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Bifurcation Analysis and Dimension Reduction of a Predator-Prey Model for the L-H Transition

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The L-H transition denotes a shift to an improved confinement state of a toroidal plasma in a fusion reactor. A model of the L-H transition is required to simulate the time dependence of tokamak discharges that include the L-H transition. A 3-ODE predator-prey type model of the L-H transition is investigated with bifurcation theory of dynamical systems. The model is recognized as a slow-fast system.

INTRODUCTION

The L- and H-modes
de are confinement states of a toroidal plasma, referring to states of low and high confinement, respectively. The transition from the L- to the H-mode is called the L-H transition. The T-mode is a transient, intermediate mode between the L- and H-mode, characterized by an oscillatory behavior. The L-H transition still lacks a first principle explanation. Modeling the L-H transition might contribute to a better understanding of the underlying mechanisms.

THE 3-ODE L-H TRANSITION MODEL

We consider the minimal 3-ODE predator-prey type L-H transition model suggested by Kim and Diamond [2, 3]. The model ignores spatial dependencies. The dependent variables in the model are:

- the drift wave turbulence level \( \varepsilon \),
- the shear of the zonal flow \( V_{zi} \), and
- the gradient of the ion pressure \( \nabla N \).

The model can be formulated as

\[
\begin{align*}
\frac{d}{dt} \varepsilon &= \mathcal{E} (N - a_1 \varepsilon - a_2 c_1 N^3 - a_3 V_{zi}^2), \\
\frac{d}{dt} V_{zi} &= V_{zi} (b_1 \varepsilon - a_1 c_2 N^3 - b_2), \\
\frac{d}{dt} N &= Q(t) - N (c_1 \varepsilon + c_2),
\end{align*}
\]

where \( a_1, b_1, c_1, i = 1, 2, 3 \) are parameters and \( Q(t) \) is the heating power. Introducing new variables and time, \( u = a_1^{1/3} c_1^{2/3} \varepsilon, v = a_1^{1/3} c_1^{2/3} V_{zi}^2, s = a_1^{1/3} c_1^{2/3} N, \)

results in the non-dimensionalized system

\[
\begin{align*}
u &= w (w - u - v - w^2), \\
\dot{w} &= \mu_1 v (1 + \mu_1 w^2 - \mu_2), \\
\dot{v} &= \mu_2 (\sigma - w - 1 + \mu_1 w).
\end{align*}
\]

Here, \( \mu_1, \mu_2, \sigma > 1 \) are new parameters and \( \sigma \) is the rescaled heating power.

BIFURCATION ANALYSIS

The nullclines are

\[
\begin{align*}
\mathcal{N}_u &= \{ u = 0 \} \cup \{ u = w (1 - w^2) - v \}, \\
\mathcal{N}_v &= \{ v = 0 \} \cup \{ v = \mu_1 (1 + \mu_1 w) \}, \\
\mathcal{N}_w &= \{ w (1 + \mu_1 w^2 - \mu_2) \}.
\end{align*}
\]

Stability of the equilibrium points:

- \( L \) is a stable node when it is below \( N_u \),
- \( H \) is always a saddle (unstable),
- \( T \) is a focus point. Stability depends on the value of \( \sigma \),
- \( QH \) is stable for \( \sigma > 1 \).

Three Transition Types

The bifurcation diagram structure depends on \( \mu_1, i = 1, \ldots, 5 \). By varying \( \mu_2 \) and \( \mu_2 \), the three different transition types are observed.

The 3-ODE L-H transition model can be reduced with GSPT to a 2-ODE system.

DIMENSION REDUCTION WITH GSPT

Put \( \mu_2 = \varepsilon \), where \( 0 < \varepsilon \ll 1 \).

- \( u \) and \( v \) are slow variables,
- \( w \) is a fast variable.

For \( \varepsilon > 0 \), but sufficiently small, solutions converge to the slow manifold,

\[
M_\varepsilon = M_0 + \varepsilon M_1 + \varepsilon^2 M_2 + \cdots
\]

The reduced system of the flow is found by taking the limit \( \varepsilon \to 0 \):

\[
\begin{align*}
\dot{u} &= u (w - u - v - w^2), \\
\dot{v} &= \mu_2 w (1 + \mu_1 w^2 - \mu_2), \\
\dot{w} &= 1 + \mu_1 w.
\end{align*}
\]

The reduced system contains the same dynamics as the full system.

CONCLUSION

Kim and Diamond’s 3-ODE L-H transition model was investigated with bifurcation theory.

- The model contains three types of transitions: an oscillating, a sharp with hysteresis, and a smooth transition.
- The system can be reduced to a 2-ODE system

A spatio-temporal L-H transition model has been proposed by Miki et al. [4].

References: