THE DIPOLE MOMENT OF A WALL-CHARGED VOID IN A BULK DIELECTRIC

I. W. McAllister

Electric Power Engineering Department
Building 325
The Technical University of Denmark
DK-2800 Lyngby, Denmark

INTRODUCTION

In the Pedersen approach to PD transients [1-3], the occurrence within a void of a partial discharge results in a deposition of charge on the void wall. External to the void, this charge distribution (both +ve and -ve charges) may be viewed as a dipole. The related change in field produces a change in the charge induced on the detecting electrode. In turn this latter change leads to the familiar PD transient. In the present paper, the dipole moment of a wall-charged void is examined with reference to the spatial extent of the surface charge density σ and the distribution of this charge. Thereafter the salient factors influencing the void dipole moment are discussed.

BASIC ASPECTS OF DIPOLE MOMENTS

The dipole moment $\vec{\mu}$ of a distribution of charge is defined as

$$\vec{\mu} \equiv \int \vec{r} dQ \tag{1}$$

where $\vec{r}$ is a radius vector which locates the position of the charge element dQ. If the net charge associated with the distribution is zero, i.e. if

$$\int dQ = 0 \tag{2}$$

then $\vec{\mu}$ is independent of the coordinate origin, i.e. the origin of $\vec{r}$. Depending on the charge configuration, dQ is given by $\rho d\Omega$, σdA or αds, where ρ, σ and α are the volume

Authorized licensed use limited to: Danmarks Tekniske Informationscenter. Downloaded on July 07,2010 at 10:54:29 UTC from IEEE Xplore. Restrictions apply.

charge density, the surface charge density and the line charge density, respectively. $d\Omega$, dA and ds represent the volume element, the surface element and the element of length.

From (1), it is evident that $\vec{\mu}$ is dependent only upon the existence of a charge distribution and is independent of the surrounding media.

Polarizable Media

The polarization of dielectric bodies by the application of an electric field also produces a dipole moment $\vec{\mu}_P$ which is defined as

$$\vec{\mu}_P \equiv \iiint \vec{P} d\Omega \tag{3}$$

where $\vec{P}$ represents the dielectric polarization, i.e. the dipole moment density. The volume integral is undertaken with respect to the volume of the body. For gaseous voids ($\varepsilon_r \cong 1$), $\vec{P} \cong 0$ and thus $\vec{\mu}_P \cong 0$; i.e. a void in which no discharges have occurred has, at that instant, essentially zero dipole moment.

DIPOLE MOMENTS OF WALL-CHARGED VOIDS

We consider a spherical void of radius R embedded in a bulk dielectric. Hence with reference to spherical coordinates r,θ,ψ having the sphere centre as origin, the void wall is represented by the surface $r = R$.

The discharge development in the void is assumed to be axially symmetric, and hence the wall charge distribution $\sigma(\theta)$ will exhibit the same symmetry. Thus to study the influence of the wall charge distribution on the void dipole monent, we will consider spherical caps of charge and evaluate the associated $\vec{\mu}$. The caps are symmetrically located around the z-axis and subtend a polar angle β, see Fig.1.

Owing to symmetry, the void dipole moment will have only a z-component, and thus we have

$$\mu_z = \int z\sigma(\theta) dA \tag{4}$$

Authorized licensed use limited to: Danmarks Tekniske Informationscenter. Downloaded on July 07,2010 at 10:54:29 UTC from IEEE Xplore. Restrictions apply.

or, in relation to spherical coordinates and the cap geometry,

$$\mu_z = 2\pi R^3 \int_0^{\pi} \sigma(\theta)\cos\theta\sin\theta d\theta \tag{5}$$

Although the net charge in the void is zero, we can quantify the charge involved in the dipole moment by referring to the magnitude of the charge Q_h (positive or negative) associated with a hemispherical wall. This can be evaluated from

$$Q_h = 2\pi R^2 \int_0^{\pi/2} \sigma(\theta)\sin\theta d\theta \tag{6}$$

With reference to the caps of charge, we will consider 3 surface charge distributions, viz., for $0 \leq \theta \leq \beta$,

$$\sigma(\theta) = \sigma_o \tag{7}$$

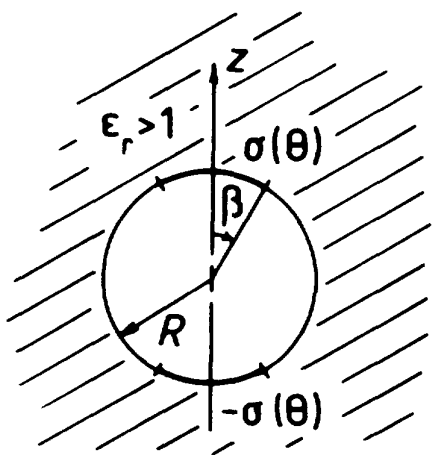

Fig.1. Geometry of wall-charged void.

Authorized licensed use limited to: Danmarks Tekniske Informationscenter. Downloaded on July 07,2010 at 10:54:29 UTC from IEEE Xplore. Restrictions apply.

β	$\pi/2$	$\pi/4$	$\pi/6$	$\pi/12$
$\sigma(\theta)$	$\frac{\mu_z}{\mu_{max}}$	$\frac{\mu_z}{\mu_{max}}$	$\frac{\mu_z}{\mu_{max}}$	$\frac{\mu_z}{\mu_{max}}$
σ_o	0.50	0.85	0.93	0.98
$\sigma_o \cos n\theta$	0.68	0.91	0.96	0.99
$\sigma_o \cos^2 n\theta$	0.75	0.93	0.97	0.99

Table 1. Void dipole moment for different wall-charge distributions.

$$\sigma(\theta) = \sigma_o \cos n\theta \qquad (8)$$

$$\sigma(\theta) = \sigma_o \cos^2 n\theta \qquad (9)$$

for $\beta = \frac{\pi}{2n}$ and $n = 1,2,3$ & 6

For $(\pi-\beta) \leq \theta \leq \pi$, the caps of charge are of the opposite polarity. Upon substituting for $\sigma(\theta)$ in (5) and (6) and evaluating the various integrals, we obtain the corresponding μ_z and Q_h.

It is obvious that, for a given distribution of surface charge, the greater the surface area involved the greater will be the magnitude of the associated dipole moment. Hence to illustrate the influence of both the functional and spatial distribution of σ, the maximum dipole moment μ_{max}, which may be associated with a cap of charge, has been taken as a reference value. For a fixed value of Q_h, μ_{max} is obtained by considering Q_h to be related to two point charges placed at the maximum distance apart in the axial direction. Thus for a spherical void μ_{max} is given by

$$\mu_{max} = 2RQ_h \qquad (10)$$

Hence for any cap of charge, the influence of Q_h can be suppressed by considering the ratio (μ_z/μ_{max}), see Table 1.

Authorized licensed use limited to: Danmarks Tekniske Informationscenter. Downloaded on July 07,2010 at 10:54:29 UTC from IEEE Xplore. Restrictions apply.

In this Table, it is illustrated that, for a hemispherical configuration ($\beta = \pi/2$), μ_z increases by 50% as the σ-distribution becomes more non-uniform. In contrast, for small cap-angles ($\beta < \pi/12$), the dipole moments are virtually identical in magnitude. However, compared with the relevant $\beta = \pi/2$ value, relative increases in μ_z of between 30 to 100% are attained, with the greatest increase associated with the uniform σ-distribution. This trend is not unexpected, since separation between the positive and negative charge configurations increases as β reduces, and this in turn diminishes the effect of charge distribution.

DISCUSSION AND CONCLUSION

From a study of spherical voids, it is shown that, although the σ-distribution influences the dipole moment, the spatial extent of σ has a greater influence. This behaviour is not unexpected. For a void of fixed dimensions, the smaller the charged surface area, the greater is the effective separation between the positive and negative charges, and thus the greater the dipole moment.

In this preliminary study, the surface charge density has been taken as the independent variable. However, with reference to partial discharge activity, this is not the controlling parameter. In this case it is the internal field strength associated with the quenching of the discharge development; i.e. the wall charge field has to be of such a magnitude that this process occurs. The internal field distribution is known to be strongly influenced by the spatial extent of the wall charge, e.g., for $\beta = \pi/2$ and $\sigma(\theta) = \sigma_o \cos\theta$, the internal field is uniform [4], while the other limit may be taken as the field between two point charges, see [5].

Consequently, it will be necessary to establish fully the relationship between the internal field and the surface charge density, prior to employing the general expression to evaluate the dipole moment associated with partial discharge activity.

REFERENCES

[1] A. Pedersen, "Partial Discharges in Voids in Solid Dielectrics. An Alternative Approach", 1987 Annual

Authorized licensed use limited to: Danmarks Tekniske Informationscenter. Downloaded on July 07,2010 at 10:54:29 UTC from IEEE Xplore. Restrictions apply.

Report - Conference on Electrical Insulation and Dielectric Phenomena, IEEE Publication 87CH2462-0, pp.58-64, 1987.

[2] G.C. Crichton, P.W. Karlsson and A. Pedersen, "Partial Discharges in Ellipsoidal and Spheroidal Voids", IEEE Trans. Elect. Insul., Vol.24, pp.335-342, 1989.

[3] A. Pedersen, G.C. Crichton and I.W. McAllister, "The Theory and Measurement of Partial Discharge Transients", IEEE Trans. Elect. Insul., Vol.26, pp.487-497, 1991.

[4] I.W. McAllister and G.C. Crichton, "Analysis of the Temporal Electric Fields in Lossy Dielectric Media", IEEE Trans. Elect. Insul., Vol.26, pp.513-528, 1991.

[5] I.W. McAllister, "Decay of Charge Deposited on the Wall of a Gaseous Void", IEEE Trans. Elect. Insul., Vol.27, pp.1202-1207, 1992.

Authorized licensed use limited to: Danmarks Tekniske Informationscenter. Downloaded on July 07,2010 at 10:54:29 UTC from IEEE Xplore. Restrictions apply.