Successful scientific applications of large-scale molecular dynamics often rely on automated methods for identifying the local crystalline structure of condensed phases. Many existing methods for structural identification, such as common neighbour analysis, rely on interatomic distances (or thresholds thereof) to classify atomic structure. As a consequence they are sensitive to strain and thermal displacements, and preprocessing such as quenching or temporal averaging of the atomic positions is necessary to provide reliable identifications. We propose a new method, polyhedral template matching (PTM), which classifies structures according to the topology of the local atomic environment, without any ambiguity in the classification, and with greater reliability than e.g. common neighbour analysis in the presence of thermal fluctuations. We demonstrate that the method can reliably be used to identify structures even in simulations near the melting point, and that it can identify the most common ordered alloy structures as well. In addition, the method makes it easy to identify the local lattice orientation in polycrystalline samples, and to calculate the local strain tensor. An implementation is made available under a Free and Open Source Software license.