On devising Boussinesq-type models with bounded eigenspectra: One horizontal dimension

The propagation of water waves in the nearshore region can be described by depth-integrated Boussinesq-type equations. The dispersive and nonlinear characteristics of the equations are governed by tuneable parameters. We examine the associated linear eigenproblem both analytically and numerically using a spectral element method of arbitrary spatial order p. It is shown that existing sets of parameters, found by optimising the linear dispersion relation, give rise to unbounded eigenspectra which govern stability. For explicit time-stepping schemes the global CFL time-step restriction typically requires $\Delta t \propto p^{-2}$. We derive and present conditions on the parameters under which implicitly-implicit Boussinesq-type equations will exhibit bounded eigenspectra. Two new bounded versions having comparable nonlinear and dispersive properties as the equations of Nwogu (1993) and Schäffer and Madsen (1995) are introduced. Using spectral element simulations of stream function waves it is illustrated that (i) the bounded equations capture the physics of the wave motion as well as the standard unbounded equations, and (ii) the bounded equations are computationally more efficient when explicit time-stepping schemes are used. Thus the bounded equations were found to lead to more robust and efficient numerical schemes without compromising the accuracy.

General information
State: Published
Organisations: Department of Applied Mathematics and Computer Science, Scientific Computing, Chalmers University of Technology
Authors: Eskilsson, C. (Ekstern), Engsig-Karup, A. P. (Intern)
Keywords: (Nonlinear dispersive water waves, Boussinesq-type equations, Spectral/ hp element method, Eigenvalue analysis, Time integration, Implicitly-implicit equations)
Number of pages: 20
Pages: 261-280
Publication date: 2014
Main Research Area: Technical/natural sciences

Publication information
Journal: Journal of Computational Physics
Volume: 271
ISSN (Print): 0021-9991
Ratings:
BFI (2015): BFI-level 1
BFI (2014): BFI-level 1
ISI indexed (2013): ISI indexed yes
BFI (2013): BFI-level 1
BFI (2012): BFI-level 1
ISI indexed (2012): ISI indexed yes
BFI (2011): BFI-level 1
ISI indexed (2011): ISI indexed yes
BFI (2010): BFI-level 1
BFI (2009): BFI-level 1
BFI (2008): BFI-level 1
Original language: English
DOIs:
10.1016/j.jcp.2013.08.048
Source: dtu
Source-ID: u::9548
Publication: Research - peer-review › Journal article – Annual report year: 2014