A DETAILED INVESTIGATION OF THE CORRECTED BEM METHOD AND THE POTENTIAL FOR IMPROVING BLADE DESIGN

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Abstract:

Improved BEM models with corrections for wake rotation and expansion are superior to the standard BEM method in predicting flow properties in the tip and root sections of blades for horizontal axis wind turbines. Turbines optimized for improved aerodynamics using the advanced models are. Special attention is on effects not captured by standard BEM methods and if the design can be improved based on this. The accuracy of the newly developed advanced BEM model is discussed.

Keywords: BEM, BEM corrections, optimization, wake rotation, wake expansion

0 Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>Rotor radius</td>
<td>m</td>
</tr>
<tr>
<td>P</td>
<td>Power</td>
<td>W</td>
</tr>
<tr>
<td>T</td>
<td>Thrust</td>
<td>N</td>
</tr>
<tr>
<td>V₀</td>
<td>Undisturbed wind speed</td>
<td>m/s</td>
</tr>
<tr>
<td>Vₐₕ</td>
<td>Axial wind speed in rotorplane</td>
<td>m/s</td>
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<tr>
<td>Ω</td>
<td>Rotational speed</td>
<td>s⁻¹</td>
</tr>
<tr>
<td>λₜ</td>
<td>Tip speed ratio</td>
<td>-</td>
</tr>
<tr>
<td>λₙ</td>
<td>Local speed ratio</td>
<td>-</td>
</tr>
<tr>
<td>a</td>
<td>Axial induction factor</td>
<td>-</td>
</tr>
<tr>
<td>aₑₒᵣ</td>
<td>Effective axial induction</td>
<td>-</td>
</tr>
<tr>
<td>aʰ</td>
<td>Tangential induction factor</td>
<td>-</td>
</tr>
<tr>
<td>k₁</td>
<td>a(Cₚ/F)_ correlation constants</td>
<td>-</td>
</tr>
<tr>
<td>k₂</td>
<td>-</td>
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</tr>
<tr>
<td>k₃</td>
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<td>-</td>
</tr>
<tr>
<td>Cₚ</td>
<td>Power coefficient</td>
<td>-</td>
</tr>
<tr>
<td>Cₗ</td>
<td>Thrust coefficient</td>
<td>-</td>
</tr>
<tr>
<td>Cₚₑₒᵣ</td>
<td>Flapwise moment coefficient</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>About % radial position</td>
<td>-</td>
</tr>
<tr>
<td>Cₗₑₒᵣ</td>
<td>Flapwise moment coefficient</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>About rotor centre</td>
<td>-</td>
</tr>
<tr>
<td>Cₚₗ</td>
<td>Local power coefficient</td>
<td>-</td>
</tr>
<tr>
<td>Cₗₗ</td>
<td>Local thrust coefficient</td>
<td>-</td>
</tr>
<tr>
<td>F</td>
<td>Prandtl’s tip loss factor</td>
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1 Introduction

The blade element momentum (BEM) method in its original form has in recent years been subjected to a thorough investigation with the purpose of determining its accuracy. Among other issues this has led to the conclusion that the power production from the inner part of the blade is underestimated and conversely overestimated on the outer part. The cause for this is believed to be the failure of the BEM method to accurately predict the axial velocities, for a given loading, correct. For wind turbines in normal operation the rotation of the wake causes an acceleration of the axial flow on the inner part and the wake expansion causes a deceleration on the outer part. Both effects are not captured in the standard BEM model. These issues have been treated in detail in Madsen et al. [1] & [2] and the results are the BEM corrections for the...
axial flow. If these are implemented, the method is denoted corrected or improved blade element momentum method (BEM$_{cor}$).

In Madsen et al. [1] & [2] the main focus is on the derivation of the corrections and the corrected velocities are validated against an actuator disc (ACD) calculation. This is done for a defined loading, thus no iterations are performed. In the following the corrections have been implemented in a full BEM$_{cor}$ algorithm in order to validate the results when the loading is updated according to the velocities in the rotor plane. Notice that the results of BEM$_{cor}$ are sensitive to the order in which various properties are updated when iterating. A full description of the implementation is found in Madsen et al. [1].

The present work is a fundamental study of the importance of the BEM corrections, i.e. how large is the influence from wake rotation and expansion. The best and worst case scenarios are therefore deliberately sought and the blade-designs may deviate from what is reasonable from a manufacturers point of view. E.g. the chord is unconstrained and very large near the blade root, but still sensible at a tip speed ratio of 8. However, an unconstrained optimization is necessary in order to make sure the full potential of the wake rotation is utilized. Blades optimized for maximum power are studied, but also more realistic blades. The latter are defined by optimizing for maximum power to thrust ratio which results in a reasonable low thrust level.

Outline of the article:

- Section 2 gives an introduction to BEM$_{cor}$, and the optimization algorithm is described.
- In section 3 the BEM$_{cor}$ method is validated.
- Section 4 is a study of unconstrained optimization for maximum power using BEM$_{cor}$ and BEM.
- Section 5 is a comparison study of BEM and BEM$_{cor}$ calculations for a series of blades designed using BEM$_{cor}$ and unconstrained optimization of the power to thrust ratio.

2 Method

2.1 The modified BEM method

A full description of the classic BEM method can be found in O.L. Hansen [4] or Glauert [5]. In the following, only the changes introduced in BEM$_{cor}$ is described, which basically applies to a few equations. For a full description, see Aa. Madsen et al [1].

The corrected BEM model differs from the classic BEM model mainly in the calculation of the axial induction $a$, which is a measure of the deceleration of the flow in the rotor plane relative to far upstream.

A normal force coefficient $C_y$ is determined by projecting the lift $l$ and drag $d$ (see Figure 1)

$$C_y = \frac{l \cos \phi + d \sin \phi}{\frac{1}{2} \rho V_0^2 c}$$

$v_{rel}$ is the relative speed of air, $\phi$ is the flowangle relative to the rotorplane, $\rho$ is the density and $c$ is the chord length. In the classic BEM model the axial induction is then determined

$$a = \frac{1}{\frac{8\pi r F \sin^2 \phi}{C_y c N_B} + 1}$$

Where $F$ is Prandtl’s tip loss factor, $r$ is the radius and $N_B$ is the number of blades. The axial velocity $v_a$ in the rotorplane is

$$\frac{v_a}{V_0} = 1 - a$$

Where $V_0$ is the undisturbed wind speed. Equation (1) is only valid if $a$ is smaller than approx. $a<0.3$. If $a>0.3$ an empirical relation must be used.

In BEM$_{cor}$ equations (1) and (2) are replaced. First, instead of (1) the following correlation formula is used.
The optimization objective is either the power
\[ \text{obj} = -C_p \]  
(7)
Or the power to thrust ratio
\[ \text{obj} = -C_p \exp \left[ -\frac{C_p - C_{p,\text{design}}}{2 \cdot 0.05^2} \right] \]  
(8)
In the latter case the exponential is a penalty function included to keep \( C_p \) close to the desired value \( C_{p,\text{design}} \).

3 Validation of BEM\textsuperscript{cor}

The corrected BEM is validated against the optimum rotor presented in Johansen et al. [3], for which actuator disc data is available. The design data for the turbine is summarized in Table 1.

| \( R \) | 63.0 m |
| \( N_b \) | 3 |
| 2D profile | Risoe B1-15 (15% thickness) |
| \( \lambda = \Omega R/V_0 \) | 8 |
| Design \( c_l \) | 1.4 |
| Design \( \alpha \) | 8.0 degree |
| Design \( l/d \) | 110 |

Table 1 Summary of parameters for optimum \( C_p \) rotor in Johansen et al. [3]

The local power coefficient is defined as
\[ C_p = \frac{\Omega \nu_{rel}^2 C_x c N_B}{V_0^2 2\pi} \]  
(9)
Where \( C_x \) is the local edgewise force coefficient
\[ C_x = \frac{1}{2} \rho \nu_{rel}^2 c \sin \theta - d \cos \theta \]
\[ \frac{1}{2} \rho \nu_{rel}^2 c \sin \theta - d \cos \theta \]
Figure 3 shows \( C_p \) calculated using BEM, BEM\textsuperscript{cor} and ACD respectively. BEM\textsuperscript{cor} correlates well with the ACD results over most of the blade. Figure 4 shows the local thrust coefficient \( C_t \). Again, the results from BEM\textsuperscript{cor} are closer to the ACD which is especially important on the inner part of the blade.

2.2 Optimization algorithm

The optimization algorithm is an unconstrained steepest descent method with steplength determined by the golden section method. See e.g. Sun and Yuan [9]. The design variables are 36 discrete values of \( a_{eff} \) distributed over the blade using a cosine spacing to pack the blade elements closer near the root and tip. Since \( a_{eff} \) is a design variable the chord is varied in order to change the axial forces accordingly. In each BEM\textsuperscript{cor} iteration \( \alpha \) is determined using (6), then \( C_t \) is determined using (3) and finally \( c \) is found from (4). All other properties are updated according to the normal BEM\textsuperscript{cor} algorithm as described in Aa. Madsen et al [1]. The twist is defined to be the inflow angle corrected by the design angle of attack.
Notice that BEM methods cannot converge for very small radii and therefore will integrated results differ slightly from the ACD results. More important is the distribution which correlates well with BEM corr.

Table 2 summarizes the global power and thrust coefficients calculated using various methods. The power and thrust coefficients are defined in the usual way

\[
C_P = \frac{P}{\frac{1}{2} \rho V_0^2 \pi R^2}, \quad C_T = \frac{T}{\frac{1}{2} \rho V_0^2 \pi R^2}
\]

(10)

Where \(P\) is the power and \(T\) the thrust. BEMcorr predicts larger values of \(C_P\) and \(C_T\) than BEM, and these values are closer to the results reported in Johansen et al. [3] which are based on accurate methods (i.e. ACD and CFD).

<table>
<thead>
<tr>
<th>Method</th>
<th>(C_P)</th>
<th>(C_T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BEM</td>
<td>0.505</td>
<td>0.858</td>
</tr>
<tr>
<td>BEMcorr</td>
<td>0.510</td>
<td>0.865</td>
</tr>
<tr>
<td>Ellipsys3D (CFD)</td>
<td>0.515</td>
<td>0.872</td>
</tr>
<tr>
<td>Lifting Line</td>
<td>0.514</td>
<td>0.868</td>
</tr>
<tr>
<td>Actuator Disc</td>
<td>0.510</td>
<td>0.870</td>
</tr>
</tbody>
</table>

Table 2 Performance of optimum rotor

Figure 5 shows the results, without tiploss and drag, compared to the results presented by de Vries [8]. The curve representing BEMcorr is close to the results by Glauert. This result must be seen in the light of the discussion in de Vries [8] and will not be treated further here.

Figure 6 compares results for BEM and BEMcorr if tip loss is included, and both with and without drag. In the former case a constant lift to drag ratio of 110 is defined along the blade. With drag included there is a well defined optimum at \(\lambda = 8\). Notice that BEMcorr shows values which are approx. 1% higher than BEM, thus it is possible to obtain a higher power if BEMcorr is used.

### 4 Design for optimum \(C_P\)

The maximum \(C_P\), which can be obtained using BEMcorr has been determined for \(\lambda\)-values from 2 to 12 using the optimization algorithm described earlier. The optimization is unconstrained and large chords are therefore allowed as well as the use of thin profiles on the inner part of the blade. Such a design is not feasible from a manufacturer’s point of view but has been selected in order to obtain maximum power. Table 3 summarizes the design parameters.

<table>
<thead>
<tr>
<th>(R)</th>
<th>1.0 m</th>
</tr>
</thead>
<tbody>
<tr>
<td>(N_b)</td>
<td>3</td>
</tr>
<tr>
<td>(c)</td>
<td>Unconstrained</td>
</tr>
<tr>
<td>2D profile</td>
<td>Risoe B1-15 (15% thickness)</td>
</tr>
<tr>
<td>(\lambda)</td>
<td>2,..., 12</td>
</tr>
<tr>
<td>Design (C_l)</td>
<td>1.4</td>
</tr>
<tr>
<td>Design (\alpha)</td>
<td>8.0 degree</td>
</tr>
<tr>
<td>Design (ld)</td>
<td>or 110.0</td>
</tr>
</tbody>
</table>

Table 3 Design parameters for optimization for maximum \(C_P\)

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Figure 5 Optimum \(C_P\) values using various models for inviscous flow without tip loss. Data is reproduced from de Vries [8]
5 Design for optimum $C_P/C_T$

In this section the differences between BEM and BEM$_{cor}$ are treated. First, in order to take maximum advantage of the effect of wake rotation, a series of turbines are designed using BEM$_{cor}$. The design objective is maximum $C_P/C_T$ ratio for 12 values of $C_P$ from $C_P=0.450$ to 0.505.

Table 4 summarizes the design parameters. A tip speed ratio of 8 has been selected and drag is included. Figure 1 shows the optimized chord distributions for $C_P=0.450$, 0.475 and 0.505.

![Figure 1](image1.png)

Table 4 Design parameters for optimization for maximum $C_P/C_T$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_R$</td>
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<tr>
<td>$R$</td>
<td>1.0 m</td>
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<tr>
<td>$c$</td>
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<td>Risoe B1-15 (15% thickness)</td>
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<tr>
<td>$j$</td>
<td>8</td>
</tr>
<tr>
<td>Design $C_D$</td>
<td>1.4</td>
</tr>
<tr>
<td>Design $C_P$</td>
<td>8.0 degree</td>
</tr>
<tr>
<td>Design $l/d$</td>
<td>110.0</td>
</tr>
</tbody>
</table>

For the 12 obtained designs the power, thrust and root flap moment is now calculated using BEM and BEM$_{cor}$ respectively. The root flap moment is considered important because of the differences in the distribution of forces predicted by BEM and BEM$_{cor}$. The results are given in terms of a dimensionless flap coefficient $C_F$ defined in appendix A.

Figure 8 illustrates the difference in performance predicted by BEM and BEM$_{cor}$. Values of $C_T$ and $C_F$ are plotted against $C_P$ for the 12 turbines. The curves showing BEM results are shifted to the left, i.e. the $C_P$ values are smaller than if calculated using BEM$_{cor}$. This error in the BEM calculation is approx -0.8%. The exact errors are shown in Table 5, where the errors on $C_T$ and $C_F$ are also stated. The results are given in percentage error of the BEM-value relative to BEM$_{cor}$. The maximum errors are found for the turbine with the highest loading where the $C_P$ -error is -0.9% and the $C_F$ -error is -0.7%. The errors for less loaded turbines are smaller but still significant. The error on the flap moment is in all cases small, i.e. -0.1%.

![Figure 8](image2.png)

For the 12 obtained designs the power, thrust and root flap moment is now calculated using BEM and BEM$_{cor}$ respectively. The root flap moment is considered important because of the differences in the distribution of forces predicted by BEM and BEM$_{cor}$. The results are given in terms of a dimensionless flap coefficient $C_F$ defined in appendix A.

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![Figure 8](image3.png)

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![Figure 8](image4.png)
5.1 Decomposition of $C_p$

In order to study how the various velocity components and the viscous drag influences the power, $C_p$ is decomposed into 4 contributions from axial velocities and 1 contribution from the drag.

The following dimensionless values are defined

$$\lambda = \frac{\Omega r}{V_0}, \quad \nu_{rel}^* = \frac{\nu_{rel}}{V_0}, \quad \nu_a^* = \frac{\nu_a}{V_0}, \quad \nu_i^* = \frac{\nu_i}{V_0}$$

Where the tangential velocity $\nu_i$ is found from the tangential induction factor $a'$

$$\nu_i^* = \lambda (1 + a')$$

The relative velocity $\nu_{rel}^*$ and the flowangle $\phi$ are

$$\nu_{rel}^* = \sqrt{\nu_i^* + \nu_a^*} \quad \tan \phi = \frac{\nu_a^*}{\nu_i^*}$$

The contribution to power due to lift forces is by direct integration

$$P(v_a^*) = N \frac{\Omega^2}{2 \rho} \int_{blade} \nu_{rel}^* C_l \sin \phi \, dr \quad (11)$$

If inviscous flow is assumed the bound circulation $\Gamma$ is related to the lift

$$\Gamma = \frac{1}{2} \lambda C_l \nu_{rel}^* \quad (11)$$

can therefore be written as

$$P(v_a^*) = N \frac{\Omega^2}{2 \rho} \int_{blade} \nu_{rel}^* \Gamma \, dr$$

Rewriting to dimensionless quantities yields

$$C_p(v_a^*) = \frac{2N \lambda}{\pi} \int r \nu_a^* \Gamma^* \, dr \quad (12)$$

The definition of the dimensionless bound circulation $\Gamma^*$ follows directly from the derivation of (12) by collecting the remaining dimensional quantities

$$\Gamma^* = \frac{\Gamma}{V_0 R} \quad (13)$$

Because all other quantities in (12) are constant for fixed tip speed ratio and geometry, it follows that $\Gamma^*$ must also follow a functional relationship of the form

$$\Gamma^* = f(\lambda)$$

Notice that $\nu_a$ is also a function of $\lambda$. Since (12) is linear it can be split into the following four contributions corresponding to the 4 velocity components composing $\nu_a^*$

$$C_p(1-a) = \frac{2N \lambda}{\pi} \int r \Gamma^* (1-a) \, dr \quad (14)$$

$$C_p(\Delta v_a) = \frac{2N \lambda}{\pi} \int r \Gamma^* \Delta v_a \, dr \quad (15)$$

$$C_p(-\Delta v_a) = \frac{2N \lambda}{\pi} \int r \Gamma^* (-\Delta v_a) \, dr \quad (16)$$

$$C_p(\Delta v_m) = \frac{2N \lambda}{\pi} \int r \Gamma^* \Delta v_m \, dr \quad (17)$$

The power contribution due to drag is

$$P(d) = -N \frac{\Omega^2}{2 \rho} \int r \nu_{rel}^* c \cos \phi \, dr \quad (18)$$

Where the drag coefficient $C_d$ is defined by the lift to drag ratio

$$C_d = \frac{\int \nu_{rel}^* c \cos \phi \, dr}{\int \nu_{rel}^* \Gamma \, dr} \quad (19)$$

Combining (18) with (19) and rewriting to nondimensional quantities yields

$$C_p(d) = \frac{2N \lambda}{\pi} \int \nu_{rel}^* c \Gamma^* \, dr \quad (20)$$

$C_p$ is now decomposed into the contributions from (14), (15), (16), (17) & (20), i.e.

$$C_p = C_p(1-a) + C_p(\Delta v_a) + \ldots$$

$$+ C_p(-\Delta v_a) + C_p(\Delta v_m) + C_p(d) \quad (21)$$

In section 5 the blade series was defined by optimization using BEM_{cor}. A decomposition of the obtained $C_p$-values is seen in Figure 9. The individual BEM corrections contributes with less than (+-)0.2% to the total $C_p$ and the wake expansion correction almost exactly cancels the correction for unchanged mass flow. The contribution from wake rotation is very small. I.e. the optimization-algorithm does not take advantage of the positive effects from wake rotation. The increase in power which can be obtained using BEM_{cor} (see section 4) is therefore not due to any specific flow property introduced by the BEM corrections. It is simply a result of the more accurate method used and the following optimization.

Even though the above results indicates a weak effect due to wake expansion and rotation, this may not be the case on poorly designed turbines. It is also noticed that the correction for unchanged mass flow is important and that it increases the power production on the main part of the blade. This is unfortunate because it is not based on a thorough study.
The implementation of BEM corrections in a BEM algorithm, including the updating of forces, has been validated.

An optimization for maximum power has shown that a $C_P$ of 0.51 can be obtained using BEM$_{cor}$ and a constant lift to drag ratio of 110. If BEM is used the maximum is approx 1% lower, i.e. $C_P = 0.505$.

A comparison of 12 blades has shown that the error using BEM was as high as 0.9% on $C_P$ and 0.7% on $C_T$, when comparing with BEM$_{cor}$. The error on the root flap moment was small, approx 0.1%.

The blades were designed using BEM$_{cor}$. It was found that the optimization algorithm did not take advantage of the positive effects from wake rotation, thus no significant positive effect can be obtained by designing for a strong rotation of the wake.

In future work the presented designs will be compared to results for a real turbine, the NM80.

References


Appendix A. Dimensionless flap moment coefficient

The dimensionless flapwise force on a blade segment is

$$C_y = \frac{\cos \varphi \cdot d + \sin \varphi}{\frac{\nu_f}{r_0} r c}$$

Where $l$ and $d$ are 2D lift and drag forces. $\nu_f$ is the relative speed of air, $\varphi$ is the flowangle relative to the rotorplane, $\rho$ is the density and $c$ is the chord. $r, r_0$ and $c$ are non-dimensionalized

$$(r - r_0) = \frac{(r - r_0)}{R}, \quad c = \frac{c}{R}$$
$r_0$ is the radius about which the moment is taken (e.g. the blade root). The flapwise moment can be found by direct integration

$$M_F = \int_{\text{blade}} C_y \frac{1}{2} \rho \nu_{\text{rel}}^2 c (r - r_0) dr$$

Collecting all nondimensional quantities on the right hand side yields the dimensionless flap moment

$$\frac{M_F}{\frac{1}{2} \rho \nu_0^2 R^3} = \int_{\text{blade}} C_y^* v_{\text{rel}}^* c^* (r - r_0)^* dr^* \quad (22)$$

The flap coefficient is defined as the left hand side multiplied by the number of blades $N_B$ and divided by $\pi$

$$C_{F,\text{rel}} = \frac{M_F N_B}{\frac{1}{2} \rho \nu_0^2 R^3 \pi} \quad (23)$$

% refers to the value of $r_0/R$. If omitted, it is understood that $r_0=0$. Notice that the denominator is the moment from the force on the rotor disc due to the stagnation pressure, had the total force been concentrated at the tip.

For a fixed turbine geometry, all flow quantities on the right hand side of (23) depends on the tip speed ratio only. A functional dependency, equivalent to that of the power and thrust coefficients, therefore holds

$$C_{F,\text{rel}} = f(\lambda)$$