Combining this with (6.10) and assuming that the feature is migrating with unchanging shape we can obtain:

\[ \frac{\partial X}{\partial t} = -a \frac{\partial X}{\partial x} \quad (6.12) \]

Eqs. 6.10 and 6.12 combined gives:

\[ \frac{\partial q}{\partial x} = a \frac{\partial X}{\partial x} \quad (6.13) \]

which can be integrated to

\[ q = aX + q_0 \quad (6.14) \]

This result implies that the longshore sediment transport \( q \) is proportional to the local volume of the undulation from the dune foot plus a constant, \( q_0 \).

To estimate this sediment volume per alongshore unit length, \( X \), the thickness of the active layer across the beach and the shore face must be estimated. Alternatively the active width of the undulation can be determined to provide the same information. These layers are those layers which take part in the undulations movement as shown in figure 6.2. Also the limiting depth on the shore face, at which this active layer is a substantial shoreline feature must be identified. In addition to this, information on the migration speed \( a \) of the features needs to be determined, this was determined in section 6.3.

The local sediment volume has been estimated from the coastal profiles extracted from the KDI database as follows: For every coastal profile the distance from the mean shoreline to each point on the profile is calculated. Next the area \( A \) between two limiting depths is found by integrating the found distances over the vertical, thus:

\[ A = \int_{h^-}^{h^+} y(z)dz \quad (6.15) \]

\[ y = sign(e - e_m) \sqrt{(e - e_m)^2 + (n - n_m)^2} \quad (6.16) \]

where \( sign() \) is the sign function, \( y \) is the distance from each point on the profile to the mean shoreline position, \( e \) and \( n \) are the east and north coordinates (in UTM Zone 32) of each point on the profile, \( e_m \) and \( n_m \) are the east and north coordinates of the mean shoreline position. \( h^- \) and \( h^+ \) are the limiting depths between which the volume is found. \( A \) is the shaded area in figure 6.30, where the shaded area left of the vertical line is counted as negative in the calculation.
6.5. SEDIMENT FLUX DUE TO UNDULATIONS

The calculation was performed at each profile along the shoreline. Then the volume $V$ of sediment in the shoreline features could be estimated as:

$$V = \int_{L} (A(x) - \min(A(x))) \, dx$$

(6.17)

where $V$ is the total volume of sediment in the shoreline features, $x$ is the longshore coordinate, $L$ is the length of the stretch of shoreline in question and $\min(A(x))$ is the smallest area found along the stretch of shoreline.

The mean flux of sediment in the shoreline features is then determined as:

$$Q_f = \frac{aV}{L}$$

(6.18)

Where $a$ is the migration speed of the feature, i.e. the speed at which it moves alongshore, and $Q_f$ is the mean sediment flux including pore volume.

The shoreline undulations were only vaguely identified at Husby; therefore the calculation of the transported sediment volume has only been performed at Srd. Holmslands Tange. The limiting depths certainly have an impact on the result. A natural choice of the limiting depth off-shore is the smallest depth at which no correlation to the shoreline exists; that is the smallest depth contour where shoreline undulations are not recognized. The same is true for the limiting depth on-shore; the natural choice for the limiting depth on shore is the contour on the beach which is not correlated to the shoreline contour. The criteria for when a depth contour was believed to be...
correlated well enough with the shoreline is chosen as $\bar{S} > 0.5$, where $\bar{S}$ is the time averaged value of $S$ for the phase shifted contours. With this choice it is found that $h^+ = +2$ m and $h^- = -5$ m, see figure 6.21. The second important parameter for the calculation is the celerity or migration speed of the shoreline undulations; in the previous section this was estimated to be 370 m/year at Srd. Holmslands Tange. Using these numbers the sediment flux at all time frames is found to vary between 100,000 m$^3$/year and 150,000 m$^3$/year equivalent to 4.5-7.1 % of the total longshore sediment transport on the beach face, estimated as 2.1 million m$^3$/year, by Kystdirektoratet (2001). Of the transported sediment in the undulations, between 5% and 20% of the sediment transport occurs above the 0-m contour (between +2 m and 0 m) and the remaining 80% to 95% occurs below the 0-m contour (between 0 m and -5 m).

### 6.6 Comparison with Stability Analysis

To investigate if the observed undulations at Srd. Holmslands Tange is in line with the shoreline instability theory, these are compared to results from the stability analysis by Falqués and Calvete (2005) using the wave climate measured at point 2041, see figure 6.3.

Falqués and Calvete (2005) used the CERC formulae to investigate whether a small perturbation would increase in time as function of the following parameters: angle of the incoming waves at the closure depth, height of the incoming waves and steepness of the incoming waves.

In table 6.3 is shown the most unstable undulation length for a closure depth of 10 m and a beach profile steepness of $\beta = 0.01$, as function of wave steepness, $s$, and angle of the incoming waves; values are taken from figure 6 Falqués and Calvete (2005). It is observed that the undulations will only occur if the angle of the incoming waves is larger than 50° for the steep waves ($s = 0.4$) and larger than 60° for the less steep waves ($s = 0.2$). This 50° is closely related to the maximum in the longshore sediment transport at 45° in-coming waves in the CERC-formula approach.

The coastal profile at Srd. Holmslands Tange is similar to the profile with a steepness of $\beta = 0.01$; this profile has a water depth of around 5m 700 m off-shore, the water depth 700 m off-shore is around 6 m at Srd. Holmslands
Tange. The present average wave climate \((H_s=1.3 \text{ m and } T_p = 4.6 \text{ s})\) gives a wave steepness of 0.25. The average angle of the incoming waves for the shoreline at Srd. Holmslands Tange is 48° using the measured wave climate from point 2041 and a shoreline orientation of 263° from true north. Using Snell’s law for wave refraction and linear wave theory this average wave direction, which is measured in 25 m water depth, becomes 46° in 10 m water depth (using the mean wave period \(T_p = 4.6 \text{ s}\)).

We are just on the limit in between a stable and an unstable shoreline. In a real environment you need to consider both wave irregularity and wave directionality as well as impact from wind and current. Also an accurate sediment transport description may change the critical angle either up or down. Including all these factors will require a very comprehensive analysis and very accurate field data. At least the stability analysis by Falqués and Calvete (2005) indicate the right order of magnitude of the observed longshore undulation lengths.

6.7 Discussion

Migrating shoreline undulations are observed at both locations on the Danish West coast: Srd. Holmslands Tange and Husby. The analysis of the offshore extent of the undulations showed that the undulations at Srd. Holmslands Tange extend further offshore than the undulations at Husby.

Srd. Holmslands Tange has a substantial amount of wave energy arriving at the shoreline from a very oblique angle. Therefore the undulations at Srd. Holmslands Tange are thought to be caused by the instability mechanism described in the introduction. At Husby a lesser part of the wave energy arrives from an oblique angle. This corresponds well with the fact that the offshore extent of the undulations is much larger at Srd. Holmslands Tange than at Husby, since it is known that the instability mechanism will only cause the shoreline to be unstable if the shoreline undulations reach a certain minimum distance off shore. The undulations at Husby can therefore not be caused by the described instability mechanism, but must be caused by some other mechanism such as shoreline interaction with rips in the longshore breaker bars, as proposed by Kystdirektoratet (2005).

Comparing the wave climate in the preceding 6 months period for each time frame with the correlations between the shoreline position and the depth contours for the Srd. Holmslands Tange Area (comparing figures 6.5 and 6.22), it is not possible to find convincing evidence that the correlations are better when the waves have been arriving with an oblique angle to the shoreline, than when the opposite is the case. The possible reason for this is that the morphology reacts so slowly to the changes in the hydrodynamics that a simple relationship cannot be found.

Introducing a phase shift of the depth contours in the alongshore di-
rection was seen to improve the correlations between undulation on the shoreline and undulations on the depth contours. The biggest improvement was found for the first 5 time frames compared with the last 6 time frames. During the first 5 time frames, more wave energy arrived at the shoreline from oblique directions than during the last 6 time frames. Thus the phase shift is most pronounced during periods of oblique wave incidence.

The longshore bar system at Srd. Holmslands Tange consists of three longshore bars which all migrate southward, that is in the direction of the longshore sand transport and the migration direction of the shoreline undulations. The three longshore bars were observed to form an angle with the coast, indicating that as the bars migrate southward they also migrate offshore.

The correlation between the offshore position of the longshore bars and the shoreline features was existing though weak; i.e. large scatter was seen in the plots, resulting in low correlation coefficients.

Comparing the wave climate in the preceding 6 months period for each time frame with the correlations between the shoreline position and position of the middle bar, i.e. comparing figures 6.29 and 6.5, it is seen that the correlations are much better when the waves arrive with an oblique angle to the shoreline. This suggests that the position of the shoreline is influenced by the position of the longshore bar when the waves arrive at the shoreline with an oblique angle, whereas when the waves arrive normal to the shoreline the location of the bar does not influence the position of the shoreline. However this is still an open question since one can also argue that the shoreline position will have an impact on the bar position.

The comparison of the observed longshore undulations with results from the stability analysis by Falqués and Calvete (2005), showed that we are right on the limit between a stable and an unstable shoreline. Thus the comparison could not confirm that the mechanism responsible for the shoreline undulations at Srd. Holmslands Tange is the proposed instability. However the predicted undulation lengths by Falqués and Calvete (2005) fall in the correct range of the observations.

6.8 Conclusion

The offshore extent of migrating shoreline undulations is determined for two different sections of shoreline on the Danish West Coast; one section being exposed to very obliquely incident waves and the other exposed to less obliquely incident waves. On the shoreline with the most obliquely incident waves the undulations extend from the +2 m contour to at least the -5 m contour. On the other stretch the undulations are confined to a narrow region close to the shoreline.

The volume of sediment which is transported in the migrating undu-
lations extending offshore is found to vary between 100,000 m$^3$/year and 150,000 m$^3$/year equivalent to 4.5-7.1% of the total longshore sediment transport. Between 5% and 20% of the sediment transport due to the migrating undulations occurs above the 0-m depth contour and between 80% and 95% happens below the 0-m depth contour.

A weak relation between the migrating undulations and the position of the longshore breaker bars is found. This relation is stronger during periods of obliquely incident waves than during period of less obliquely incident waves.
Chapter 7

Miscellaneous Ideas

During the ph.d. study many different ideas were tested. Some lead to the model used in the previous chapter and some were abandoned due to lack of time. In this chapter various ideas and findings are presented. Perhaps it can be of value to someone in the future. The chapter is divided into the following sections:

- Instability and evolution of very short undulations.
- Time-lagged one-line evolution model.
- Iterative model for the estimation of longshore shoreline undulations.

7.1 Instability and Evolution of Very Short Undulations

In this section the instability of undulations shorter than the surf zone width are investigated. The possible existence of these short unstable shoreline undulation was pointed out by Uguccioni et al. (2006). The present work extends the theoretical work of Uguccioni by calculating the equilibrium form of shore longshore beach undulations assuming a linear beach profile and constant wave conditions. The stability analysis by Uguccioni is first extended. Using this analysis perturbations with much shorter lengths (i.e. O(50-500 m)) are found to be unstable for steep beach profiles. The nonlinear evolution of the most unstable perturbation is then modeled using a shoreline evolution in order to determine the equilibrium shape of the short shoreline undulations.

7.1.1 Methods

The methods used are very similar to those presented in chapter 2. Therefore only a brief description is offered here: The basic idea is that a computer
model is used to compute the sediment transport field in the near shore zone. This sediment transport field is then integrated to give the longshore sediment transport. The shape of the longshore sediment transport can then be used to either evaluate the stability of the initial shoreline in the stability analysis, or be used to move the shoreline in the evolution model.

**Stability Analysis**

The stability analysis is performed as described in section 2.3.6 with the small change that the beach profile in the present section is a straight line with a given gradient, $\beta$, to the depth of closure beyond which the depth becomes constant, see figure 7.1. Furthermore, in the present section, the beach profile is shifted on-shore and off-shore due to the shoreline perturbation, meaning that the shoreline perturbation affects the bathymetry all the way out to where the depth becomes constant, see figure 7.1.

**Non-linear Evolution Model**

In the non-linear evolution model the change in shoreline position is computed as in a traditional one-line model, namely through the sediment conservation equation for shorelines:

$$\frac{\Delta y}{\Delta t} = \frac{1}{D_{cl}} \frac{\Delta q}{\Delta x}$$

(7.1)

where $\Delta y$ is the change in shoreline position during one morphological time step, $\Delta t$, $t$ is the time, $x$ is the longshore coordinate, $q$ is the volumetric longshore sediment transport including pore volume and $D_{cl}$ is the closure depth. The volume of sediment due to gradients in the longshore sediment transport is distributed evenly over the beach profile, leading to a shift in the entire profile position as shown in figure 7.1. The beach profile shifts back and forth without changing its shape. The underlying assumptions are:

- The starting beach profile is in equilibrium with the cross-shore transport
- The cross-shore transport will return the beach profile to the equilibrium profile within one morphological time step.
- The curvature of the shoreline has no impact on the equilibrium profile

The hybrid model consists of the following steps:

1. Compute the local wave, flow and sediment transport fields using Mike21FM on the bathymetry.
7.1. EVOLUTION OF SHORT UNDULATIONS

2. Integrate the sediment transport across the beach profile to determine the longshore sediment transport.

3. Update the bathymetry using equation 7.1.

4. Return to 1.

The initial bathymetry consists of a sinusoidal perturbation with its length equal to the most unstable length found during the stability analysis for the given wave climate and beach profile. The mesh is constructed of quadrilaterals with a constant resolution in the longshore direction, whereas the cross shore resolution is finer in the surf zone than outside. The sediment transport is integrated across north-south beach profile lines. This means that the integration lines are not always perpendicular to the shoreline. An upwind scheme is used to compute the gradients in the longshore transport when updating the bathymetry; a backward Euler scheme is used for discretizing the time. The time step is chosen such that the maximum movement of the coastline during one morphological time step is 1 m.

As a verification that an equilibrium solution has been reached, a number of statistical properties (the variance, the skewness and the kurtosis) are calculated for the shoreline. When these properties do not change in time the equilibrium solution has been reached. In theory the statistical properties should be constant in logarithmic time since it takes exponentially long time for an equilibrium solution to be reached. This has not always been fulfilled due to the long simulation time it takes.

7.1.2 Results

In this section the results are presented. Many of the parameters needed for the computations in Mike21FM are common for all calculations. These
Figure 7.2: Growth rate as a function of perturbation length for $H_s = 3$ m, $T_p = 7$ s and $\beta = 1/20$.

parameters are seen in table 7.1.

Stability Analysis

In the following results from the stability analysis are presented. The stability analysis has been run twice, each time with different parameters allowed to vary. In the first analysis $H_s = 3$ m, $T_p = 7$ s and $\beta = 1/20$, only $L$ and $MWD$ were varied. In the second analysis $\beta$ and $H_s$ were also varied in order to get a more complete picture of the instability.

First Analysis

The results from the first analysis are seen in figure 7.2. From the figure it is clear that $L = 300$ m is the most unstable length for oblique waves ($MWD > 50^\circ$). Surprisingly $MWD = 35^\circ$ is unstable for some lengths, whereas $MWD = 30^\circ$ is not unstable for any lengths.

Second Analysis

In the second analysis more parameters were varied; these are seen in table 7.2.

The growth rate as a function of perturbation length is shown in figure 7.3 to 7.5 for different values of $\beta$ for $H_s = 2$ m. For $\beta = 1/20$ all mean wave directions have a most unstable perturbation length, whereas for $\beta = 1/100$ only the most oblique waves cause the shoreline to be unstable.

In table 7.3 the results are summarized in such a way that the most unstable perturbation length is shown for the possible combinations of wave climates and beach profiles; if all lengths are stable for the given combination
7.1. EVOLUTION OF SHORT UNDULATIONS

SW model:
Spectral discretization  directional sector
Number of directions  54
Min direction  225 °
Max direction  135 °
Solution technique  Newton-Raphson
Max # of iterations  500
RMS norm of residual  1e-6
Max norm of residual  0.001
Relaxation factor  0.1
Wave breaking  Specified gamma
Constant value  0.8
Alpha  1
Gamma  1
Effect on frequency  No
Bottom friction  No

HD model:
Flood and dry:  On
Drying depth  0.005 m
Flooding  0.05 m
Wetting  0.1 m
Density  Barotropic
Eddy formulation:  Smagorinsky
Constant value  0.28
Bed resistance:  Manning
Constant value  32

ST model:
Porosity  0.4
d50  0.2 mm
Sediment grading  1.1

Table 7.1: Common parameters for Mike21 FM, see DHI (2009a) for detailed explanation of parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_s$</td>
<td>1 &amp; 2 m</td>
</tr>
<tr>
<td>$T_p$</td>
<td>7 s</td>
</tr>
<tr>
<td>$MWD$</td>
<td>30, 40, 50 &amp; 60 °</td>
</tr>
<tr>
<td>$\beta$</td>
<td>1/20, 1/50 &amp; 1/100</td>
</tr>
<tr>
<td>$L$</td>
<td>50, 75, 100, 150, 200, 250, 300, 400, 500, 750 &amp; 1000 m</td>
</tr>
</tbody>
</table>

Table 7.2: Values for parameters for the second stability analysis.
Figure 7.3: Growth rate as a function of perturbation length for different MWD and for $\beta = 1/20$, $H_s = 2 m$

Figure 7.4: Growth rate as a function of perturbation length for different MWD and for $\beta = 1/50$, $H_s = 2 m$
7.1. EVOLUTION OF SHORT UNDULATIONS

Figure 7.5: Growth rate as a function of perturbation length for different MWD and for $\beta = 1/100$, $H_s = 2$ m

Table 7.3: Length in m of the most unstable perturbation (if any unstable lengths).

<table>
<thead>
<tr>
<th>$H_s$, [m]</th>
<th>$\beta$</th>
<th>1</th>
<th>0.05</th>
<th>0.02</th>
<th>0.01</th>
<th>2</th>
<th>0.05</th>
<th>0.02</th>
<th>0.01</th>
</tr>
</thead>
<tbody>
<tr>
<td>$MWD$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30°</td>
<td></td>
<td>75</td>
<td>-</td>
<td>-</td>
<td>150</td>
<td>-</td>
<td>-</td>
<td>150</td>
<td>-</td>
</tr>
<tr>
<td>40°</td>
<td></td>
<td>75</td>
<td>-</td>
<td>-</td>
<td>150</td>
<td>200</td>
<td>-</td>
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</tr>
<tr>
<td>50°</td>
<td></td>
<td>75</td>
<td>-</td>
<td>-</td>
<td>200</td>
<td>150</td>
<td>200</td>
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<td>200</td>
<td>150</td>
<td>150</td>
<td>150</td>
<td>150</td>
</tr>
</tbody>
</table>

the field is blank. From the table it is seen that for $H_s = 1$ m, only the steepest coastline is unstable, whereas for $H_s = 2$ m all three beach profiles are unstable for the most oblique waves.

Non-linear Evolution Model

The non-linear evolution model was run for $\beta = 1/20$, $MWD = 60^\circ$, $H_s = 1$ m, $T_p = 7$ s and $L = 300$ m. A time stack of the solution is seen in Figure 9. It is seen that a number of short modes emerge at the beginning of the simulation, after some time they die out and only the main mode is present in the solution at the end of the simulation.

In figure 7.7 the normalized variance, skewness and kurtosis as function of time step is seen. All three have been scaled between 0 and 1 as the exact values are not interesting. As can be seen from the figure, the variance and skewness both become constant at the end of the simulation whereas the kurtosis is still changing a little, indicating that the simulation should be
Figure 7.6: Time stack of the evolution of a longshore beach wave for $H_s = 3$ m, $T_p = 7$ s, $\beta = 1/20$, $L = 300$ m. There are two morphological time steps between each line in the figure. Note that two waves are shown.
run for a little while longer. In figure 7.8 and 7.9 the time stack and statistical properties of a simulation with $H_s = 2 \text{ m}, T_p = 7 \text{ s}, \beta = 1/50$ and $L = 150 \text{ m}$ are shown. It is noted that there are no secondary modes in the beginning of the simulation and that the statistical properties become constant at the end of the simulation.

7.1.3 Discussion

Stability Analysis

Two main questions arise when looking at the results from the stability analysis: Why are the short perturbations unstable? And why are some lengths unstable even for a wave angle of 35°? This is in contrast to the mechanism believed to cause the shoreline instability.

The wave fields in figure 7.10 shows a stable and an unstable situation. The situation on the left where $H_s = 1 \text{ m}$ is stable whereas the situation on the right where $H_s = 2 \text{ m}$ is unstable. When looking at the wave height variation it is clear that the two waves start out refracting in the same way (see the contour near $y = 600 \text{ m}$). The larger wave starts to break around $y = 400 \text{ m}$, whereas the smaller wave keeps refracting without breaking until $y = 150 \text{ m}$. This has the effect that a large part of the wave front, located right at the boundary at the contour at 600 m, reaches the downstream end of the perturbation for $H_s = 1 \text{ m}$ before breaking, whereas it reaches the upstream side of the perturbation for $H_s = 2 \text{ m}$ before breaking. This
Figure 7.8: Time stack of the evolution of a longshore beach wave for $H_s = 2$ m, $T_p = 7$ s, $\beta = 1/50$, $L = 150$ m. There are two morphological time steps between each line in the figure. Note that two waves are shown.
explains why the longshore transport is largest on the upstream side of the perturbation in the case of $H_s = 2$ m, whereas the transport is largest on the downstream side for $H_s = 1$ m; i.e. the longshore transport is largest where the breaking wave height is largest given that the wave direction is the same. The wave directions are in fact the same in the two cases because the refraction in the model is linear; i.e. only the wave length influences the refraction and wave breaking does not influence the wave length.

The above analysis makes it probable that the reason the short perturbations are unstable for short lengths is the refraction of the waves by more than one perturbation before breaking. When the wave front has passed one perturbation it is a matter of coincidence in the geometry whether the largest part of the wave front starts breaking upstream or downstream of the perturbation and thereby renders the perturbation stable or unstable. This further explains why short perturbations can be unstable even for non-oblique wave incidence.

It is possible for the waves to refract on more than one perturbation because the beach profile is shifted instantaneously all the way out to closure depth. The underlying assumption is that the cross-shore transport will modify the beach profile so it keeps its equilibrium form whether the longshore transport deposits or erodes sediment. This assumption may be valid on long beaches with features much larger than the surf-zone width, due to the large timescale of these features, but in the present case where the features examined are shorter than the surf-zone width the assumption is an oversimplification, leading to results which are hard to believe, as seen
in the present section.

This leaves the problem of how to avoid these short instabilities in one-line models. In the present work two approaches have been tried. The first one is presented in chapter 2. In this approach the crossshore coordinate for the beach profile is taken as the shortest distance to any point on the shoreline. Thereby short shoreline undulations are only seen on the coastal profile close to the shoreline and therefore short undulations are not unstable in this model. The second approach, that was tried, is presented in the next section, it involves introducing a time lag in the one-line model so the entire profile is not shifted on-shore or off-shore instantaneously as in the present model, but the offshore movement of sediment in the coastal profile is delayed.

**Non-linear Evolution Model**

The non-linear evolution model was able to simulate the growth of the longshore beach waves from a sinusoidal perturbation to a dynamic equilibrium. In this respect the model was successful. However because the instability was afterwards found to be due to an oversimplification in the model the results are not worth discussing in detail.
7.2 Time Lagged One-line Model

In the present section the non-linear evolution of an unstable shoreline undulation is modeled with a one-line model with a time lag. The idea in this model is to delay the offshore movement of sediment in the beach profile. The motivation is the findings presented in the previous section that an instantaneous movement of the entire beach profile in the one-line model leads to short spurious undulations on the shoreline. Introducing a delay in the offshore movement of sediment removes the spurious short undulations as will be seen in the following. First a description of the model is presented; then the results are presented and discussed.

7.2.1 Model Description

The modeling system used is the same as the one described in section 2.3.1. However in the present model an extra equation is used when updating the shoreline. The change to the beach profile is calculated as:

\[
\Delta Y_{N+1} = \sum_{n=1}^{N} a_n \exp (-k_n y) \tag{7.2}
\]

\[
k_n = \frac{b}{t_{N+1} - t_n} \tag{7.3}
\]

\[
\Delta y = \Delta Y_{N+1} - \Delta Y_N \tag{7.4}
\]

where \(\Delta Y_{N+1}\) is the total change of the beach profile from a straight uniform coast to the present time step, \(N + 1\). \(\Delta y\) is the change from the previous time step to the present time step. \(b\) is a constant which determines how fast the beach profile changes towards the equilibrium profile. \(t_n\) is the time step when the sediment was deposited and \(t_{N+1}\) is the present time step. \(a_n\) is found from the continuity equation for sediment which now reads:

\[
\Delta q_n = \int_0^y a_n \exp(-k_n y) dy \tag{7.5}
\]

\[
\Delta q_n = \frac{a_n}{k_n} (1 - \exp(-k_n y_c)) \leftrightarrow \tag{7.6}
\]

\[
a_n = \frac{\Delta q_n k_n}{1 - \exp(-k_n y_c)} \tag{7.7}
\]

where \(\Delta q_n\) is the sediment deposited at time step \(t_n\) including pore volume, i.e. \(\Delta q_n = \frac{1}{(1-n) \Delta x} \Delta q \Delta x\) for time \(t_n\). As time goes, \(k_n\) becomes larger and therefore the sediment is distributed more and more evenly over the beach profile. Figure 7.12 shows the distribution at different time steps.

The spatial gradient \(\Delta q\) is computed by first computing the gradient of \(q\) on the grid level using a central difference finite difference scheme as shown in figure 7.13. It is noted that the Mike21FM model is a finite volume model and the sediment transport \(q\) is therefore known at the element centers from this model. Next the computed gradients are integrated as:

\[
\Delta q_i = \sum_{j=1}^{J} \Delta y_{i,j} \frac{q_{i+\frac{1}{2},j} - q_{i-\frac{1}{2},j}}{\Delta y_{i,j}} \tag{7.8}
\]
Figure 7.11: Sketch showing $\Delta x$, $\Delta q$, $\Delta y$ and $dy$. $q$ is the longshore sediment transport.

Figure 7.12: Example of the distribution function for sediment over the beach profile for different times.

Figure 7.13: The computation of $\Delta q$. 
where $\Delta q_i$ is the computed gradient of the $i$'th shoreline node. $J$ is the number of nodes across the coastal profile. $\Delta \sigma_{i,j}$ is the distance between neighboring element centers (the $(i - \frac{1}{2}, j)$ center and $(i + \frac{1}{2}, j)$ center).

The mesh is constructed so the grid lines extending off-shore (the shore-normal grid lines) are perpendicular to the shoreline close to the shoreline, but close to the north boundary they are pointing straight north. This is done by solving the Laplace equation for the coastline angle and moving the nodes in the mesh so the angle of the shore-normal grid lines obey the solution. The boundary condition on the shoreline is the coastline angle and the boundary condition on the north boundary is $\pi$ (i.e. pointing north). The procedure is solved iteratively until all nodes in the mesh obey the Laplace solution for the angle.

The bathymetry is set according to the Dean profile, see Bruun (1954a) which reads:

$$d = Ay^{2/3}$$

where $d$ is the depth, $A$ is a constant depending on the diameter of the sediment and $y$ is the cross shore coordinate. In the present case $y$ was taken to follow the shore-normal grid lines of the mesh. The distance between the nodes on a shore-normal grid line was small close to the shoreline and larger away from the shoreline as seen in figure 7.15, this is in order to resolve the cross shore variation of the longshore sediment transport which takes place close to the shoreline, i.e. in the surf zone.

The actual update of the coastal profile and the shoreline is done by moving the nodes in the mesh according to the $\Delta y$ found from equation 7.4 along the shore-normal grid lines of the mesh. Then the Laplace equation for the new mesh is solved and all nodes are then moved to obey the Laplace solution for the shoreline angle. In figure 7.14 an example of the movement of nodes along a shore-normal grid line is shown. Cells that become smaller than a certain area are removed from the mesh during the routine. This means that the neighbour cell closest to the shoreline is made into a triangle. An example of a mesh with removed cells is seen in figure 7.15.

### 7.2.2 Results and Discussion

The model was run using the parameters shown in table 7.1 and the following wave parameters: $H_s = 1$ m, $MWDF = 60^\circ$, $T_p = 5$ s and $DSI = 100$. The mesh parameters were $D_{ocl} = 9$ m and $A = 0.08$. The distance between shoreline points was 120 m and the distance between nodes on the shore-normal grid lines was 10 m in the surf and 100 m further from the shoreline. The morphological time step was 33 years and $b = 10$ days in equation 7.3. The reason for this extremely large time step is that the closure depth is large compared to the wave height. The chosen combination is not realistic since waves with $H_s = 1$ m cannot move the sediment out to -9 meters. The reason this has not been corrected is that the model presented in the
Figure 7.14: Example of the movement of mesh nodes along the shore-normal grid-lines according to the found $\Delta y$.

Figure 7.15: Example where some cells have been removed from the mesh because their area was too small.
present section was abandoned for reasons explained later on, and it was not
demed reasonable to make new computations with an abandonned model.
The results are still interesting as the combination of closure depth and wave
height should not affect the shape of the emerging shoreline, but rather the
time scales and length scales. A time stack of the evolution of the shoreline
is shown in figure 7.16. The reason the simulation was stopped at this point
is that the model breaks down when the spit on the downstream side of the
undulation grows more than what is shown on the figure. The breakdown
of the model happens in the mesh update routine. No solution was found
to the breakdown and it was decided to abandon this type of model.

The reason for abandoning the model was that the mesh in the model
is a curvilinear mesh. This has many advantages, e.g. the calculation of
derivatives is quite easy. Furthermore due to the way sediment is moved
offshore as time passes in the model, it is a large advantage to have the
curvilinear mesh. In fact a formulation of this model was not found possible
on a general unstructured mesh where the structure of the element table
changes as the model evolves. This is due to the history effect in the model.

However the curvilinear mesh is not very good at discretizing the domain
when a spit grows in the domain and in the end this was the reason for
abandoning the model even though a great amount of time and effort was

Figure 7.16: The time stack of the evolution of the shoreline.
spent on making it. It is noted that the present model can be used to simulate the evolution of the shoreline up to and slightly past the formation of the spit and therefore many problems in the field of shoreline morphology can be simulated using this type of model.
7.3 An Iterative Model

In this section a different approach to model the equilibrium shape of shoreline undulations is tested. The basic idea being tested is to use an iterative procedure to find the equilibrium shape of a shoreline undulation instead of evolving the shape from a sinusoidal shape in the time domain as was done in the previous sections.

7.3.1 Model Description

The modelling framework remains the same in the present section as in section 2.3.1. However, in the present section the continuity equation for sediment is not used to update the shoreline. Instead the conservation equation is used to obtain a relation between the equilibrium shape and the shape of the long shore sediment transport. This relation is then used to find the equilibrium shape. Similar relationships are well known from e.g. the study of sand bars in rivers (see Fredsoe (1982)) and from the growth of a shoreline spit, see Petersen et al. (2008).

An equilibrium shape has been found when the long shore shoreline feature migrates along the shoreline without changing its shape. Generally a feature traveling with unchanging shape can be described by $h = h(x - ct)$, where $h$ is the local height of the feature, $x$ is the horizontal coordinate, $t$ is the time and $c$ is the migration speed of the feature, see figure 7.17.

![Figure 7.17: Sketch showing a shoreline undulation moving without changing its form with migration speed $a$ during one time step $\Delta x$](image)

Taking the total derivative of $h$ gives:

\[
\frac{dh}{dt} = \frac{\partial h}{\partial t} + \frac{\partial h}{\partial x} \frac{\partial x}{\partial t} \quad (7.10)
\]

\[
\frac{dh}{dt} = \frac{\partial h}{\partial t} + c \frac{\partial h}{\partial x} \quad (7.11)
\]

Since the feature is not changing its shape the left hand side is zero, i.e. $\frac{dh}{dt} = 0$, therefore we can combine with 7.1 to obtain:

\[
-c \frac{\partial h}{\partial x} = -\frac{1}{D_{cld}} \frac{\partial q}{\partial x} \leftrightarrow \quad (7.12)
\]

\[
q = c \cdot D_{cld} \cdot h + q_0 \quad (7.13)
\]
This implies that the longshore sediment transport is proportional to the local undulation width plus a constant in order for the shoreline undulation to travel without changing shape. We shall use equation 7.13 to try to reach this equilibrium solution through an iterative procedure.

**Iterative Procedure**

An initial mesh is constructed. It consists of a straight shoreline with a step. An example of an initial mesh is shown in figure 7.18, the southern boundary is the shoreline. The beach profile is linear with a slope of $\beta$, see figure 7.1. The cross shore coordinate is the north-south distance to the shoreline.

Next the sediment transport field is computed using the Mike21FM framework as explained in section 2.3.1, with the difference that the east and west boundaries are not periodic. The boundary condition used in the SW module is a zero gradient boundary condition on both the East and West boundary. On the northern boundary the wave conditions are specified as in the previous sections. The waves are always specified to come from western directions, so the downstream side of the step on the initial shoreline is the right (East) side of the step. In the HD model the boundary condition is zero surface elevation on both the East and West boundary.

Having found the sediment transport field, the longshore sediment transport is found by integrating the sediment transport field across the surf zone. Thereby the longshore sediment transport is known along the entire shoreline. To ensure stability in the model is has been necessary to use a filter on this longshore sediment transport. The standard filter:

$$
\hat{f}_i = 0.25 f_{i-1} + 0.5 f_i + 0.25 f_{i+1}
$$

(7.14)

where $\hat{f}_i$ is the filtered value of $f_i$, has been used 10 times at each timestep.

Now the positions of points on the shoreline are updated. This is done using equation 7.13. Only the points on the shoreline which are located...
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Part of shoreline being updated

Figure 7.19: Definition sketch of the parameters used to compute a downstream (East) of the step are updated. The point closest to the step which is part of the update is found as the shoreline point where the longshore sediment transport is smallest (due to the shaddowing effect of the step this point is in the region just downstream of the step). The longshore sediment transport at this points is $q_0$ in equation 7.13, which further implies that $h = 0$ in equation 7.13 at the point. The last parameter to be found is $c$, which is determined as:

$$c = \frac{q_{top} - q_{min}}{h_{top} - h_{min}}$$  (7.15)

where $q_{top}$ is the longshore sediment transport at the top of the step, $h_{top}$ is the value of $h$ at the top of the step, $q_{min}$ is the minimum longshore sediment transport, and $h_{min}$ is the value of $h$ at this location. The parameters are shown in figure 7.19. Once $c$ and $q_0$ are found, the shoreline is updated by:

$$h = \frac{q - q_0}{c \cdot D_{cll}}$$  (7.16)

After the shoreline has been updated an extra shoreline point is added at the downstream end of the shoreline to ensure that a sufficiently long straight section of shoreline exists in order to avoid boundary effects.

During the first iteration only the first shoreline point after $x_{min}$ is allowed to move freely. The shoreline downstream of the updated section of shoreline is a straight line which continues the slope of the updated section, an example is shown in figure 7.20. Once the shoreline has been updated, the mesh and the bathymetry are updated and the sediment transport can be calculated on the new mesh and bathymetry for the second iteration.

During the second iteration both the first and the second point after $x_{min}$ is allowed to move, everything is the same as for the first iteration. During the following iterations an extra point is included in the number of points that are updated for each extra iteration. In figure 7.21 the updated shoreline is seen after the tenth iteration.

7.3.2 Results and Discussion

The model was run using the parameters seen in table 7.1. The wave parameters were $H_s = 1$ m, $T_p = 5$ s, $DSI = 100$ and $MW D = (30^\circ, 40^\circ, 45^\circ, 50^\circ$
Figure 7.20: Example of the updated shoreline after the first iteration.

Figure 7.21: Example of the updated shoreline after the tenth iteration.
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and $60^\circ$) using the initial mesh seen in figure 7.18. The shoreline and the longshore sediment transport for each of the 5 simulations is seen in figures 7.22 to 7.26.

For figures 7.22 to 7.24 (for $MWD = 30^\circ$, $40^\circ$ and $45^\circ$) the model explodes shortly after the shown iteration. This could be because these wave directions are in the stable regime, i.e. any perturbation will erode on the shoreline when the waves arrive at the shoreline with an angle which is not oblique.

For figures 7.25 and 7.26 a number of similar shoreline undulation are seen downstream of the step. Except for the last undulation in figure 7.25, this undulation is wider than the previous ones. This is an imprint of the way the shoreline changes from, iteration to iteration: the undulation furthest from the step changes its shape more from iteration to iteration than the undulations closer to the step. This behaviour is seen in figure 7.27 which shows an iteration stack of the simulation with $MWD = 50^\circ$. It is observed that the leftmost undulation changes more from one iteration to the next, than the other undulations. Further it is seen that the length of the undulation is largest right after an undulation is made and then shortens as the next undulation is created and expanded.

These results are very promising, however there is one large problem and this is the intense filtering of the longshore sediment transport signal which is needed to ensure stability in the model. To get a feel for the amount of filtering which is applied to obtain the presented results, an example of the raw and filtered long shore sediment transport along the shoreline after 600 iterations for the simulation with $MWD = 60^\circ$ is shown in figure 7.28. It is seen that the peak value of the raw signal is 20% larger than the peak for
Figure 7.23: The shoreline and the longshore sediment transport after 550 iterations for MW D = 40°

Figure 7.24: The shoreline and the longshore sediment transport after 400 iterations for MW D = 45°
Figure 7.25: The shoreline and the longshore sediment transport after 1500 iterations for MW D = 50°

Figure 7.26: The shoreline and the longshore sediment transport after 800 iterations for MW D = 60°
the filtered signal. This means that deviations in the shape of the shoreline of at least 20% can be expected between the true shoreline shape and the shoreline shape computed in the present section.

Attempts have been made to slowly remove the filtering, after having reached an equilibrium solution with the heavy filtering. All these attempts were unfortunately unsuccessful.

It was tried to reduce the width of the step on the initial shoreline so the step was 400 m compared with the 800 m which was used in the other simulations. Figure 7.29 shows the shoreline after 2064 iterations with an initial step on the shoreline of 400 m. It is observed that the width of the undulations downstream of the step grow in farther away from the step, so it seems that the model is trying to make the undulations wider to compensate for the smaller initial width. However the width of the last undulation is around 725 m and when comparing this value with the width of the last undulation in figure 7.26 which was 990 m it is seen that the width of the initial step does influence the solution. One possible reason for this is that the $a$ in equation 7.13 is computed according to equation 7.15, i.e. using the sediment transport upstream of the step and just downstream of the step. Thus $c$ is fixed from the initial mesh.
7.3. AN ITERATIVE MODEL

Figure 7.28: Example showing both the filtered and non-filtered longshore transport.

Figure 7.29: The shoreline and the long shore sediment transport after 2064 iterations.
7.3.3 Conclusion

In the present section a different approach was tried to obtain a shoreline shape in dynamic equilibrium with the hydrodynamic forcing. It was seen that such an approach was possible, but only if heavy filtering was used on the longshore sediment transport signal. Due to the heavy filtering the results are not reliable and are therefore not discussed in detail. With more time and effort perhaps the approach can be more successful.
The focus of the present thesis is numerical modeling of longshore shoreline undulations. The foundation for the modeling effort is that the shoreline undulations can be explained by the shoreline instability theory described in section 2.2. Other possible causes for alongshore shoreline undulations have not been thoroughly investigated, since the aim has been to see how far the shoreline instability theory could bring us in the understanding of longshore shoreline undulations. This is discussed in the following.

Initial Formation of Shoreline Undulations

According to the shoreline instability theory the shoreline undulations grow from small random perturbations on an otherwise straight and uniform shoreline. In chapter 2 this theory is used to make a numerical investigation into the most unstable length of the initial shoreline undulations. The most unstable length is determined as function of different model parameters. This is not a new idea; previous work includes Falqués and Calvete (2005) and Ugccioni et al. (2006). The results from these previous works were confirmed in the present investigation: A most unstable undulation length was found to exist, the length was found to scale with break zone width, and the length was found to increase when increasing the effect of the inertia terms in the longshore current.

Some additional results were also found. Firstly, the most unstable length increases when the amplitude of the undulation increases, secondly an increase in the the directional spreading of incoming waves also increases the length and thirdly, introducing a phase lag between the undulation on the shoreline and the undulation on the off-shore contour generally stabilizes the shoreline.

It is clear that it is the effect of the curvature of the shoreline on the incoming waves, which is responsible for the shoreline instability to have a preferred undulation length. This effect can also explain why the most
unstable undulation length increases, when the amplitude of the undulation increases. The morphology is thereby keeping the shoreline curvature constant as the undulation amplitude increases.

When the numerical stability analysis was compared to some natural shoreline undulations, the results were mixed (see chapter 5). On the west coast of Namibia the stability analysis was able to predict the length of the observed undulation for realistic model parameters (section 5.3.2). On two stretches of shoreline on the Danish west coast (section 5.4.2) the stability analysis predicted shoreline undulations which were more than twice as long as the observed undulations when using realistic values for the wave conditions and the beach profile. At Srd. Holmslands Tange the average wave conditions resulted in a shoreline which was right on the limit for shoreline instability. So the stability analysis predicted the shoreline to be stable in this case if the wave climate was weighted with the significant wave height cubed.

Previous applications of stability analyses to real shorelines have also had mixed successes. Falqués (2006) used his stability analysis from Falqués and Calvete (2005) on the Dutch coast and found the coast to be stable for the given wave climate. Whereas Medellin et al. (2009) used the same stability analysis to predict the observed lengths for the undulations on the El Puntal spit at Santander, Spain. The stability analysis from the present work has also been applied to this shoreline, but without succes as the model predicts this shoreline to be stable for the given wave climate and coastal profile.

From the above it is seen that the shoreline instability mechanism cannot explain all of the observed rhythmic shoreline behaviour. Thus other mechanisms must come into play or interact with the shoreline instability mechanism. Possible interacting mechanisms include rip currents and offshore sand waves.

**Growth and Width of Shoreline Undulations**

Right after the initial formation of shoreline undulations, the amplitude of the undulations grow without changing their shape. After a certain growth an assymetric shape begins to develop with an upstream gentle alongshore slope and a steep downstream one (see chapter 3). During the growth, the maximum in the longshore sediment transport is located upstream of the crest of the undulation and the minimum of the longshore sediment transport is located upstream of the trough of the undulation. Thereby there is accretion on the crest and erosion at the trough. As the width of the undulation increases, the lead of the crest over the maximum in the longshore transport decreases, thereby the accretion at the crest also decreases. The mechanism responsible for the shift is the curvature of the shoreline around
the crest which increases together with the undulation width.

It is difficult to compare the growth of the shoreline undulations in the model with observations since these data, to the authors knowledge, do not exist. Of the previous modeling efforts, it only makes sense to compare with Falqués et al. (2009), unfortunately no details regarding the growth of the undulations are specified in this work which makes a comparison not possible.

Factors which could affect the growth of the undulations, which are not included in the present shoreline evolution model include: the steepening of the beach profile at the crest of the undulations and flattening of the profile at the trough which is clearly observed on the undulations in Namibia (see figure 5.5); the phase difference between the undulations on the shoreline compared with the undulation on the off-shore contour observed at Srd. Holmlands Tange (see figure 6.20); the sloping water depth beyond the depth of closure and interactions between the longshore undulations and longshore breaker bars.

Perhaps it is due to these missing factors that the undulations in the present work all grow to relatively large width. All equilibrium widths are obtained for aspect ratios of at least 0.2, meaning that the width is 20% of the length of the undulation. This fits well with some of the observations on the west coast of Namibia where the aspect ration was 0.18, but is in contrast to observations on the west coast of Denmark, where the aspect ratios of undulations are around 0.04 at Srd. Holmslands Tange.

**Shape and Dimensions of Shoreline Undulations**

Three different shoreline shapes are identified in the present work, these are undulations with no spit, undulations with flying spits and undulations with reconnecting spits (chapter 3). The type of shape is found to depend mainly on the variability of the incoming wave conditions. For time-invariant wave climates it is found that spits always develop on the downstream end of the shoreline undulation. For varying wave climates, the formation of spits is suppressed quite easily; with just 20% wave energy coming from directions different than the main wave direction, no spit was formed.

Comparing these findings with observed undulations it is difficult to confirm the exact percentages of wave energy from different directions needed to hinder the development of spits. However, on the west coast of Namibia the wave climate is very constant, and here the spits are observed on two out of three undulations, with strong indications that a spit previously existed on the third undulation at an earlier in time. This indicates that the consequent formation of spits in the shoreline evolution model, when the wave climate is constant, is correct.

Comparing the limit for spit development with the equivalent limit found
by Ashton and Murray (2006a), it is found that formation of spits is 
suppressed at a lower percentage of waves from directions other than 
the main wave direction in the present model than in their model. This 
is explained by the large growth rates of the spit in the model by 
Ashton and Murray (2006a), which occurs because the curvature of 
the shoreline is ignored in their model as explained by Petersen et al. (2008).

When a spit does develop it is found that the migration direction of 
the spit depends on the direction of the incoming waves and on the 
curvature of the shoreline at the crest. For a larger shoreline curvature 
the spit grows more towards the shoreline than for a small shoreline curvature. Both the 
steepness of the beach profile and the length of the undulation affects 
the shoreline curvature, such that a steep beach profile and a short undulation 
length results in a large shoreline curvature at the undulation crest.

For a constant wave climate and multiple undulations in the domain, 
there is a strong indication that it is somewhat random which exact undula-
tion lengths grow and become dominant. In the simulations in section 
4.3 four undulations with different lengths around a mean length were 
observed. The shortest undulation was 28% shorter than the mean length, 
and the longest undulation was 24% longer than the mean length. While 
there was no indication that the shorter undulations would merge to increase 
their length, the spits on the shorter undulations are migrating towards the 
trough of the undulations and it seems likely that they will reconnect to the 
shoreline. This latter behaviour was also observed when running simulation 
with a single undulation in a model domain which was slightly shorter than 
the most unstable undulation length for the given wave climate (see section 
3.4 for $MWD = 55^\circ$).

For a varying wave climate and multiple undulation in the domain, the 
length of the dominant undulation increases as time goes by. This is due 
to the combined effect of the increase in the length of the most unstable 
undulation when the undulation amplitude increases and the effect of shad-
owing of the incoming waves by neighboring undulations. This result is in 
agreement with Ashton and Murray (2006a), however the undulations in 
their work are more flat in the trough than the undulation in the present 
work. This discrepancy is perhaps due to the difference in the sea bed be-
\text{\textendash}yond the depth of closure, which is flat in the present model and sloping in 
their model.

Model Improvements

In the present work the main focus has been the study of shoreline undula-
tions in a relatively simple model. This has proven beneficial in the way 
that much of the observed behaviour of shoreline undulations can be de-
scribed using this simple model. Thereby much of the observed behaviour is
understood. However there are some weak points in the present work where the room for improvement can justify further work.

Resolution

First of all some of the shoreline features are not resolved well enough in the present work. As long as the shoreline undulations are the only features the model has to describe, the used resolution is sufficient to obtain reliable results, but when spits begin to develop on the downstream end of the undulation these features are only described by 10-15 points. The effect of this relatively coarse resolution on the development of the shape and dimensions of the spit could be studied more than was done in the present work. This was due to time constraints and the large computational cost related to increasing the resolution. A fortran version of the model (matlab was used for the present work) will run much faster and can perhaps permit such a study.

Constant Water Depth

Secondly the constant water depth beyond the depth of closure is a simplification which could have a significant impact on the final width of the undulations and perhaps on the shape of the troughs of the undulations.

Changes in Coastal Profile

Thirdly the analysis of data from the Danish west coast showed that a phase lag exist between undulations on the shoreline and undulations on the off-shore depth contours. Furthermore the sea map from Namibia clearly showed that the coastal profile is steeper at the crest and shallower in the troughs of the undulations. These effects are not included in the one-line model approach and could have some effect on the results.

Where the first two points can be overcome without changing the basics of the model, the last point will be more difficult to implement in a justified manner because the existing cross-shore sediment transport descriptions are still not accurate enough for long term morphological modeling. Therefore some type of behaviour oriented model (e.g. Splinter et al. (2011)) will have to be implemented for the cross-shore morphology.

Wind Driven Sand Transport

Lastly the assumption of zero sediment flux over the model boundary at the shoreline is evaluated. Bagnold estimated the the potential aeolian sediment transport on a dry flat beach as (see Hesp (1999), original reference
is Bagnold (1941)):  

\[ q_a = C \sqrt{\frac{d}{d_{\text{ref}}}} U_f^3 \rho_a g \]  

(8.1)

where \( q_a \) is the sediment transport rate in kg/m/s, \( C \) is 1.8, \( d \) is grain diameter, \( d_{\text{ref}} = 0.025 \text{ m} \) is a reference grain diameter, \( U_f \) is the friction velocity, \( \rho_a \) is the density of air and \( g \) is the gravitational acceleration.

We can find the friction velocity using (see Owen (1964)):  

\[ U_f = \frac{u_z}{\kappa \ln(z/z_0)} \]  

(8.2)

\[ z_0 = 0.02 \left( \frac{U_f^2}{g} \right) \]  

(8.3)

where \( U_f \) is the friction velocity, \( u_z \) is the wind speed at height \( z \) and \( z_0 \) is the apparent roughness height. For a wind speed of 10 m/s at 10 m height, \( U_f \) is found to be 0.35 m/s, which results in a sediment transport rate of 36 m\(^3\)/m/year.

If it was blowing 10 m/s all year, and if all the wind-blown sediment was lost from the beach, the shoreline would recede 9 m/year (using a closure depth of 4 m as in most of the present work). Assuming that, the wind blows more than 10 m/s 10 % of the year (in Denmark we have up to 170 days per year with 10 m/s or more, see Danish Meteological Institute (2011)), results in a shoreline retreat of about 1 m/year. This corresponds to 10-30% of the growth rate of undulations in table 3.2.

Usually the airblown sand is blown into the backshore dune system, and only a minor part is therefore lost from the nearshore system. In the cases where a spit has formed, there are no dunes to back up the shoreline, and the airblown sediment can easily blow into the embayment behind the spit. So clearly the aeolian transport can have a large impact on the shoreline evolution.

**Improvement of the Volume Correction**

For the results presented in this thesis, a small error existed in the implementation of the sediment volume correction scheme described in section 2.3.8. This has the effect that small amounts of sediment are added to the model domain. The added volume has been determined to be around 3-4 % of the total eroded and deposited volume. By improving the scheme, the added volume due to numerical errors is reduced to around 0.5 %.

Most of the remaining 0.5 % added volume happens at those morphologic time steps where a new mesh is created; either because the old mesh is so distorted that moving nodes in the correction step creates non-convex or inverted elements, or because the number of points on the shoreline has changed. Creating a new mesh changes the location of the nodes in the mesh and therefore also changes the volume of sediment, thereby introc
Table 8.1: Comparison of morphologic parameters for models with different sediment volume errors. Note that Cur is the model with < 0.1% error volume error, New is the model with 0.5% volume error and Old is the model with 3% volume error.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Old</th>
<th>New</th>
<th>Cur</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spit migration direction</td>
<td>3.9°</td>
<td>3.8°</td>
<td>3.9°</td>
</tr>
<tr>
<td>Width of undulation, $w_u$</td>
<td>1170 m</td>
<td>1200 m</td>
<td>1200 m</td>
</tr>
<tr>
<td>Width of spit, $w_s$</td>
<td>500 m</td>
<td>450 m</td>
<td>450 m</td>
</tr>
</tbody>
</table>

a volume error. This volume error can be corrected by computing it as the difference between the volume of the old and the new mesh. The volume of a mesh is determined by finding the volume between the mesh and the mesh projected onto the $y, z$ plane (this mesh is found by setting the $y$-coordinate of all nodes in the mesh to zero).

In case a new mesh was made due to inverted or non-convex element, the found volume error is spread out along the entire shoreline. In case nodes were added or removed to the shoreline, the volume error is spread out along the shoreline edges which were affected by the adding or removing of nodes.

In case these corrections are applied the sediment volume error is reduced to less than 1 per thousands of the total eroded and deposited volume.

The effect of the added volume on the evolution of the shoreline has been studied by running the simulation from section 3.3.1 (named Old in the following) using a model where the mistake was fixed (named New in the following) and using a model where the above sketched improvements were implemented (name Cur in the following). The two new models were started right before the development of a spit had begun in the original simulation. The length of the undulation was $L = 5000$ m and the mean wave direction was $MWD = 60°$, all other parameters were set according to table 3.1.

As can be seen in table 8.1 the effect of improving the sediment conservation in the model is small. The migration direction of the spit is affected by 0.1°, the width of the spit is affected by around 10% and the width of the undulation is affected by 2.5%. In appendix D time stack of the shoreline evolution for the three cases can be seen together with plots showing the evolution of the morphologic parameters.

To put the volume errors into perspective, they can be compared with the effect of the assumption of zero sediment flux at the shoreline estimated in the previous section. The 3% error is equivalent to around 0.4 m/year of shoreline advancement, thus it is seen to be smaller, but on the same order of magnitude as the estimated effect of the zero sediment flux assumption (this was 1 m/year). The 0.5% error is seen to be one order or magnitude smaller than the effect of the zero sediment flux assumption at the shoreline.

The small effect of the volume error on the resulting shoreline shape and
dimensions - coupled with the relatively long computational time for the simulations - is the reason why the simulations have not been remade after the correction of the volume error was implemented.
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Appendix A

Parameters Used for Sediment Table Generation

In table A.1 the parameters used for generating the sediment transport tables are presented, see DHI (2011b) for a description of the presented parameters. Note that the table consists of two parts, one which contains the sediment transport in the direction of the current, and one which contains the sediment transport perpendicular to the direction of the current. In the present work the sediment transport perpendicular to the current has been set to zero in all cases because only the longshore transport is used.
Table A.1: Model parameters used for generation of sediment transport tables.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tolerance in Concentration</td>
<td>0.0001</td>
</tr>
<tr>
<td>Maximum no Wave Periods</td>
<td>150</td>
</tr>
<tr>
<td>No of Steps per Period</td>
<td>140</td>
</tr>
<tr>
<td>Relative Sediment Density</td>
<td>2.65</td>
</tr>
<tr>
<td>Critical Shields Parameter</td>
<td>0.05</td>
</tr>
<tr>
<td>Include Effect of Ripples</td>
<td>True</td>
</tr>
<tr>
<td>Include Bed Slope Effects</td>
<td>False</td>
</tr>
<tr>
<td>Include Helical Effects</td>
<td>False</td>
</tr>
<tr>
<td>Include Streaming Terms</td>
<td>True</td>
</tr>
<tr>
<td>Include Density Currents</td>
<td>False</td>
</tr>
<tr>
<td>Include Undertow</td>
<td>True</td>
</tr>
<tr>
<td>Use Empirical Cb</td>
<td>False</td>
</tr>
<tr>
<td>Sand Transport Model</td>
<td>2</td>
</tr>
<tr>
<td>Water Temperature</td>
<td>10°C</td>
</tr>
<tr>
<td>Gamma1</td>
<td>1</td>
</tr>
<tr>
<td>Gamma2</td>
<td>0.8</td>
</tr>
<tr>
<td>Wave Theory</td>
<td>Sinusoidal</td>
</tr>
<tr>
<td>Order of Solution</td>
<td>5</td>
</tr>
<tr>
<td>Use Irregular Waves</td>
<td>True</td>
</tr>
<tr>
<td>Dissipation Factor Beta</td>
<td>0.15</td>
</tr>
</tbody>
</table>
Appendix B

Time Stacks for Constant Wave Forcing

Time stacks for results from chapter 3.

B.1 Wave Spreading Effects

The evolution of the shoreline after the formation of a spit is shown in figure B.1 for \( DSI = 25 \) and in figure B.2 for \( DSI = 40 \) with the remaining parameters set according to table 3.1.
Figure B.1: The time stack of the evolution of an undulation after the formation of a spit at the downstream end, for the case with a direction spreading index of DSI = 25, and the remaining parameters set according to tabel 3.1. Note that 2 undulations are shown.

Figure B.2: The time stack of the evolution of an undulation after the formation of a spit at the downstream end, for the case with a direction spreading index of DSI = 40, and the remaining parameters set according to tabel 3.1. Note that 2 undulations are shown.
B.2 Undulation Length Effects

Figure B.3: The time stack of the evolution of an undulation after the formation of a spit at the downstream end, for the case with a mean wave direction of $MW \theta = 55^\circ$ and undulation length $L = 5500$ m, and the remaining parameters set according to table 3.1. Note that 2 undulations are shown.

For the case with a mean wave direction of $50^\circ$ the time stack of the evolution of the shoreline is shown in figure B.6 for the period after the formation of a spit has begun at the downstream end of the undulation. As seen on the figure a spit is formed on the downstream end which grows towards the undulation trough and eventually merges with the shoreline in the trough.
Figure B.4: The time stack of the evolution of an undulation after the formation of a spit at the downstream end, for the case with a mean wave direction of $MWD = 60^\circ$ and undulation length $L = 4500 \text{ m}$, and the remaining parameters set according to table 3.1. Note that 2 undulations are shown.
Figure B.5: The time stack of the evolution of an undulation after the formation of a spit at the downstream end, for the case with a mean wave direction of $MWD = 65^\circ$ and undulation length $L = 4000$ m, and the remaining parameters set according to tabel 3.1. Note that 2 undulations are shown.
Figure B.6: The time stack of the shoreline evolution after the formation of a spit at the downstream end for a simulation with a mean wave direction of $\text{MWD} = 50^\circ$, a directional spreading index of $\text{DSI} = 10$ and an undulation length of 7000 m.
Appendix C

Evolution of Morphologic Parameters for Namibia

For the three combinations shown in table 5.2, the evolution of the undulation width, \( w_u \), and the total undulation width, \( w_{u,\text{tot}} \), are shown in figure C.1. Further the evolution of the width of the spit is shown in figure C.2 and the alongshore migration of the tip of the spit and the crest if the undulation is shown in figure C.3.
Figure C.1: The evolution of the undulation width for the three combinations from table 5.2. The full lines show the undulation width, \( w_u \), and the dashed lines show the total undulation width, \( w_{u,tot} \).

Figure C.2: The evolution of the width of the spit, \( w_s \), for the three combinations from table 5.2.
Figure C.3: Alongshore migration of the crest of the undulation (dashed lines) and the tip of the spit (full lines), for the three combinations from table 5.2.
Appendix D

Effect of Volume Error

D.1 Shoreline Evolution

For the three models shown in table 8.1, time stacks of the shoreline evolution is shown in figure D.1. Note that Cur is the model with < 0.1% error volume error, New is the model with 0.5% volume error and Old is the model with 3% volume error.

It is also noted that it looks like the spit is migrating faster for the Old model (with 3% error), but this is only in the beginning of the shown evolution, as seen in figure D.4 after year 135 the spits in the three models have the same alongshore migration rate. Further it can be observed that the added sediment for the Old model, primarily has the effect of making the spit slightly wider than in the other two model runs.

D.2 Evolution of Morphologic Parameters

For the three models shown in table 8.1, the evolution of the undulation width, $w_u$, and the total undulation width, $w_{u,tot}$, are shown in figure D.2. Further the evolution of the width of the spit, $w_s$, is shown in figure D.3 and the alongshore migration of the tip of the spit and the crest of the undulation is shown in figure D.4. The evolution of the curvature at the crest of the undulation is shown in figure D.5.
Figure D.1: Time stack of the shoreline evolution for the three models from table 8.1.
Figure D.2: The evolution of the undulation width for the three models from table 8.1. The full lines show the undulation width, \( w_u \), and the dashed lines show the total undulation width, \( w_{u,\text{tot}} \).

Figure D.3: The evolution of the width of the spit, \( w_s \), for the three models from table 8.1.
Figure D.4: Alongshore migration of the crest of the undulation (dashed lines) and the tip of the spit (full lines), for the three models from table 8.1.

Figure D.5: The evolution of the shoreline curvature at the crest of the undulation for the three models from table 8.1.