Solution of Constrained Optimal Control Problems Using Multiple Shooting and ESDIRK Methods

In this paper, we describe a novel numerical algorithm for solution of constrained optimal control problems of the Bolza type for stiff and/or unstable systems. The numerical algorithm combines explicit singly diagonally implicit Runge-Kutta (ESDIRK) integration methods with a multiple shooting algorithm. As we consider stiff systems, implicit solvers with sensitivity computation capabilities for initial value problems must be used in the multiple shooting algorithm. Traditionally, multi-step methods based on the BDF algorithm have been used for such problems. The main novel contribution of this paper is the use of ESDIRK integration methods for solution of the initial value problems and the corresponding sensitivity equations arising in the multiple shooting algorithm. Compared to BDF-methods, ESDIRK-methods are advantageous in multiple shooting algorithms in which restarts and frequent discontinuities on each shooting interval are present. The ESDIRK methods are implemented using an inexact Newton method that reuses the factorization of the iteration matrix for the integration as well as the sensitivity computation. Numerical experiments are provided to demonstrate the algorithm.