

Multiscale optimization of saturated poroelastic actuators

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A multiscale method for optimizing the material micro structure in a macroscopically heterogeneous saturated poroelastic media with respect to macro properties is presented. The method is based on topology optimization using the homogenization technique, here applied to the optimization of a bi-morph saturated poroelastic actuator.

Introduction

Poroelasticity is mostly used in the soil-mechanics community but it is also applicable to describe bio-mechanical problems such as scaffold or tissue modeling. In general it describes the fluid motion and interaction with a saturated porous elastic material e.g. a foam. By customizing the design of the material microstructure it is possible to make smart materials that e.g. bend or twist when pressurized[1]. Optimizing periodic porous materials by topology optimization with respect to one or some of the homogenized coefficients have been presented for stiffness[2], permeability[3] and the combination of stiffness and permeability[4]. This optimization towards an *optimized* material might be of no use if the relationship between the obtained effective properties and the application scale objective is not linear. This is the case in poroelasticity where the stiffness, the pressure coupling, the storage and the permeability all influence the deformation. In this context the steady state solution of a saturated pressure loaded poroelastic structure is investigated as this reduces the problem such that only the pressure coupling and the stiffness influence the deformation.

The method is applied to the design of a bi-morph poroelastic actuator shown in Figure 1.

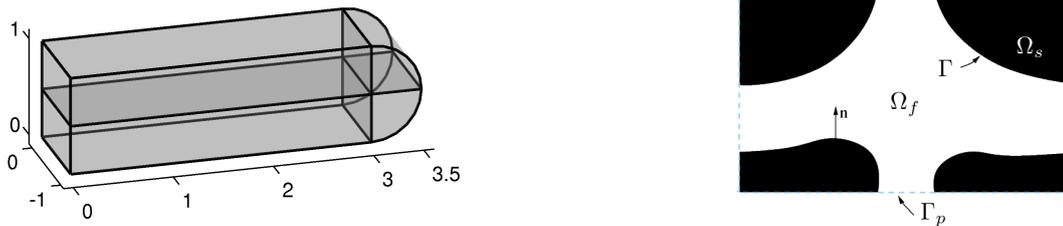


Figure 1: Bi-morph poroelastic actuator. Two individually sealed porous slabs with different material is assembled with a much stiffer half cylinder that will interact with the specimen. Left face is fixed. Objective values are integrated along the right edge ($x = 3.5, z = 0.5$). Right: 2D Schematic of the fluid-structure interaction problem.

Theory

The basis for the poroelastic theory is the solid-fluid interaction in a representative unit element (RVE) which in this case is the periodic unit cell. At the micro scale the solid material (Ω_s) is governed by the Navier equation and the fluid (Ω_f) is assumed incompressible and governed by the Navier-Stokes equation. Over the interacting boundary (Γ) there is a continuity in velocities and stresses in between the solid and the fluid. By assuming a sufficiently large difference in the

length scales of the macro and micro problems and assuming a neither too fast moving fluid nor too slow, it is possible to expand the governing equations by a two scale asymptotic expansion as described in [5, 6].

The resulting equations are separable such that one set describes the micro scale (homogenization) whereas the other describes the macro scale. Assuming that the permeability is large enough, the dynamic effects from the flow can be neglected and the resulting macroscopic equation reduce to

$$\operatorname{div}(E : \varepsilon(\mathbf{u}) - \alpha p) = \mathbf{0} \quad (1)$$

where E is the homogenized stiffness tensor, α is the pressure coupling tensor, $\varepsilon(\mathbf{u})$ is the linear strain for the displacement and p is the pressure.

Results

The optimization problem is formulated as a min-max problem for the vertical deflection under pressurization of either the upper or the lower domain. In actuator problems it is necessary to apply a spring to the output boundary such that the actuating force can be tuned. For the current example 3 springs are added, one in each coordinate direction. The springs have the stiffness $k = 10^{-4}$. The properties of the elastic base material is $E = 1$, $\nu = 0.3$ and the applied pressure is $p = 10^{-4}$. The initial material designs are crosses cf. Figure 2 which are altered by topology optimization such that the final materials also shown in the figure appear. By optimizing the material micro structure the vertical deflection is increased 50 fold from $u_3 = -3.4 \cdot 10^{-3}$ to $u_3 = -0.18$, however certain issues with respect to simultaneous horizontal extension still need to be addressed.

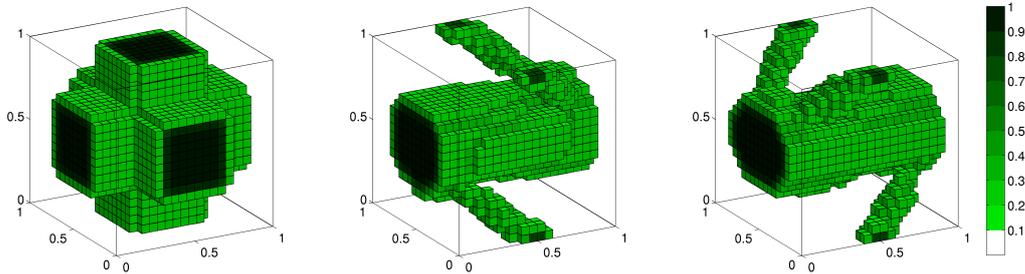


Figure 2: Left: Initial cross micro structure. Center: Optimized upper material microstructure. Right: Optimized bottom material microstructure.

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