The period adding and incrementing bifurcations: from rotation theory to applications - DTU Orbit (15/01/2019)

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This survey article is concerned with the study of bifurcations of piecewise-smooth maps. We review the literature in circle maps and quasi-contractions and provide paths through this literature to prove sufficient conditions for the occurrence of two types of bifurcation scenarios involving rich dynamics. The first scenario consists of the appearance of periodic orbits whose symbolic sequences and ‘rotation’ numbers follow a Farey tree structure; the periods of the periodic orbits are given by consecutive addition. This is called the period adding bifurcation, and its proof relies on results for maps on the circle. In the second scenario, symbolic sequences are obtained by consecutive attachment of a given symbolic block and the periods of periodic orbits are incremented by a constant term. It is called the period incrementing bifurcation, in its proof relies on results for maps on the interval.

We also discuss the expanding cases, as some of the partial results found in the literature also hold when these maps lose contractiveness. The higher dimensional case is also discussed by means of quasi-contractions. We also provide applied examples in control theory, power electronics and neuroscience where these results can be applied to obtain precise descriptions of their dynamics.