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We consider the question whether, given a countable family of lattices \((\Gamma_j)_{j \in J}\) in a locally compact abelian group \(G\), there exist functions \((g_j)_{j \in J}\) such that the resulting generalized shift-invariant system \((g_j(\cdot - \gamma))_{j \in J, \gamma \in \Gamma_j}\) is a tight frame of \(L^2(G)\). This paper develops a new approach to the study of generalized shift-invariant system via almost periodic functions, based on a novel unconditional convergence property. From this theory, we derive characterizing relations for tight and dual frame generators, we introduce the system bandwidth as a measure of the total bandwidth a generalized shift-invariant system can carry, and we show that the so-called Calderón sum is uniformly bounded from below for generalized shift-invariant frames. Without the unconditional convergence property, we show, counter intuitively, that even orthonormal bases can have arbitrary small system bandwidth. Our results show that the question of existence of frame generators for a general lattice system is rather subtle and depends on analytical and algebraic properties of the lattice system.

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