Velocity potential formulations of highly accurate Boussinesq-type models

The highly accurate Boussinesq-type equations of Madsen et al. (Madsen, P.A., Bingham, H.B., Schaffer, H.A., 2003. Boussinesq-type formulations for fully nonlinear and extremely dispersive water waves: Derivation and analysis. Proc. R. Soc. Lond. A 459, 1075-1104; Madsen, P.A., Fuhrman, D.R., Wang, B., 2006. A Boussinesq-type method for fully nonlinear waves interacting with a rapidly varying bathymetry. Coast. Eng. 53, 487-504; Jamois et al. (Jamois, E., Fuhrman, D.R., Bingham, H.B., Molin, B., 2006. Wave-structure interactions and nonlinear wave processes on the weather side of reflective structures. Coast. Eng. 53, 929-945) are re-derived in a more general framework which establishes the correct relationship between the model in a velocity formulation and a velocity potential formulation. Although most work with this model has used the velocity formulation, the potential formulation is of interest because it reduces the computational effort by approximately a factor of two and facilitates a coupling to other potential flow solvers. A new shoaling enhancement operator is introduced to derive new models (in both formulations) with a velocity profile which is always consistent with the kinematic bottom boundary condition. The true behaviour of the velocity potential formulation with respect to linear shoaling is given for the first time, correcting errors made by Jamois et al. (Jamois, E., Fuhrman, D.R., Bingham, H.B., Molin, B., 2006. Wave-structure interactions and nonlinear wave processes on the weather side of reflective structures. Coast. Eng. 53, 929-945). An exact infinite series solution for the potential is obtained via a Taylor expansion about an arbitrary vertical position $z=(z)^{\text{cap}}$ over cap. For practical implementation however, the solution is expanded based on a slow variation of $(z)^{\text{cap}}$ over cap and terms are retained to first-order. With shoaling enhancement, the new models obtain a comparable accuracy in linear shoaling to the original velocity formulation. General consistency relations are also derived which are convenient for verifying that the differential operators satisfy a potential flow and/or conserve mass up to the order of truncation of the model. The performance of the new formulation is validated using computations of linear and nonlinear shoaling problems. The behaviour on a rapidly varying bathymetry is also checked using linear wave reflection from a shelf and Bragg scattering from an undulating bottom. Although the new models perform equally well for Bragg scattering they fail earlier than the existing model for reflection/transmission problems in very deep water.