In this paper we explore tramp ship routing and scheduling. Tramp ships operate much like taxies following the available demand. Tramp operators can determine some of their demand in advance by entering into long-term contracts and then try to maximise profits from optional voyages found in the spot market. Routing and scheduling a tramp fleet to best utilise fleet capacity according to current demand is therefore an ongoing and complicated problem. Here we add further complexity to the routing and scheduling problem by incorporating voyage separation requirements that enforce a minimum time spread between some voyages. The incorporation of these separation requirements helps balance the conflicting objectives of maximising profit for the tramp operator and minimising inventory costs for the charterer, since these costs increase if similar voyages are not performed with some separation in time. We have developed a new and exact branch-and-price procedure for this problem. We use a dynamic programming algorithm to generate columns and describe a time window branching scheme used to enforce the voyage separation requirements which we relax in the master problem. Computational results show that our algorithm in general finds optimal solutions very quickly and performs much faster compared to an earlier a priori path generation method. Finally, we compare our method to an earlier adaptive large neighbourhood search heuristic and find that on similar-sized instances our approach generally uses less time to find the optimal solution than the adaptive large neighbourhood search method uses to find a heuristic solution.