We present a numerically efficient technique to evaluate the Green's function for extended two-dimensional systems without relying on periodic boundary conditions. Different regions of interest, or "patches," are connected using self-energy terms which encode the information of the extended parts of the system. The calculation scheme uses a combination of analytic expressions for the Green's function of infinite pristine systems and an adaptive recursive Green's function technique for the patches. The method allows for an efficient calculation of both local electronic and transport properties, as well as the inclusion of multiple probes in arbitrary geometries embedded in extended samples. We apply the patched Green's function method to evaluate the local densities of states and transmission properties of graphene systems with two kinds of deviations from the pristine structure: bubbles and perforations with characteristic dimensions of the order of 10-25 nm, i.e., including hundreds of thousands of atoms. The strain field induced by a bubble is treated beyond an effective Dirac model, and we demonstrate the existence of both Friedel-type oscillations arising from the edges of the bubble, as well as pseudo-Landau levels related to the pseudomagnetic field induced by the nonuniform strain. Second, we compute the transport properties of a large perforation with atomic positions extracted from a transmission electron microscope image and show that current vortices may form near the zigzag segments of the perforation.