The geomagnetic field gradient tensor: Properties and parametrization in terms of spherical harmonics

We develop the general mathematical basis for space magnetic gradiometry in spherical coordinates. The magnetic gradient tensor is a second rank tensor consisting of $3 \times 3 = 9$ spatial derivatives. Since the geomagnetic field vector $B$ is always solenoidal ($\nabla \cdot B = 0$) there are only eight independent tensor elements. Furthermore, in current free regions the magnetic gradient tensor becomes symmetric, further reducing the number of independent elements to five. In that case $B$ is a Laplacian potential field and the gradient tensor can be expressed in series of spherical harmonics. We present properties of the magnetic gradient tensor and provide explicit expressions of its elements in terms of spherical harmonics. Finally we discuss the benefit of using gradient measurements for exploring the Earth's magnetic field from space, in particular the advantage of the various tensor elements for a better determination of the small-scale structure of the Earth's lithospheric field.