Simulation of Nonlinear Gain Saturation in Active Photonic Crystal Waveguides

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Abstract—In this paper we present a theoretical analysis of slow-light enhanced traveling wave amplification in an active semiconductor Photonic crystal waveguides. The impact of group index on nonlinear modal gain saturation is investigated.

Keywords-Photonic crystal waveguides; Semiconductor Optical Amplifier, Slow light, Gain saturation

I. INTRODUCTION
Photonic crystal (PhC) structures have been proposed as a potential waveguide infrastructure for high-density photonic integrated circuits (PICs). Optical amplification is one of the fundamental functionalities, required for compensating attenuation and coupling losses and thus increasing the number of integrated devices. A major advantage in combining PhC waveguides and active III-V semiconductors may be the possibility to drastically decrease the component length via enhanced light-matter interaction enabled by slow-light (SL) propagation [1]. The investigation of group velocity related gain enhancement was initiated in Bragg slabs [2]. It is natural to extend such idea to PhC line defect waveguides with guided modes within the bandgap [3]. Comparing with successful progresses of PhC Lasers [4], the attempts of realizing the PhC travelling wave semiconductor optical amplifiers (SOAs) [5] lead to various challenges, e.g. heating in membranes.

Instead of computational intensive 3-dimensional ab-initio simulations of Maxwell-Bloch equations [6], one-dimensional coupled-wave analysis based on a perturbative approach has been widely used to investigate the impact of SL effect on optical properties of passive PhC waveguides, e.g. Kerr nonlinearity [7,8] or disorder-induced scattering [9,10]. Meanwhile, the carrier dynamics of active semiconductor material can be well-described by macroscopic rate equation approaches [11]. However, so far, only a simple rate equation analysis [5] with heuristic inclusion of group velocity has been suggested to qualitatively investigate the gain characteristics of PhC travelling wave SOAs.

In this work, we present a theoretical analysis of SL-enhanced CW light amplification in active PhC membrane waveguides (as a travelling wave optical amplifier) as shown in Fig. 1. In particular, we focused on the carrier-depletion-induced modal gain saturation based on a rigorous carrier dynamics analysis.
\[ g_{\text{sat}} = g_0 (1 + f \tau / f_r) \], where the maximum material gain is \[ g_0 = 2n_0 \rho_0 / c \]. The corresponding imaginary refractive index \( n_{\text{ir}} \) and imaginary susceptibility constant \( \chi_{\text{perm}} = -2n_{\text{ir}} \). By neglecting variation of field envelope within a supercell and approximating spatial averaging of propagation coefficient, i.e., zero-order Fourier coefficients, Eq. (1) is equivalent to the one-dimensional propagation equations previously used for studying SL enhancement of gain/absorption [3,5] in small-signal regime. Our approach also permits the inclusion of different loss mechanisms [9,10].

The corresponding occupation factor \( f_n(r,t) \) and total carrier density \( N_n(r,t) \) in the conduction and valence band \((\alpha = c,v)\) of active semiconductor material are determined by the following distributed rate equations:

\[
F(r) \rho_{\alpha}(r,t) = -R_{\text{inv}} + R_{\text{rel},\alpha},
\]
\[
F(r) N_{\alpha}(r,t) = -R_{\text{inv}} + R_{\text{pump}} - F(r) N_n / \tau, \quad \alpha = c,v
\]

Here \( \rho_{\alpha} \) is the local carrier density, \( \tau \) is the carrier lifetime of total carrier density. \( R_{\text{pump}} \) is the injection rate of carriers by optical/electrical stimulation. Stimulated emission rate \( R_{\text{inv}} \) and ultrafast carrier relaxation term \( R_{\text{rel},\alpha} \) are defined as follow:

\[
R_{\text{inv}} = -\frac{1}{h \omega} \sum_{\alpha \neq \beta} \text{Im} [\chi_{\text{perm}}(\omega)] \rho(\omega) \rho(\omega) \]

\[
R_{\text{rel},\alpha} = \rho_{\alpha} \int \int \int \int \int f_{\text{eq}}(r) - f(r) + \frac{f(r)}{\tau_{\alpha}}, \quad f_{\text{eq}}(r) = \frac{\alpha(r)}{d} \int \int \int \int \int F(r)(f_{\text{eq}}(r) + f(r) - 1).
\]

Here \( h \) is Planck’s constant. Time constant \( \tau_{\text{rel},\alpha} \) describes ultrafast carrier-carrier scattering, which contributes to gain as spectral hole-burning (SHB) effects. \( f_{\text{eq}} \) is the quasi-equilibrium occupation factor. \( \alpha(r) \) is a normalization factor for the change rate of electron-hole pairs due to stimulated emission. The active material distribution function on both sides of equation is used to keep the conservation of carrier and photon numbers. Thus based on the same principle of conservation, the confinement factors used for the stimulated emissions in effective rate equations analysis [5] should be carefully customized for each PhC waveguide design. Severe spatial carrier depletions within PhC waveguides due to SL-enhanced stimulated emission are expected, which quantitatively depend on the details of carrier dynamics as well as PhC waveguide geometry and carrier transport properties [13].

We use Eq. (2) to directly calculate spatial carrier distribution within a supercell depleted by a guided Bloch mode with a given CW power flux level \( P_{\text{cw}} = |\psi|^2 P_{\text{z}} \). Then effective population inversion factors in quasi-equilibrium \( f_{\text{eq}} \) and non-equilibrium \( f_{\text{eq}} \) are evaluated to facilitate the coupled-wave analysis by Eq. (1):

\[
f_{\text{eq}} = \left[ \frac{\int \int \int \int \int F(r)(f_{\text{eq}}(r) + f(r) - 1) dV}{\int \int \int \int \int \int F(r) dV} \right], \quad f_{\text{eq}} = \left[ \frac{\int \int \int \int \int F(r)(f_{\text{eq}}(r) + f(r) - 1) dV}{\int \int \int \int \int \int F(r) dV} \right].
\]

Thus the effective modal gain averaged within a supercell can be obtained:

\[
g_{\text{sat}}(P_{\text{sat}}) = g_0 \cdot 2n_0 \rho_0 \cdot f_{\text{eq}} \cdot \frac{1}{4c} \int \int \int \int \int E_0 F(r) dV
\]

Furthermore, semi-analytical formulas can be approximated to evaluate 1D propagation effects by Eq. (1):

\[
f_{\text{eq}} \approx f_{\text{eq}}(0) \left[ 1 + \frac{P_{\text{sat}}}{P_{\text{sat}}} \right], \quad f_{\text{eq}} \approx f_{\text{eq}}(0) \left[ 1 + (\epsilon_{\text{SBH}}) P_{\text{sat}} \right]
\]

Here a nonlinear gain suppression factor \( \epsilon_{\text{SBH}} \) in unit of \([W^{-1}]\) and saturation power \( P_{\text{sat}} \) are extracted to characterize the saturation contributions from ultrafast carrier-carrier scattering as well as total carrier density depletion.

The simulation examples and results presented in the paper were obtained in 2D using commercial finite element method (FEM) software COMSOL. For simplicity, we used W1 PhC waveguides with a single quantum well (QW), which is described by free carrier gain model [14] only considering electron distribution and homogeneous carrier pumping over PhC waveguides. Typical simulation parameters are given in Table 1. In this case imaginary index perturbation \( n_{\text{ir}} = 0.006 \) corresponds to a material gain \( g_0 = 486 \text{cm}^{-1} \) (full population inversion) at the wavelength of 1550nm.

### III. RESULTS

#### A. Modal Gain in Small-signal Regime

Fig. 2 illustrates the effective modal gain in active PhC W1 waveguides in the small-signal regime, which is determined by Eq. (5). Fig.2 (left) shows the calculated group index based on FEM eigen-frequency calculation of a 2D supercell. Due to different loss mechanisms in practical SL PhC waveguides [9,10] and fundamental limitations to gain enhancement close to photonic bandgap [15], we limit our discussions to slow light modes with group index up to around 50 and relative small imaginary refractive index change. The inset shows different electric field intensity distribution \( |\psi|^2 \) as a function of group index, which will determine the strength of light-matter interaction and carrier depletion in Eq. (1) & (2).

As the carrier density in the active material increases beyond the transparency value \( N_{\text{tr}} \), positive modal gain might be achieved to compensate propagation loss. Fig. 2(right) shows that the estimated maxima linear modal gain can be increased from \( \sim 15 \text{cm}^{-1} \) to \( 200 \text{cm}^{-1} \) as group index increased from 4 to 52. The device length to realize a linear optical enhancement increases from 400 to 780 microns.

### Table I. Simulation Parameters

<table>
<thead>
<tr>
<th>( n_{\text{ir}} )</th>
<th>( a = 340 \text{nm} )</th>
<th>( n_{\text{tr}} = 0.006 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>11.19</td>
<td>340</td>
<td>52</td>
</tr>
<tr>
<td>air hole radius 0.26a</td>
<td>Single QW vertical confinement 8/290</td>
<td></td>
</tr>
</tbody>
</table>
B. Modal Gain in Large-signal Regime

Fig. 3 Impact of group index on CW modal gain saturation of active PhC waveguides. (a) Effective population inversion factor $f_{\text{opt}}$ and corresponding nonlinear gain suppression factor $\Delta g_{\text{SHB}}$ as a function of input power $P_{\text{in}}$. Examples of (b) carrier density $N_c$ (normalized by transparency density $N_{tr}$) and (c) population inversion factor $f_{\text{m}}$ when effective modal gains are halved in a supercell. From left to right: $n_g=4,16,52$.

Fig. 3 shows examples of CW modal gain saturation for active PhC waveguides for different group index. As shown in Fig.3(a), when the input power increases, the effective population inversion $f_{\text{opt}}$ declines toward zero. The saturation power of slow-light mode is lower than the one of the fast light mode due to the SL-enhanced stimulated emission. The slow-light mode has significantly higher gain suppression factor $\Delta g_{\text{SHB}}$ than the fast-light mode. This is mainly due to the fact that slow-light mode has higher electric field intensity than fast-light mode with a given input power. For the given parameters, the saturation power of slow-light mode ($n_g=52$), which also limits the saturation output power of overall PhC SOAs, is around -10dBm. Fig.3(b)&(c) shows examples of carrier density distribution and population inversion factor when the effective modal gains are halved in a supercell. The patterns of carrier density and population inversion factor are related to the electric field intensity as shown in the inset of Fig. 2(left).

Fig. 4 Calculated saturation power $P_{\text{sat}}$ and the corresponding nonlinear gain suppression factor $\Delta g_{\text{SHB}}$ as a function of group index with different initial carrier densities $N_c$, normalized to transparency density $N_{tr}$. Dashed lines are fitting curves with dependence on group index.

By expanding Eq. (6) and keeping only the first two terms, we might find a nonlinear response proportional to $\Delta g_{\text{SHB}}$ and $P_{\text{sat}}^{-1}$, which contributes to an effective third-order susceptibility term with extra implicit dependence on group index in propagation equation Eq. (1). Fig. 4 shows the calculated saturation power and gain suppression factor as a function of group index. The dashed lines are fitting curves to indicate that both $\Delta g_{\text{SHB}}$ and $P_{\text{sat}}^{-1}$ roughly increase linearly as a function of group index despite deviations in the fast light region. Similar to Kerr nonlinearity [7], the large carrier-induced third-order susceptibility in active PhC waveguides might be also roughly scaled up with $n_g^2$. Considering the lower saturation power, PhC SOAs with SL enhancement is more attractive for nonlinear signal processing, e.g. Four-wave mixing (FWM), rather than linear optical amplification [5].

IV. CONCLUSION

We presented a theoretical investigation of CW modal gain saturation in active PhC waveguides with a set of rigorously derived carrier dynamics and couple-wave equations. Simulation indicates that SL-enhanced PhC SOAs as a travelling wave amplifier has higher peak modal gain at the expense of lower saturation power. The corresponding carrier-induced third-order susceptibility might be roughly scaled up quadratically with group index, which make active PhC waveguides potentially promising for nonlinear optical signal processing.