Wavelet frames and their duals

This thesis is concerned with computational and theoretical aspects of wavelet frame analysis in higher dimensions and, in particular, with the study of so-called dual frames of wavelet frames. A frame is a system of "simple" functions or building blocks which deliver ways of analyzing signals. The signals are then represented by linear combinations of the building blocks with coefficients found by an associated frame, called a dual frame. A wavelet frame is a frame where the building blocks are stretched (dilated) and translated versions of a single function; such a frame is said to have wavelet structure. The dilation of the wavelet building blocks in higher dimension is done via a square matrix which is usually taken to be integer valued. In this thesis we step away from the "usual" integer, expansive dilation and consider more general, expansive dilations.

In most applications of wavelet frames it is essential to have a dual frame with the same structure, but this is not always the case. We explore the relationship between dual frames of a wavelet frame. We show the existence of a "nice" wavelet frame for which the canonical choice of a dual frame is not a wavelet system. At the same time, this "nice" wavelet frame has infinitely many other "nice" dual wavelet frames.

To avoid the possible lack of wavelet structure of a dual frame, we develop a construction procedure for pairs of dual frames which both have wavelet structure. Using this simple procedure we construct pairs of dual, bandlimited wavelet frames with good time localization and other attractive properties. Furthermore, the dual wavelet frames are constructed in such a way that we are guaranteed that both frames will have the same desirable features. The construction procedure works for any real, expansive dilation.

A quasi-affine system is a variant of the wavelet system that has been used successfully in the study of properties of wavelet systems for integer dilations. We extend the investigation of such quasi-affine systems to the class of rational, expansive dilations and introduce a new family of oversampled quasi-affine systems. We show that the wavelet system is a frame if, and only if, the corresponding family of oversampled quasi-affine systems are frames with uniform frame bounds. We also prove a similar equivalence result between pairs of dual wavelet frames and dual quasi-affine frames. We then characterize when the canonical dual frame of an oversampled quasi-affine frame is also a quasi-affine system. Finally, we uncover some fundamental differences between the integer and rational settings by exhibiting an example of a quasi-affine frame such that its wavelet counterpart is not a frame.

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