Mathematical Modeling and Dimension Reduction in Dynamical Systems

Processes that change in time are in mathematics typically described by differential equations. These may be applied to model everything from weather forecasting, brain patterns, reaction kinetics, water waves, finance, social dynamics, structural dynamics and electrodynamics to name only a few. These systems are generically nonlinear and the studies of them often become enormously complex. The framework in which such systems are best understood is via the theory of dynamical systems, where the critical behavior is systematically analyzed by performing bifurcation theory. In that context the current thesis is attacking two problems.

The first is concerned with the mathematical modelling and analysis of an experiment of a vibro-impacting beam. This type of dynamical system has received much attention in the recent years and they occur frequently in mechanical applications, where they induce noise and wear which decrease the life time of machines. From the modelling point of view these systems are often particularly rich in nonlinear dynamics. In the present study a mathematical model is derived. Amongst other outcomes the model was successfully applied to predict a nonlinear phenomenon, namely the existence of isolas of subharmonic orbits. These were then verified in the practical experiment in the lab. The second problem that is addressed in the current thesis is a problem that has developed as a consequence of the increasing power of computers which has created the demand for analysis of ever more advanced and complex systems. These complex systems are computationally very demanding and proper analysis of the qualitative behavior of the systems becomes difficult. In general it is not possible to construct bifurcation diagrams for these so-called high-dimensional models efficiently. In order to overcome this obstacle much research is going into the direction of development of robust methods to perform dimension and model reduction such as to pave the way for a qualitative analysis of the highdimensional problems by analyzing the low-dimensional models.

In this thesis we demonstrate how to reduce the dimension of a certain class of dynamical systems by construction of k-dimensional submanifolds using the so-called graph transform. The method is suitable for a specific class of problems with spectral gaps, these are often observed. In particular the method is applied to a mechanical system. Furthermore the method has some unique and promising properties compared to other methods.

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